Problem Set 1 **Physics 607** **(due Sept. 3, 2001)**

1. Derive the relations

$$\begin{aligned} J^2 &= J_+ J_- + J_z^2 - J_z \,, \\ J^2 &= J_- J_+ + J_z^2 + J_z \,. \end{aligned}$$

2. Show that the normalization factor c in the equation $\Theta_{l,-l}(\theta) = c\, \sin^l \theta$ is

$$c = \frac{1}{2^l l!} \sqrt{\frac{(2l+1)!}{2}},$$

and thereby verify that Eq. (1.30) is correct.

3. Write a MAPLE program to obtain the first 10 Legendre polynomials using Rodrigues' formula.

4. Legendre polynomials satisfy the recurrence relation

$$l P_l(x) = (2l-1) x P_{l-1}(x) - (l-1) P_{l-2}(x).$$

Write a MAPLE program to determine $P_2(x), P_3(x), \cdots, P_{10}(x)$ (starting with $P_0(x) = 1$ and $P_1(x) = x$) using the above recurrence relation.

5. Write a MAPLE program to generate the associated Legendre functions and $P_l^m(x)$. Determine all $P_l^m(x)$ with $l \leq 4$ and $1 \leq m \leq l$.