

Problem Set 2**Physics 607****(due Sept. 10, 2001)**

1. Use the MAPLE routine CGC.MAP to evaluate $C(1, 3/2, J; m_1, m_2, M)$ for all possible values of (m_1, m_2, J, M) . Show that the resulting values satisfy the two orthogonality relations.

2. Prove

$$\langle l_1 || C^k || l_2 \rangle = (-1)^{l_1 - l_2} \langle l_2 || C^k || l_1 \rangle,$$

where C_q^k is the tensor operator

$$C_q^k \stackrel{\text{def}}{=} \sqrt{\frac{4\pi}{2k+1}} Y_{kq}(\theta, \phi).$$

3. Prove

$$[J^2, \boldsymbol{\sigma} \cdot \mathbf{r}] = 0,$$

$$[J_z, \boldsymbol{\sigma} \cdot \mathbf{r}] = 0,$$

where $\mathbf{J} = \mathbf{L} + \frac{1}{2}\boldsymbol{\sigma}$

4. Prove

$$Y_{JJM}(\hat{r}) = \frac{\mathbf{L}}{\sqrt{J(J+1)}} Y_{JM}(\hat{r})$$