Breit correction to the parity-nonconservation amplitude in cesium

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Including the Breit interaction leads to a 0.6% reduction in the magnitude of the 6s-7s parity-nonconservation (PNC) amplitude in 133Cs, confirming a result recently obtained by A. Derevianko [Phys. Rev. Lett. 85, 1618 (2000)]. A revised value of the theoretical PNC amplitude for 133Cs is given; the corresponding value of the weak charge shows no noticeable deviation from the standard model.

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I. INTRODUCTION

Parity nonconservation (PNC) in atoms, described in the standard model of the electroweak interaction by exchange of Z bosons between bound electrons and nuclear quarks, leads to nonvanishing electric-dipole matrix elements between atomic states with the same parity. The dominant part of PNC matrix elements (arising from the vector nucleon current) is proportional to a conserved weak charge $Q_{W}$, which is sensitive to physics beyond the standard model such as the existence of additional neutral $Z'$ bosons.

Measurements of the 6s-7s PNC amplitude in 133Cs, following the procedure described by Bouchiat and Bouchiat [1], were carried out at the 2% level of accuracy by Gilbert and Wieman [2]. When combined with calculations of the PNC amplitudes, which were estimated to be accurate to 1% [3–5], the measurements led to an experimental value for the weak charge,

$$Q_{W}^{(133)Cs} = -71.04 \pm 1.81,$$  \hspace{1cm} (1.1)

where the uncertainties in the calculations are added in quadrature with the experimental errors. This experimental value differed from the weak charge predicted by the standard model [6,7] (which includes radiative corrections)

$$Q_{W}^{(133)Cs} = -73.09 \pm 0.03.$$  \hspace{1cm} (1.2)

by 1.1 standard deviation.

In recent years, there has been a substantial improvement in the experimental determination of $Q_{W}$ in cesium, due primarily to the precise measurement of the ratio of the 6s-7s PNC amplitude to $\beta$, the vector part of the Stark polarizability, by Wood et al. [8]. Another factor responsible for the improvement is a measurement of $\beta$ by Bennett and Wieman [9], in terms of $M_{ud}$, the off-diagonal hyperfine matrix element between the 6s and 7s states [10–12]. A reevaluation of the accuracy of the atomic structure calculations used in the determination of $Q_{W}$ was also carried out in Ref. [9] based on comparisons of theoretical polarizabilities, hyperfine constants, and transition matrix elements with recent measurements, suggesting that the uncertainty in the theoretical PNC amplitude should be reduced from 1% to 0.4%. Combining the recent experiments with the revised estimate of the theoretical uncertainty led to the value

$$Q_{W}^{(133)Cs} = -72.06 \pm 0.44,$$  \hspace{1cm} (1.3)

where the theoretical uncertainty $\pm 0.34$ is added in quadrature with the experimental error $\pm 0.28$. This value differs by 2.3 standard deviations from the theoretical value in Eq. (1.2). Although the 0.6% accuracy of the experimental value of the weak charge is comparable to the accuracy of other experimental parameters used in tests of the standard model cited in Ref. [6], 2.3$\sigma$ is one of the two largest differences with standard model predictions. Implications of this relatively large difference for physics beyond the standard model have been the subject of several recent investigations [13–15].

The contribution of the Breit interaction to the 6s-7s PNC amplitude in 133Cs was recently investigated by Derevianko [16] and found to be substantially larger than previously estimated. The increased contribution accounted for a substantial part of the 2.3$\sigma$ discrepancy discussed above. In the present paper, we evaluate the Breit correction both in the lowest-order Dirac-Hartree-Fock (DHF) approximation and including higher-order correlation effects. The present result for the Breit correction in the DHF approximation, 0.3% of the PNC-DHF amplitude\footnote{We use units $iea_0 \times 10^{-11}(-Q_{W}/N)$ throughout for the PNC amplitude.} or $+0.002$, agrees precisely with the earlier estimate in [4,5]. Furthermore, our result for the Breit correction to the correlated PNC amplitude agrees well with the value found in Ref. [16], 0.6% of the correlated amplitude or $+0.0054$, confirming the principal conclusion of [16] and practically removing the deviation from the standard model claimed in [9].

II. CALCULATION

In the “frozen-core” DHF approximation, the perturbation $\psi^{HF}_v$ to a valence electron state $\psi_v^{HF}$ induced by the weak interaction $h_{\text{PNC}}$ satisfies the inhomogeneous DHF equation

$$\left(h_0 + V^{HF} - \varepsilon_v^{HF}\right) \psi_v^{HF} = -h_{\text{PNC}} \psi_v^{HF}.$$  \hspace{1cm} (2.1)

In this equation, $V^{HF}$ is the HF potential of the closed xenon-like core and $\varepsilon_v^{HF}$ is the eigenvalue of the unperturbed DHF
TABLE I. Contributions, in the PNC-DHF approximation, to the 6s-7s PNC amplitude in $^{133}$Cs. The two terms in Eq. (2.2) and their sum $E^{\text{HF}}_{\text{PNC}}$ are evaluated using the Coulomb DHF potential and the Coulomb + Breit DHF potential.

| Type               | $\langle \psi_{1s}^{\text{HF}}|D|\psi_{6s}^{\text{HF}} \rangle$ | $\langle \psi_{1s}^{\text{HF}}|D|\psi_{6s}^{\text{HF}} \rangle$ | $E^{\text{HF}}_{\text{PNC}}$ |
|--------------------|-------------------------------------------------|-------------------------------------------------|-----------------|
| Coul. only         | 0.27492                                          | -1.01439                                        | -0.73947        |
| Coul. + Breit      | 0.27411                                          | -1.01134                                        | -0.73722        |
| $\Delta\%$         | -0.29%                                           | -0.30%                                          | -0.30%          |

The Breit interaction is included in $V^{\text{HF}}$, $\delta V^{\text{HF}}_{\text{PNC}}$, and $\delta V^{\text{HF}}_{\text{D}}$. Results for the correlated PNC amplitude calculated without and with the Breit interaction are shown in Table II. We see that the Breit corrections to each term in Eq. (2.5) and to the sum are 0.6%. The Coulomb $E_{\text{PNC}}$ amplitude is very close to the results of [3–5]. The difference is caused by some additional small corrections that are not considered in this work, including structural radiation, normalization, and double core polarization by simultaneous action of weak interaction and photon electric field.

It is interesting to examine the origin of the 0.6% Breit correction. To this end, we decompose each term in Eq. (2.5) into a sum over intermediate states,

$$E^{\text{PNC}}_{\text{Br}} = \sum_n \frac{\langle \psi_{1s}^{\text{Br}}|D|\psi_{6s}^{\text{Br}} \rangle}{E_{\text{PNC}}^{6s} - E_{n\text{p}}} + \frac{\langle \psi_{1s}^{\text{Br}}|D|\psi_{6s}^{\text{Br}} \rangle}{E_{\text{PNC}}^{6s} - E_{n\text{p}}}.$$  

This $E_{\text{PNC}} = h_{\text{PNC}} + \delta V^{\text{HF}}_{\text{PNC}}$, $\tilde{D} = D + \delta V^{\text{HF}}_{\text{D}}$, and $6s$, 7s, and $n\text{p}$ designate Brueckner orbitals. Each term in the sum over states has three factors subject to Breit corrections, the matrix element of $h_{\text{PNC}}$, the matrix element of $\tilde{D}$, and the energy denominator. The contributions of the Breit interaction from these three factors are $-0.6\%$, $-0.4\%$, and $0.4\%$, respectively. The corresponding contributions in the PNC-DHF approximation (2.2) are $-0.3\%$, $-0.3\%$, and $0.3\%$, respectively. Thus, the Breit corrections to the sum arising from corrections to dipole matrix elements or to energy denominators remain very close in correlated and uncorrelated calculations, whereas Breit corrections arising from the weak matrix elements approximately double in the correlated calculation.

The corrections to the correlated PNC amplitude from $h_{\text{PNC}}$, $D$, and energies found in [16] were $-0.5\%$, $-0.4\%$, and $0.3\%$, respectively, giving a total of $-0.6\%$ in agreement with the present result. We disagree, however, with the assertion in [16] that one should ignore the Breit correction to the energies and consider only the $-0.5\%$ correction to PNC amplitudes. That assertion was based on the incorrect assumption that experimental energies, which implicitly include Breit corrections, were used in Refs. [3–5]. In both [3] and [4,5], theoretical Coulomb energies were used to evaluate the theoretical PNC amplitude.
Although the Breit interaction accounts for a substantial part of the difference between theory and experiment, it should be emphasized that, at the fraction of a percent level, there are various other small theoretical corrections that must be considered. Among these are higher-order many-body Coulomb corrections, corrections due to differences in neutron and proton distributions discussed in Ref. [18], and those from higher order in $Z \alpha$ radiative corrections discussed in Ref. [19]. Since these small corrections may add coherently, it appears premature to assign an uncertainty smaller than 1% to the theoretical PNC amplitude.

The experimental value of the weak charge $Q_W$ is found by dividing the experimental PNC amplitude by the theoretical amplitude expressed in terms of $Q_W$. We take the value of the theoretical Coulomb amplitude to be $-0.9075$, which is the average of $-0.908$ from [3] and $-0.907$ from [4,5]. (The underestimated Breit correction 0.002 has been removed from the value $-0.905$ given in [4,5] to obtain the value $-0.907$ for the Coulomb amplitude.) Adding the Breit correction to this average leads to a revised theoretical value for the PNC amplitude:

$$E_{PNC} = -0.902.$$  (2.7)

The corresponding theoretical amplitude $-0.9065$ used in Ref. [9] differs by 0.5% from the present value. Combining the present theoretical amplitude with the experimental amplitude from [8,9] leads to a value of $Q_W$ that differs from the standard model by $1.5\sigma$ if 0.4% theoretical accuracy is still assumed. However, if a more realistic 1% theoretical uncertainty is assumed, the value of the weak charge becomes

$$Q_W^{(133)Cs} = -72.42 \pm (0.28)_{\text{exp}} \pm (0.74)_{\text{theor}},$$  (2.8)

which is larger than the value given in [9] by 0.5% and shows no significant deviation from the standard model. Corrections arising from the difference between neutron and proton distributions, discussed in Ref. [18], have not been included in Eq. (2.8). The value of $Q_W$ given in Eq. (2.8) differs from the value $-72.65$ from [16]. The difference is explained by the fact that nucleon distribution corrections from [18] were included in [16], and a 0.9% Breit correction, in which Breit corrections to energy denominators were improperly omitted, was assumed.

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