Nuclear Spin-Dependent Contributions to Atomic PNC: Combined Effect of Coherent Z Exchange and the Hyperfine Interaction

Walter Johnson *Notre Dame University*
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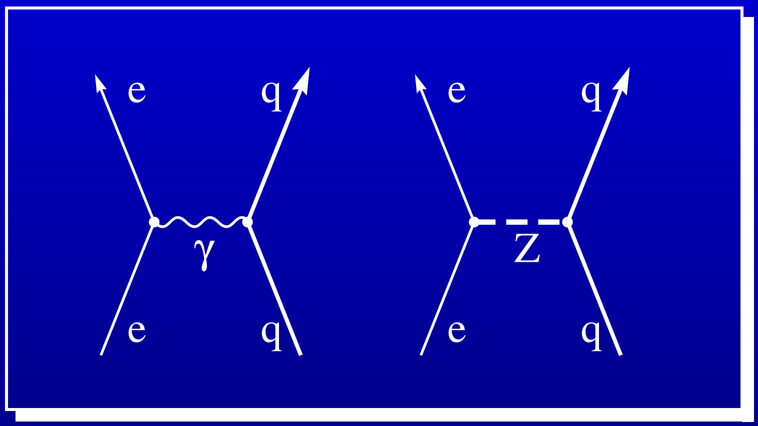
The contribution to PNC in atoms from the combined effect of coherent Z exchange and the hyperfine interaction, considered more than a decade ago by Bouchiat and Piketty, is evaluated in third-order many-body perturbation theory. A substantially smaller value is obtained for this interference term in Cs, leading to a revised experimental anapole moment for the ^{133}Cs nucleus.[1]

[1] Collaborators: M. S. Safronova (NIST, Gaithersburg) and U. I. Safronova (Notre Dame)

Atomic Parity Nonconservation



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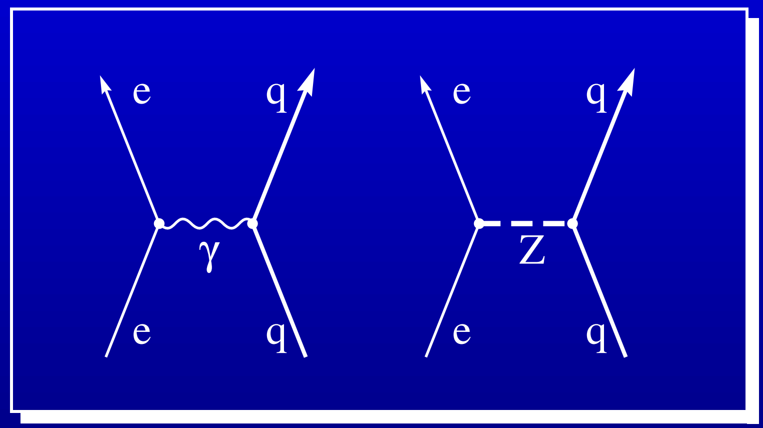

Consequence of Z exchange: Laporte's[2] rule, that electric dipole transitions take place only between states of opposite parity, is violated!

[2] Otto Laporte (1902-1971) discovered the law of parity conservation in physics. He divided states of the iron spectrum into two classes, even and odd, and found that no radiative transitions occurred between like states: O. Laporte, Z. Physik **23** 135 (1924).

Laporte: http://www.nap.edu/books/0309025494/html/268.html

Z Exchange in the Standard Model[3]

$$H_{\mathsf{PV}} = \frac{G}{\sqrt{2}} \Big[\bar{e}\gamma_\mu\gamma_5 e \left(c_{1u}\, \bar{u}\gamma_\mu u + c_{1d}\, \bar{d}\gamma_\mu d + \cdots \right) + \bar{e}\gamma_\mu e \left(c_{2u}\, \bar{u}\gamma_\mu\gamma_5 u + c_{2d}\, \bar{d}\gamma_\mu\gamma_5 d + \cdots \right) \Big]$$

where $\cdots = t,\ b,\ s,\ c$

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where $\cdots = t,\ b,\ s,\ c$

$$c_{1u} = -\frac{1}{2} + \frac{4}{3}\,\sin^2\theta_W \qquad c_{1d} = \frac{1}{2} - \frac{2}{3}\,\sin^2\theta_W$$

$$c_{2u} = -\frac{1}{2}\left(1 - 4\,\sin^2\theta_W\right) \qquad c_{2d} = \frac{1}{2}\left(1 - 4\,\sin^2\theta_W\right)$$

[3] W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 170.

Electron Axial-Vector – Nucleon Vector

Contribution of *coherent* vector nucleon current:

$$H^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 \, Q_W \, \rho(r)$$

where $\rho(r)$ is a nuclear density ($\sim$ neutron density) and

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Electron Vector – Nucleon Axial-Vector

Contribution of vector axial-vector nucleon current:

$$H^{(2)} = -\frac{G}{\sqrt{2}} \boldsymbol{\alpha} \cdot \left[c_{2p} \left\langle \phi_p^\dagger \boldsymbol{\sigma} \phi_p \right\rangle + c_{2n} \left\langle \phi_n^\dagger \boldsymbol{\sigma} \phi_n \right\rangle \right]$$

where $\langle \cdots \rangle$ designates nuclear matrix elements.

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$$c_{2p} \sim 1.25 \times c_{2u} = -0.068$$
$$c_{2n} \sim 1.25 \times c_{2d} = 0.068$$

Shell Model Estimates

$$H^{(2)} = \frac{G}{\sqrt{2}} \kappa_2 \, \alpha \cdot \mathbf{I} \, \rho(r)$$

κ_2 from "Extreme" Shell Model and from Recent Calculations.[4]

Element	A	State	κ_2	Ref. [3]
K	39	$1d_{3/2}\ (p)$	0.0272	
Cs	133	$1g_{7/2}\ (p)$	0.0151	0.0140
Ba	135	$2d_{3/2}\ (n)$	-0.0272	
Tl	205	$3s_{1/2}\ (p)$	-0.136	-0.127
Fr	209	$1h_{9/2}\ (p)$	0.0124	

[4] W. C. Haxton, C.-P. Liu, and M. J. Ramsey-Musolf, Phys. Rev. Lett. **86**, 5247 (2001).

Nuclear Anapole Moment

PNC in nucleus $\Rightarrow$ nuclear anapole:

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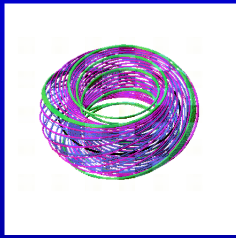

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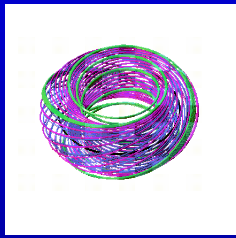

$$\boldsymbol{A} = \boldsymbol{a}\ \delta(\boldsymbol{r})$$

$$\boldsymbol{a} = -\pi \int d^3r\, r^2\, \boldsymbol{j}(\boldsymbol{r}) = \frac{1}{e}\, \frac{G}{\sqrt{2}}\, \kappa_a \boldsymbol{I}$$

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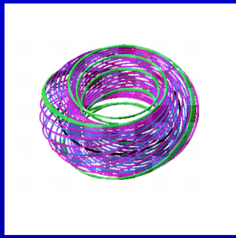

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Early estimates[5] for ^{133}Cs gave $\kappa_a = 0.063 - 0.084$. More recent estimates are found in[6]

[5] V. V. Flambaum, I. B. Khriplovich, O. P. Sushkov Phys. Letts. B **146** 367-369 (1984).

[6] V. V. Flambaum and D. W. Murray, Phys. Rev. C**56**, 1641 (1997); W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. **51**, 261 (2001)

Spin-Dependent Interference Term

According to Flambaum and Khriplovich[7] and Bronchiat and Piketty,[8] interference between the hyperfine interaction H_{hf} and $H^{(1)}$ gives another nuclear spin-dependent correction of the form

$$H^{(\mathrm{hf})} = \frac{G}{\sqrt{2}} \kappa_{\mathrm{hf}} \, \alpha \cdot \mathbf{I} \, \rho(r)$$

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$$\begin{array}{ll} {}^{133}\text{Cs:} & \kappa_{\mathsf{hf}} = 0.0078 \\ {}^{205}\text{Tl:} & \kappa_{\mathsf{hf}} = 0.044 \end{array}$$

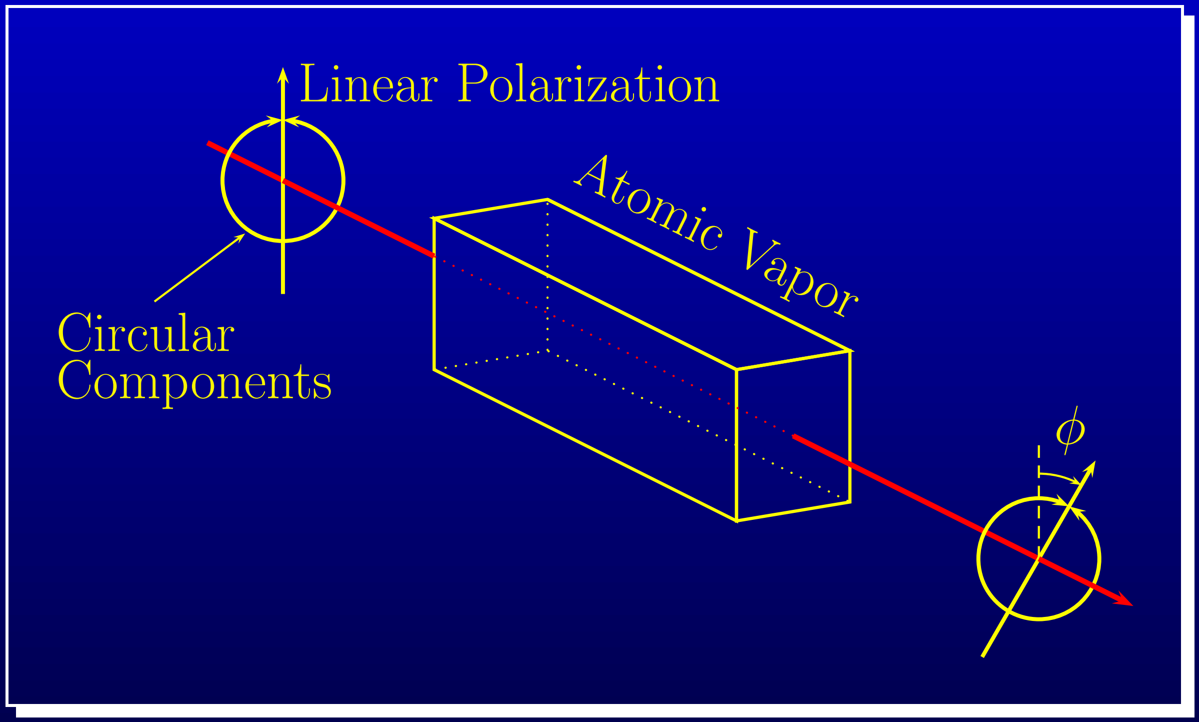
$$\kappa_{\mathsf{hf}} \sim \tfrac{1}{2}\, \kappa_2$$

⋆⋆ *Our aim is to analyze this contribution in detail.* ⋆⋆

[7] V. V. Flambaum and I. B. Khriplovich, Sov. Phys. JETP **62**, 872 (1985).
[8] C. Bouchiat and C. A. Piketty, Z. Phys. C 49, 91 (1991); Phys. Lett. B 269, 195 (1991).

Optical Rotation Experiments

Aim is to measure $E_{\text{PNC}} = \langle f|z|i\rangle \propto Q_W$:



The plane of polarization of a linearly polarized laser beam passing through a medium with $n_+ \neq n_-$ is rotated. The rotation angle $\phi \propto R_\phi = \text{Im}\,(E_{\text{PNC}})\,/M1$.

Optical Rotation Experiments-II

$$R_\phi = \mathsf{Im}\left(E_{\mathsf{PNC}}\right)/M1$$

Measured values of R_ϕ

Element	Transition	Group	$10^8 \times R_\phi$
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Oxford (95)	-15.33(45)
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Seattle (95)	-14.68(20)

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^{208}Pb	$^3P_0 - {}^3P_1$	Oxford (94)	-9.80(33)
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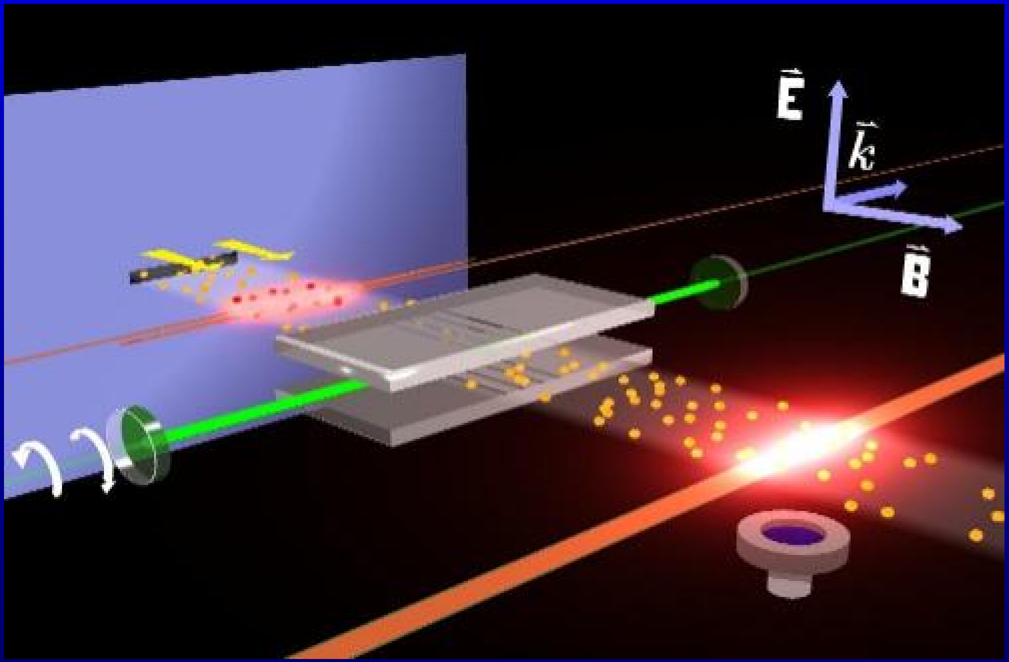
^{209}Bi	$^4S_{3/2} - {}^2D_{3/2}$	Oxford (91)	-10.12(20)

Stark-Interference Experiment



Boulder PNC apparatus: A beam of cesium atoms is optically pumped by diode laser beams, then passes through a region of perpendicular electric and magnetic fields where a green laser excites the transition from the 6S to the 7S state. The excitations are detected by observing the florescence (induced by another laser beam) with a photo-diode.

Stark-Interference Experiments II

Evolving values of $R = \mathrm{Im}\,(E_{\mathrm{PNC}})\,/\beta$ (mV/cm) for ^{133}Cs

Transition	Group	R_{4-3}	R_{3-4}
$6s_{1/2} - 7s_{1/2}$	Paris (1984)	-1.5(2)	-1.5(2)
$6s_{1/2} - 7s_{1/2}$	Boulder (1988)	-1.64(5)	-1.51(5)
$6s_{1/2} - 7s_{1/2}$	Boulder (1997)	-1.635(8)	-1.558(8)

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The vector current contribution from the last row is

$$R_{\text{Stark}} = -1.593 \pm 0.006$$

$$\text{Im}\left[E_{\text{PNC}}(6s \to 7s) \times 10^{11}\right] = -0.8376 \pm (0.0031)_{\text{exp}} \pm (0.0021)_{\text{th}}$$

Calculations of the $6s \rightarrow 7s$ Amplitude

The most recent many-body calculation[9] uses a method referred to as "perturbation theory in the screened Coulomb interaction" (PTSCI) in which important classes of many-body diagrams are summed to all orders. This gives results consistent with the SD Coupled-Cluster (SDCC) calculations.[10] Also, unpublished nonlinear Coupled-Cluster calculations were presented at the INT workshop on Fundamental Symmetries last fall by B. Das.[11]

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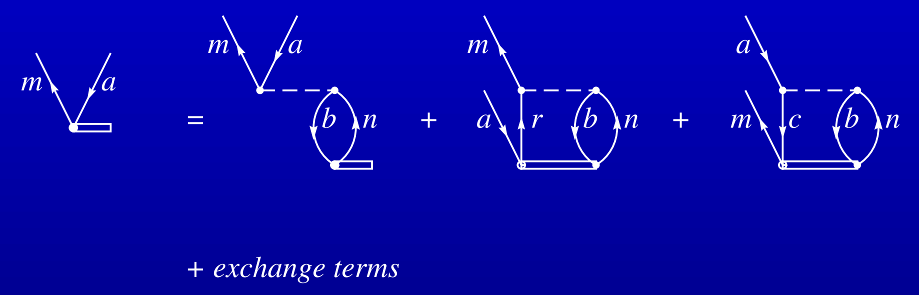
Theoretical values for E_{PNC} Units: $i(-Q_W/N) \times 10^{-11} e a_0$

PTSCI	-0.908 (5)
SDCC	-0.909 (4)
B. Das	-0.911

[9] V. A. Dzuba, V. V. Flambaum, and J. S. M. Ginges, Phys. Rev. D **66**, 076013 (2002).
[10] S. A. Blundell et al., Phys. Rev. D**45**, 1602 (1992).
[11] http://mocha.phys.washington.edu/~int_talk/WorkShops/int_02_3/People/Das_B/

Brueckner-Goldstone Diagrams for the SDCC Equations



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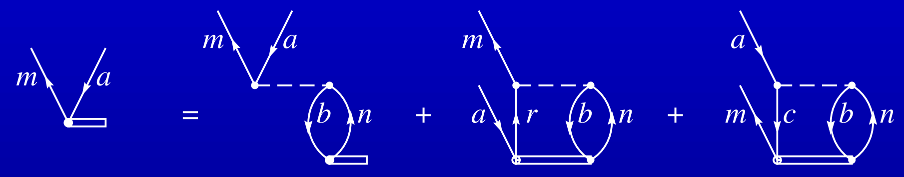
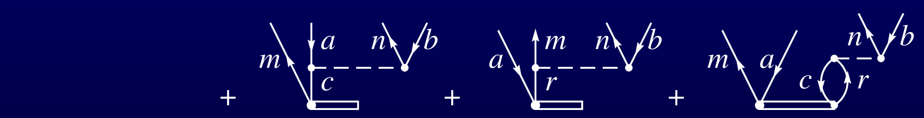


+ *exchange terms*



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Analysis of $6s \rightarrow 7s$ Amplitude in ^{133}Cs

Combining the calculations and the measurements

$$Q_W^{\mathrm{exp}}(^{133}\mathrm{Cs}) = -71.91(46)$$

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by 2.5 σ

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- Nuclear Skin Effect[15] -0.2%

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by 2.5 σ $\Rightarrow$ 0.8 σ

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- Nuclear Skin Effect[15] -0.2%

[12] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).

[13] W. R. Johnson, I. Bednyakov, and G. Soff, Phys. Rev. Lett. **87**, 233001 (2001).

[14] M. Yu. Kuchiev and V. V. Flambaum, Phys. Rev. Lett. **89**, 28302 (2002); A. I. Milstein, O. P. Sushkov, and I. S. Terekov, Phys. Rev. Lett. **89**, 28303 (2002).

[15] S. J. Pollock and M. C. Welliver, Phys. Lett. B **464**, 177 (1999); J. James and P. G. H. Sandars, J. Phys. B**32**, 3295 (1999)

Other Experiments

Element	Transition	Group
Fr	$7S_{1/2} \longrightarrow 8S_{1/2}$	Stony Brook
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Yb	$(6s^2)\,{}^1S_0 \longrightarrow (6s5d)\,{}^3D_1$	Berkeley
Yb	$(6s6p)\,{}^3P_0 \longrightarrow (6s6p)\,{}^3P_1$	Berkeley

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Ba^+	$6S_{1/2} \longrightarrow 5D_{3/2}$	Seattle

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Ba^+	$6S_{1/2} \longrightarrow 5D_{3/2}$	Seattle
Dy	$(4f^{10}5d6s)[10] \longrightarrow (4f^9 5d^2 6s)[10]$	Berkeley

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Ba^+	$6S_{1/2} \longrightarrow 5D_{3/2}$	Seattle
Dy	$(4f^{10}5d6s)[10] \longrightarrow (4f^9 5d^2 6s)[10]$	Berkeley
Sm	$(4f^6 6s^2)\ ^7F_J \longrightarrow (4f^6 6s^2)\ ^5D_{J'}$	Oxford

Angular Momentum Considerations

$$\langle F \| z \| I \rangle^{(1)} = (-1)^{j_F + F_I + I + 1} \sqrt{[F_I][F_F]} \left\{ \begin{array}{ccc} F_F & F_I & 1 \\ j_I & j_F & I \end{array} \right\}$$

$$\times \sum_{n j_n} \left[\frac{\langle j_F \| z \| n\, j_n \rangle \langle n\, j_n \| H^{(1)} \| j_I \rangle}{E_I - E_n} + \frac{\langle j_F \| H^{(1)} \| n\, j_n \rangle \langle n\, j_n \| z \| j_I \rangle}{E_F - E_n} \right]$$

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$$\times \sum_{nj_n}\left[\frac{\langle j_F\|z\|n\,j_n\rangle\langle n\,j_n\|H^{(1)}\|j_I\rangle}{E_I - E_n} + \frac{\langle j_F\|H^{(1)}\|n\,j_n\rangle\langle n\,j_n\|z\|j_I\rangle}{E_F - E_n}\right]$$

$$\langle F\|z\|I\rangle^{(2)} = \sqrt{I(I+1)}\sqrt{[I][F_I][F_F]}\,\times$$

$$\sum_{nj_n}\left[(-1)^{j_I-j_F+1}\left\{\begin{array}{ccc} F_F & F_I & 1 \\ j_n & j_F & I \end{array}\right\}\left\{\begin{array}{ccc} I & I & 1 \\ j_n & j_I & F_I \end{array}\right\}\right.$$

$$\times\frac{\langle j_F\|z\|n\,j_n\rangle\langle n\,j_n\|H^{(2)}\|j_I\rangle}{E_I - E_n}$$

$$+ (-1)^{F_I-F_F+1}\left\{\begin{array}{ccc} F_F & F_I & 1 \\ j_I & j_n & I \end{array}\right\}\left\{\begin{array}{ccc} I & I & 1 \\ j_n & j_F & F_F \end{array}\right\}$$

$$\left.\times\frac{\langle j_F\|H^{(2)}\|n\,j_n\rangle\langle n\,j_n\|z\|j_I\rangle}{E_F - E_n}\right]$$

Data Analysis for ^{133}Cs

Matrix Element (10^{-11})	$H^{(1)}$	$H^{(2)}$	$\epsilon_{F'F}$
$\langle 7s\ [3]\ \|\| z \|\| 6s\ [3] \rangle$	-2.037	-0.2250	0.1105
$\langle 7s\ [3]\ \|\| z \|\| 6s\ [4] \rangle$	-3.528	-0.7296	0.2068
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$\langle 7s\ [4]\ \|\| z \|\| 6s\ [4] \rangle$	2.981	-0.2562	-0.0859

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Q_W^{exp}	-71.91(46)
κ^{exp}	0.117(16)

Weak-Hyperfine Interference

$$Z_{wv}^{(\mathsf{hf})} = \sum_{\substack{i\neq w\\ j\neq v}} \left[\frac{(H^{(1)})_{wi}\; z_{ij}\; (H_{\mathsf{hf}})_{jv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_v)} + \frac{(H_{\mathsf{hf}})_{wi}\; z_{ij}\; (H^{(1)})_{jv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_v)} \right]$$

$$+ \sum_{\substack{i\neq v\\ j\neq v}} \left[\frac{z_{wi}\; (H^{(1)})_{ij}\; (H_{\mathsf{hf}})_{jv}}{(\epsilon_i - \epsilon_v)(\epsilon_j - \epsilon_v)} + \frac{z_{wi}\; (H_{\mathsf{hf}})_{ij}\; (H^{(1)})_{jv}}{(\epsilon_i - \epsilon_v)(\epsilon_j - \epsilon_v)} \right]$$

$$+ \sum_{\substack{i\neq w\\ j\neq w}} \left[\frac{(H^{(1)})_{wj}\; (H_{\mathsf{hf}})_{ji}\; z_{iv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_w)} + \frac{(H_{\mathsf{hf}})_{wj}\; (H^{(1)})_{ji}\; z_{iv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_w)} \right]$$

$$- \sum_{i\neq v} \frac{z_{wi}\; (H^{(1)})_{iv}}{(\epsilon_i - \epsilon_v)^2}\, (H_{\mathsf{hf}})_{vv} \;-\; (H_{\mathsf{hf}})_{ww} \sum_{i\neq w} \frac{(H^{(1)})_{wi}\; z_{iv}}{(\epsilon_i - \epsilon_w)^2}$$

Sums over Magnetic Substates

$$\langle wIF_w\|z\|vIF_v\rangle^{(\mathrm{hf})} = g_I\sqrt{I(I+1)(2I+1)[F_v][F_w]}\times$$

$$\Bigg\{ \sum_{j\neq v} (-1)^{j_v-j_w+1} \left\{ \begin{array}{ccc} F_w & F_v & 1 \\ j_j & j_w & I \end{array} \right\} \left\{ \begin{array}{ccc} I & I & 1 \\ j_j & j_v & F_v \end{array} \right\}$$

$$\left(\sum_i \left[\frac{\langle w\|H^{(1)}\|i\rangle\ \langle i\|z\|j\rangle}{(\epsilon_i-\epsilon_w)} + \frac{\langle w\|z\|i\rangle\ \langle i\|H^{(1)}\|j\rangle}{(\epsilon_i-\epsilon_v)} \right] \frac{\langle j\|t\|v\rangle}{(\epsilon_j-\epsilon_v)} \right.$$

$$\left. + \frac{\langle w\|z\|j\rangle}{(\epsilon_j-\epsilon_v)} \left[\sum_i \frac{\langle j\|t\|i\rangle\ \langle i\|H^{(1)}\|v\rangle}{(\epsilon_i-\epsilon_v)} - \frac{\langle j\|H^{(1)}\|v\rangle}{(\epsilon_j-\epsilon_v)} \langle v\|t\|v\rangle \right] \right)$$

$$+ \sum_{j\neq w} (-1)^{F_v-F_w+1} \left\{ \begin{array}{ccc} F_w & F_v & 1 \\ j_v & j_j & I \end{array} \right\} \left\{ \begin{array}{ccc} I & I & 1 \\ j_j & j_w & F_w \end{array} \right\}$$

$$\left(\frac{\langle w\|t\|j\rangle}{(\epsilon_j-\epsilon_w)} \sum_i \left[\frac{\langle j\|z\|i\rangle\ \langle i\|H^{(1)}\|v\rangle}{(\epsilon_i-\epsilon_v)} + \frac{\langle j\|H^{(1)}\|i\rangle\ \langle i\|z\|v\rangle}{(\epsilon_i-\epsilon_w)} \right] \right.$$

$$\left. + \left[\sum_i \frac{\langle w\|H^{(1)}\|i\rangle\ \langle i\|t\|j\rangle}{(\epsilon_i-\epsilon_w)} - \langle w\|t\|w\rangle\ \frac{\langle w\|H^{(1)}\|j\rangle}{(\epsilon_j-\epsilon_w)} \right] \frac{\langle j\|z\|v\rangle}{(\epsilon_j-\epsilon_w)} \right) \Bigg\}$$

Analysis for ^{133}Cs

Dipole Matrix Element	$H^{(2)}$	Hyperfine	$\sim \kappa_{\text{hf}}$
$\langle 7s\ [3]\,\|\, z \,\|6s\ [3]\rangle$	2.249[-12]	1.141[-14]	5.076[-03]
$\langle 7s\ [3]\,\|\, z \,\|6s\ [4]\rangle$	7.299[-12]	3.579[-14]	4.903[-03]
$\langle 7s\ [4]\,\|\, z \,\|6s\ [3]\rangle$	6.432[-12]	3.139[-14]	4.880[-03]
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Thus, for the $7s - 6s$ transition in ^{133}Cs, we can describe the interference term approximately as

$$H^{(\text{hf})} = \kappa_{\text{hf}}\ \boldsymbol{\alpha} \cdot \boldsymbol{I} \rho(\boldsymbol{r})$$

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- ⋆ $H^{(2)}$ is sensitive to correlations, Hyperfine term is not.
- ⋆ Hyperfine term is sensitive to negative-energy states, $H^{(2)}$ is not.

Anapole Moment of ^{133}Cs

Group	κ	κ_2	$\kappa_{\rm hf}$	κ_a
Present	0.117(16)	0.0140[1]	0.0049	0.098(16)
Haxton *et al.*	0.112(16)[2]	0.0140	0.0078[3]	0.090(16)
Flambaum and Murray	0.112(16)[4]	0.0111[5]	0.0071[6]	0.092(16)[7]
Bouchiat and Piketty		0.0084	0.0078	

[1] from Haxton *et al.*

[2] from Flambaum and Murray

[3] from Bouchiat and Piketty

[4] The spin-dependent matrix elements from Kraftmakher are used.

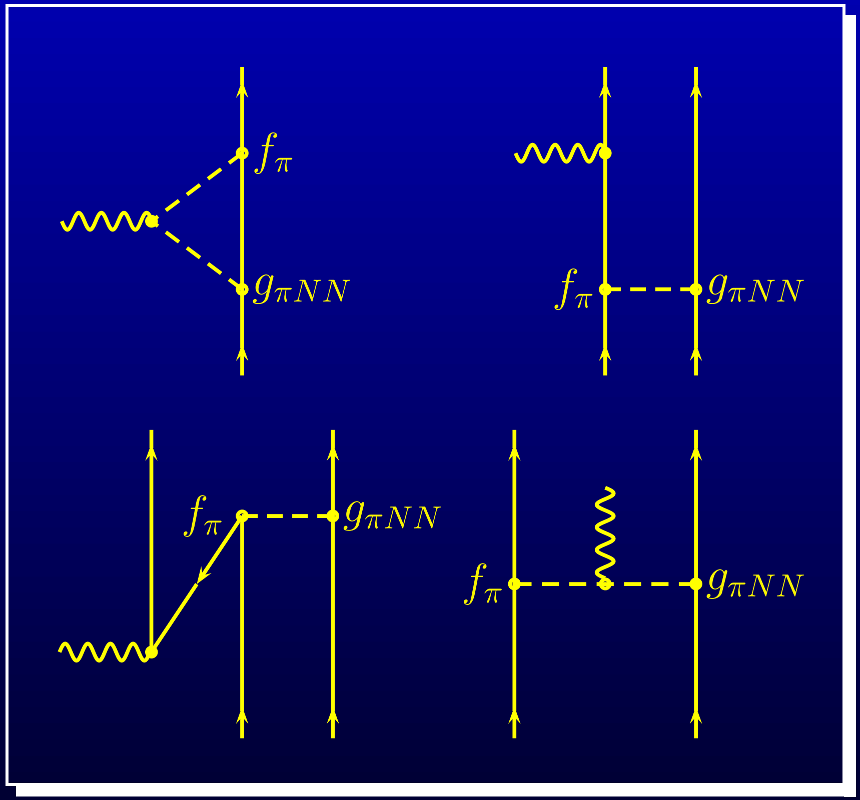
[5] Shell-model value with $\sin^2\theta_W = 0.23$.

[6] This value was obtained by scaling the analytical result from Flambaum and Khriplovich ($\kappa_{\rm hf} = 0.0049$) by a factor 1.5.

[7] Contains a 1.6% correction for finite nuclear size; the raw value is 0.094(16).

Evaluation of the Anapole Moment

The (low-energy) parity nonconserving nucleon-nucleon interaction is conventionally described by a one-meson exchange potential having one strong-interaction vertex $\{g_{\pi NN}, g_\rho, g_\omega\}$ and one weak vertex $\{f_\pi,\ h^0_\rho,\ h^1_\rho,\ h^2_\rho,\ h^0_\omega,\ h^1_\omega\}$[16]



[16] B. Desplanques, J. F. Donoghue, and B. Holstein, Ann. Phys. (NY) **124** 449 (1980)

Contributions to Cs Anapole Moment

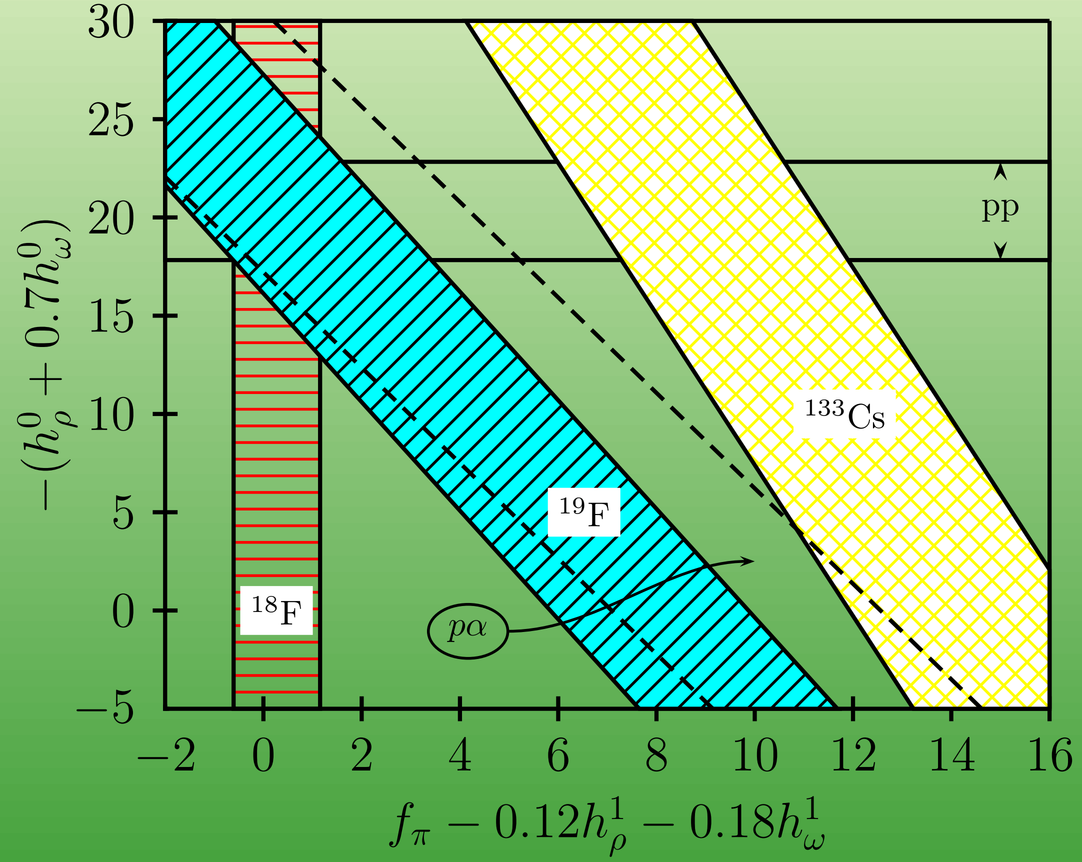
Shell-model estimates of the anapole matrix element $\langle I \| A \| I \rangle / e$ expressed as coefficients multiplied by indicated weak couplings[17]

Source	f_π	h_ρ^0	h_ρ^1	h_ρ^2	h_ω^0	h_ω^1
one-body	0.59	0.87	0.90	0.36	0.28	0.29
polariz.	51.57	-16.67	-4.88	-0.06	-9.79	-4.59
ex. cur.	8.58	0.02	0.11	0.06	-0.57	-0.57
total	60.74	-15.78	-3.87	0.36	-10.09	-4.87

$$\langle I \| A \| I \rangle / e = 60.74 f_\pi - 15.78 h_\rho^0 - 3.87 h_\rho^1 + \cdots$$

[17] W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. **51**, 261 (2001)

Constraints on Weak Coupling Constants



Microwave Experiments

The nucleon vector current does not contribute to transitions such as $|(6s\,I)F\rangle \rightarrow |(6s\,I)F'\rangle$ between different hyperfine components of an atomic level. Therefore, measurements of PNC between such levels directly measure the spin-dependent PNC amplitude.[18]

$$D = \langle (jI)F' \| ez \| (jI)F \rangle \;\; (i\kappa\; 10^{-12} e a_0)$$

Element	A	nl_j	I	D	κ_{hf}
K	39	$4s_{1/2}$	3/2	-0.222	5.01[-04]
Rb	87	$5s_{1/2}$	3/2	-1.363	7.54[-03]
Cs	133	$6s_{1/2}$	7/2	-17.24	4.52[-03]
Ba^+	135	$6s_{1/2}$	3/2	-6.169	3.59[-03]
Tl	205	$6p_{1/2}$	1/2	-30.00	1.18[-02]
Fr	211	$7s_{1/2}$	9/2	-237.9	9.34[-03]

[18] S. Aubin et al. 16th Int. Conf. on Laser Spect. (2001); S. G. Porsev and M. G. Kozlov, Phys. Rev. A **64**, 064101, (2001).

Conclusions

- Z_0-hyperfine interference in the 4-3 hyperfine transition in ^{133}Cs can be described approximately by

$$H^{(\mathsf{hf})} = \frac{G}{\sqrt{2}}\,\kappa_{\mathsf{hf}}\,\boldsymbol{\alpha}\cdot I$$

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- Microwave experiment promise to provide direct measurements of anapole moments for Cs and for other nuclei.

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