Problem Set 6 **Physics 607** **(due Oct. 13, 2001)**

1. Show that the exchange contribution to the interaction energy for the state $|ab, LS\rangle$ is

$$\eta^2 \sum_k (-1)^{l_a+l_b+S+k} \left\{ \begin{array}{ccc} l_a & l_b & L \\ l_a & l_b & k \end{array} \right\} X_k(abba)\,.$$

2. LS to jj transformation matrix:

   (a) Each nonrelativistic LS-coupled state belonging to a given J,

$$|\,[(l_1 l_2)L, (s_1 s_2)S]J\rangle\,,$$

   can be expanded as a linear combination of the nonrelativistic jj-coupled states

$$|\,[(l_1 s_1)j_1, (l_2 s_2)j_2]J\rangle\,,$$

   belonging to the same J. Write the matrix of expansion coefficients in terms of six-j symbols.

   (b) Prove that this transformation matrix is symmetric.

   (c) Give numerical values for the elements of the 2×2 matrix that gives the two (sp) states ${}^1P_1$ and ${}^3P_1$ in terms of the two states $(s_{1/2}p_{1/2})_1$ and $(s_{1/2}p_{3/2})_1$.

   (d) Give numerical values for the elements of the 3×3 matrix that gives the three (pd) states ${}^1P_1$, ${}^3P_1$ and ${}^3D_1$ in terms of the three states $(p_{1/2}d_{3/2})_1$, $(p_{3/2}d_{3/2})_1$ and $(p_{3/2}d_{5/2})_1$.