**Problem Set 7** **Physics 607** **(due Oct. *19*, 2001)**

1. For an atom with two valence electrons above a closed core, determine the number of states in the configuration $(nsn'l)$ and give $LS$ and $jj$ designations of the states. Determine the number of states in an $(nd)^2$ configuration and give $LS$ and $jj$ designations of the states.

2. Which of the following products are normally ordered?

| | | | |
|---|---|---|---|
| (a) | $a_m^\dagger a_a$ | (d) | $a_m^\dagger a_a a_n^\dagger a_b$ |
| (b) | $a_a a_m^\dagger$ | (e) | $a_c^\dagger a_d^\dagger a_a a_b$ |
| (c) | $a_a a_b^\dagger$ | (f) | $a_c^\dagger a_b a_d^\dagger a_c$ |

   Determine the expectation of each of these products in the core state.

3. In the Auger process, an initial state $|I\rangle = a_a|O_c\rangle$ with a hole in state $a$ makes a transition to a final state $|F\rangle$ with holes in states $b$ and $c$ and an excited electron in state $m$. The transition probability is proportional to the square of the matrix element $\langle F|V_I|I\rangle$. Express this matrix element in terms of the two-particle Coulomb integrals $g_{ijkl}$.

4. In the relativistic case, show that the energy in the relativistic particle-hole state obtained by coupling states $|(-1)^{j_a-m_a}a_v^\dagger a_a|O_c\rangle$ to angular momentum $JM$ is

$$E^{(1)}((j_v j_a)J) = \frac{(-1)^{J+j_v-j_a}}{[J]}\left[X_J(vaav) + [J]\sum_k \left\{ \begin{array}{ccc} j_v & j_a & J \\ j_a & j_v & k \end{array} \right\} X_k(vava)\right]$$

   provided the orbitals are evaluated in a $V^{N-1}$ HF potential. Express this matrix element in terms of Slater integrals for the case: $a = 2p_{1/2}$, $v = 3s_{1/2}$, and $J = 0$. What is the $LS$ designation of this state?