Problem Set 5 **Physics 607** **(due Oct. 1, 2001)**

1. Write out specific formulas for the radial Dirac functions $P_{n\kappa}(r)$ and $Q_{n\kappa}(r)$ of the $n = 2$ states of a hydrogenlike ion with nuclear charge Z. You may use MAPLE if you wish, however, you may find it simpler to expand the hypergeometric functions by hand.

2. Verify that the $n = 2$ radial functions determined in the previous problem are properly normalized for each of the three states.

3. Plot the radial density function $P_{n\kappa}(r)^2 + Q_{n\kappa}(r)^2$ for each of the $n = 2$ states assuming $Z = 20$.

4. Give formulas for $\langle r \rangle$ and $\langle 1/r \rangle$ for each state in problem 1. Verify that the relativistic formulas approach the proper nonrelativistic limits.

5. Suppose the nucleus is represented by a uniformly charged ball of radius R:

   (a) Show that the nuclear potential (in atomic units) is given by

   $$V_{\rm nuc}(r) = \begin{cases} -\frac{Z}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right), & r < R, \\ -\frac{Z}{r}, & r \geq R. \end{cases}$$

   (b) Determine the nuclear finite-size correction to the $n = 1$ and $n = 2$ Dirac energy levels using first-order perturbation theory. How large are these corrections for hydrogen? (Assume $R = 1.04$ fm for H and give your answer in $\rm cm^{-1}$.) How large are they for hydrogenlike uranium? (Assume $R = 7.25$ fm for U and give your answer in eV.)