Final Exam **Physics 607** **(due Dec. 14, 2001)**

1. Consider a transition from the $(1s3d)\ ^3\mathrm{D}$ state to the $(1s2p)\ ^3\mathrm{P}$ state in a helium-like ion:

   (a) Show that, in the independent-particle approximation,

   $$\langle (1s2p)\ ^3\mathrm{P}||r||(1s3d)\ ^3\mathrm{D}\rangle = \langle 2p||r||3d\rangle$$

   (b) Suppose that one can resolve the fine-structure of the initial and final states. Express the matrix elements $\langle (1s2p)\ ^3\mathrm{P}_{J_F}||r||(1s3d)\ ^3\mathrm{D}_{J_I}\rangle$ in terms of $\langle (1s2p)\ ^3\mathrm{P}||r||(1s3d)\ ^3\mathrm{D}\rangle$ or $\langle 2p||r||3d\rangle$.

   (c) The intensity of the lines from $|(1s3d)\ ^3\mathrm{D}_{J_I}\rangle$ to $|(1s2p)\ ^3\mathrm{P}_{J_F}\rangle$ is

   $$A_{I\to F} \propto \frac{S_{FI}}{[J_I]} = \frac{|\langle (1s2p)\ ^3\mathrm{P}_{J_F}||r||(1s3d)\ ^3\mathrm{D}_{J_I}\rangle|^2}{[J_I]}.$$

   By explicitly evaluating the relevant 6j symbols (using MAPLE, for example), show that the ratios of intensities for transitions $J_I \to J_F$:

   $$1\to 0 : 1\to 1 : 2\to 1 : 1\to 2 : 2\to 2 : 3\to 2$$

   are

   $$20 : 15 : 27 : 1 : 9 : 36.$$

2. Suppose we choose to describe an atom in lowest order using a potential $U(r)$ other than the HF potential.

   (a) Show that the correction to the first-order energy from the single-particle part of the potential (V_1) for a one electron atom in a state v is

   $$E_v^{(1)} = \Delta_{vv},$$

   where $\Delta = V_{\mathrm{HF}} - U$.

   (b) Show that the corresponding second-order correction is

   $$E_v^{(2)} = -\sum_{na} \frac{\Delta_{na}\tilde{g}_{avnv} + \tilde{g}_{avnv}\Delta_{an}}{\epsilon_n - \epsilon_a} - \sum_{i\neq v} \frac{\Delta_{vi}\Delta_{iv}}{\epsilon_i - \epsilon_v}.$$

   Here, i runs over a and n.