1. Show that the exchange contribution to the interaction energy for the state $|ab, LS\rangle$ is

$$\eta^2 \sum_k (-1)^{l_a+l_b+S+k} \begin{pmatrix} l_a & l_b & L \\ l_a & l_b & k \end{pmatrix} X_k(abba).$$

2. $LS$ to $jj$ transformation matrix:

(a) Each nonrelativistic $LS$-coupled state belonging to a given $J$, $|[(l_1l_2)L,(s_1s_2)S]J\rangle$, can be expanded as a linear combination of the nonrelativistic $jj$-coupled states $|[(l_1s_1)j_1,(l_2s_2)j_2]J\rangle$, belonging to the same $J$. Write the matrix of expansion coefficients in terms of six-j symbols.

(b) Prove that this transformation matrix is symmetric.

(c) Give numerical values for the elements of the $2 \times 2$ matrix that gives the two $(sp)$ states $^1P_1$ and $^3P_1$ in terms of the two states $(s_{1/2}p_{1/2})_1$ and $(s_{1/2}p_{3/2})_1$.

(d) Give numerical values for the elements of the $3 \times 3$ matrix that gives the three $(pd)$ states $^1P_1$, $^3P_1$ and $^3D_1$ in terms of the three states $(p_{1/2}d_{3/2})_1$, $(p_{3/2}d_{3/2})_1$ and $(p_{3/2}d_{5/2})_1$. 