Status of Atomic PNC: Experiment/Theory

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Abstract
Atomic PNC measurements and calculations are reviewed with emphasis on the $6s \rightarrow 7s$ transition in cesium and the corresponding value of the weak charge $Q_W(^{133}\text{Cs})$. 
Reminder

\[ H^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 Q_W \rho(r) \]

\[ Q_W = -N + Z (1 - 4 s^2) \approx -N \]

Consequence: Laporte’s rule\(^1\) violated!

\[ E_{\text{PNC}} = \langle 7s | e z | 6s \rangle \propto Q_W \times "\text{Structure Factor}" \]

\(^{1}\)O. Laporte, Z. Physik 23 135 (1924).
Highlights for $6s \rightarrow 7s$ in $^{133}\text{Cs}$

- Precise (0.35%) measurement of $E_{\text{PNC}}/\beta$.\(^2\)
- Re-measurement of $\beta$.\(^3\)
- Re-analysis of accuracy of structure calculations.\(^3\)
- Conclusion: 2.5 $\sigma$ difference of $Q_{\text{W}}^{\text{exp}}$ with standard model.\(^4\)
- Led to speculation concerning physics beyond the SM, including: new $Z'$ particles, scalar leptoquarks, four-fermion contact interactions . . .
- Led to a re-analysis of small (Breit, QED, “skin”) corrections.

\(^3\)S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. 82, 4153 (1999).
Optical Rotation Experiments

These experiments take advantage of the fact that $n_\text{−} \neq n_\text{+}$ and measure

$$R_\phi = \text{Im} (E_{\text{PNC}}) / M_1$$

where $M_1$ is the magnetic-dipole transition matrix element.

<table>
<thead>
<tr>
<th>Element</th>
<th>Transition</th>
<th>Group</th>
<th>$10^8 \times R_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{205}\text{Tl}$</td>
<td>$^2P_{1/2} - ^2P_{3/2}$</td>
<td>Oxford (95)</td>
<td>-15.33(45)</td>
</tr>
<tr>
<td>$^{205}\text{Tl}$</td>
<td>$^2P_{1/2} - ^2P_{3/2}$</td>
<td>Seattle (95)</td>
<td>-14.68(20)</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>$^3P_0 - ^3P_1$</td>
<td>Oxford (94)</td>
<td>-9.80(33)</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>$^3P_0 - ^3P_1$</td>
<td>Seattle (95)</td>
<td>-9.86(12)</td>
</tr>
<tr>
<td>$^{209}\text{Bi}$</td>
<td>$^4S_{3/2} - ^2D_{3/2}$</td>
<td>Oxford (91)</td>
<td>-10.12(20)</td>
</tr>
</tbody>
</table>
Analysis for $^{205}$Tl

The difference between the Oxford and Seattle values in the table was resolved by Majumder and Tsai\textsuperscript{5}

$$R_{\phi}(^{205}\text{Tl}) = -14.71(25).$$

Using a recent structure calculation\textsuperscript{6} (3% error)

$$Q_{W}^{\exp}(^{205}\text{Tl}) = -113(3)$$

$$Q_{W}^{SM}(^{205}\text{Tl}) = -116(1)$$

HELP! Better calculations for Tl needed.


New and improved measurement in Tl

Cronin et al.\textsuperscript{7} suggested that the optical rotation measurement of $R_\phi(6p_{1/2} \rightarrow 6p_{3/2})$ in thallium could be improved using the electromagnetically induced transparency\textsuperscript{8} of a thallium vapor.

The method was used to obtain the ratio $E2/M1$ for the $6p_{1/2} \rightarrow 6p_{3/2}$ transition\textsuperscript{7}

Details on Seattle group website.\textsuperscript{9}


\textsuperscript{9}www.washington.edu/~fortson
Stark Interference

A laser excites an $E1$-forbidden transition in an atomic beam in a presence of electric $E$ and magnetic $B$ fields. The quantity measured is the component of the transition rate arising from the interference between $E_{\text{PNC}}$ and the Stark amplitude $\beta E$ is then used to determine

$$R_{\text{Stark}} = \text{Im}(E_{\text{PNC}} / \beta)$$

This method has been used\textsuperscript{10} to measure $R$ for the $6p_{1/2} \rightarrow 7p_{1/2}$ transition in $^{205}\text{Tl}$; the value of $\beta$ for this transition was also measured.\textsuperscript{11}

The Stark interference method has also been used to study the $6s \rightarrow 7s$ transition in cesium.

The $6s \rightarrow 7s$ transition in $^{133}$Cs

<table>
<thead>
<tr>
<th>Element</th>
<th>Transition</th>
<th>Group</th>
<th>$R_{4-3}$</th>
<th>$R_{3-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{133}$Cs</td>
<td>$6s_{1/2} - 7s_{1/2}$</td>
<td>Paris (1984)</td>
<td>-1.5(2)</td>
<td>-1.5(2)</td>
</tr>
<tr>
<td>$^{133}$Cs</td>
<td>$6s_{1/2} - 7s_{1/2}$</td>
<td>Boulder (1988)</td>
<td>-1.64(5)</td>
<td>-1.51(5)</td>
</tr>
<tr>
<td>$^{133}$Cs</td>
<td>$6s_{1/2} - 7s_{1/2}$</td>
<td>Boulder (1997)</td>
<td>-1.635(8)</td>
<td>-1.558(8)</td>
</tr>
</tbody>
</table>

The vector current contribution from the last row is

$$R_{\text{Stark}} = -1.593 \pm 0.006$$

Combining this with the value of $\beta$ from Ref.[3], leads to

$$\text{Im} \left[ E_{\text{PNC}}(6s \rightarrow 7s) \times 10^{11} \right] = -0.8374 \pm (0.0031)_{\text{exp}} \pm (0.0021)_{\text{th}}$$
Calculations of the $6s \rightarrow 7s$ amplitude

The most recent many-body calculation\textsuperscript{12} uses a method referred to as “perturbation theory in the screened Coulomb interaction” (PTSCI) in which important classes of many-body diagrams are summed to all orders. This gives results consistent with the SD Coupled-Cluster (SDCC) calculations.\textsuperscript{13} The calculated values are:

$$E_{\text{PNC}} = 0.908(5), \quad \text{(PTSCI)}$$

$$0.909(4), \quad \text{(SDCC)}$$

Units: $i (Q_{W}/N) e a_{0} \times 10^{-11}$

(We omit the Breit correction that was originally included in [12] and use the error estimate for that calculation given in Ref.[3].)


Analysis of $6s \rightarrow 7s$ amplitude in $^{133}$Cs

Combining the calculations and the measurements, we find

$$Q_{W}^{\text{exp}}(^{133}\text{Cs}) = -71.90(48),$$

As mentioned previously, this disagrees by 2.5 $\sigma$ with the standard model value$^4$

$$Q_{W}^{\text{SM}}(^{133}\text{Cs}) = -73.09(3).$$
What’s Missing?

(A) Breit Interaction\textsuperscript{14}

| Type            | $\langle 7s | e_z + \delta V_{z}^{\text{HF}} | 6s \rangle$ | $\langle \tilde{7}s | e_z + \delta V_{z}^{\text{HF}} | 6s \rangle$ | $E_{\text{PNC}}$ |
|-----------------|----------------|----------------|----------------|
| Coul            | 0.43942        | -1.33397       | -0.89456       |
| Coul + Breit    | 0.43680        | -1.32609       | -0.88929       |
| $\Delta\%$      | -0.60\%        | -0.59\%        | -0.59\%        |

Another Missing Piece

(B) Vacuum-Polarization$^{15}$

RPA-level calculations

<table>
<thead>
<tr>
<th>Type</th>
<th>$\langle 7s\mid e z + \delta V^{\text{HF}}_z\mid 6s \rangle$</th>
<th>$\langle \tilde{7}s\mid e z + \delta V^{\text{HF}}_z\mid 6s \rangle$</th>
<th>$E_{\text{PNC}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coul</td>
<td>0.3457</td>
<td>-1.2726</td>
<td>-0.9269</td>
</tr>
<tr>
<td>Coul + V.P.</td>
<td>0.3471</td>
<td>-1.2778</td>
<td>-0.9307</td>
</tr>
<tr>
<td>$\Delta%$</td>
<td>0.41%</td>
<td>0.41%</td>
<td>0.41%</td>
</tr>
</tbody>
</table>

Nuclear “skin” correction

Neutrons are primarily the source of the vector atomic PNC interaction, but proton densities are used in calculations of atomic PNC.

Replacing proton densities by neutron densities leads to “skin” corrections proportional to \( \delta \rho = \rho_n - \rho_p \).

Proton and Neutron distributions\(^{16}\) for \(^{133}\)Cs.

Conclusion:17,18,19 The “skin” effect decreases the size of $E_{\text{PNC}}$ by 0.1% - 0.2%

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Conclusion (for $^{133}$Cs)

Including the (Br+VP+”skin”) corrections changes theory value to

$$E_{\text{PNC}} = -0.9053 \pm 0.0037 \ i e a_0 \times 10^{-11} (-Q_W/N)$$

Combining this with the experimental value of $E_{\text{PNC}}/\beta$, leads to an experimental value for the weak charge

$$Q_W^{\text{expt}}(^{133}\text{Cs}) = -72.15 \pm (0.26)_{\text{expt}} \pm (0.34)_{\text{theor}}$$

This still differs by 2.2 $\sigma$ from the standard-model value.
Notes

* The vertex QED correction is controversial!!

Milstein and Sushkov: The vertex QED correction is controversial!!

Kuchiev and Flambaum: The vertex QED correction is controversial!!

* The value of $\beta$ has been remeasured.$^{22}$

$$\beta = 27.22(11) \ a_0^3 \quad \beta_{BW} = 27.024(60)$$

$$\beta_{ave} = 27.06(6)$$

$$Q_W \Rightarrow -72.25 \pm (0.26)_{\text{expt}} \pm (0.34)_{\text{theor}}$$

$$\Delta Q_W \Rightarrow 2.0 \ \sigma$$

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$^{22}$ A. A. Vasilyev et al., arXiv:physics/0112071
New cesium PNC experiment

A new PNC experiment on cesium in a sapphire cell is underway at l’Ècole Normal Supérieur.\textsuperscript{23} The experimental configuration consists of two collinear laser beams, pump and probe, and an $E$ field in the same direction.

i) direct measurement of the PV asymmetry at the output of polarimeter

ii) calibration is lineshape independent

iii) the absence of Stark-M1 interference in a longitudinal field configuration, hence suppression of a source of systematics

iv) possibility of amplification when the probe beam propagates through an optically thick vapor

## Other cases

<table>
<thead>
<tr>
<th>Atom</th>
<th>Transition</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr</td>
<td>$7S_{1/2} \rightarrow 8S_{1/2}$</td>
<td>Stony Brook</td>
</tr>
<tr>
<td>Yb</td>
<td>$(6s^2)^1S_0 \rightarrow (6s5d)^3D_1$</td>
<td>Berkeley</td>
</tr>
<tr>
<td>Yb</td>
<td>$(6s6p)^3P_0 \rightarrow (6s6p)^3P_1$</td>
<td>Berkeley</td>
</tr>
<tr>
<td>Ba$^+$</td>
<td>$6S_{1/2} \rightarrow 5D_{3/2}$</td>
<td>Seattle</td>
</tr>
<tr>
<td>Dy</td>
<td>$(4f^{10}5d6s)^{10} \rightarrow (4f^95d^26s)^{10}$</td>
<td>Berkeley</td>
</tr>
<tr>
<td>Sm</td>
<td>$(4f^66s^2)^7F_J \rightarrow (4f^66s^2)^5D_{J'}$</td>
<td>Oxford</td>
</tr>
</tbody>
</table>
Francium ($Z=87$)

- $E_{\text{PNC}}(\text{Fr})[7s_{1/2} \rightarrow 8s_{1/2}] \sim 15 \times E_{\text{PNC}}(\text{Cs})$
- $^{208-221}\text{Fr}$ produced and trapped at SUNYSB$^{24}$
- $T_{1/2} \sim 20\text{m}$ for some isotopes ($^{212}\text{Fr}$)
- Spectrum established$^{25}$
- Precision measurement of lifetimes and hyperfine constants$^{26}$
- A microwave cavity experiment to measure the $I$-dependent PNC between ground-state hyperfine levels in Fr isotopes is proposed$^{27}$

Barium ion ($Z=56$)

A experiment is underway in Seattle to measure PNC in a single trapped barium ion.  

- Transition: $6s_{1/2} \rightarrow 5d_{3/2}$

- $E_{\text{PNC}} \sim 10^{-11} e a_0$ - competitive with Cs

- Seven naturally occurring isotopes, : possibility of eliminating computational uncertainties by comparing results from different isotopes

- Two odd $A$ isotopes $^{135}\text{Ba}$ and $^{137}\text{Ba}$ ($I=3/2$) can give information about $I$-dependent PNC terms (from an unpaired neutron)

- Recent progress$^{29}$ has been made on the spectroscopy of $\text{Ba}^+$

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Ytterbium ($Z=70$)

- Seven naturally occurring isotopes $^{168-176}\text{Yb}$
- Two odd $A$ isotopes $^{171}\text{Yb}$ (1/2) and $^{173}\text{Yb}$ (5/2)
- $E_{\text{PNC}}(\text{Yb})[{}^1S_0 \rightarrow {}^3D_1] \sim 100 \times E_{\text{PNC}}(\text{Cs})$\footnote{D. DeMille, Phys. Rev. Lett. 74, 4165 (1995).}
- Mixing of $(6s5d)^3D_1$ with nearby $(6s6p)^1P_1$
- Only $I$-dependent terms $\Rightarrow E_{\text{PNC}}(\text{Yb})[{}^1S_0 \rightarrow {}^3D_2]$
- Progress and details given at Berkeley website\footnote{ist-socrates.berkeley.edu/budker/}
Dysprosium (\(Z=66\))

Atomic dysprosium has two nearly degenerate levels of opposite parity \(a = (4f^{10}5d6s)[10]\) and \(b = (4f^{9}5d^{2}6s)[10]\) at 19797.96 cm\(^{-1}\) above the ground state.

- Seven naturally occurring isotopes — comparisons
- Two odd \(A\) isotopes \(^{161}\)Dy and \(^{163}\)Dy (\(I=5/2\)) — information about \(I\)-dependent PNC terms
- A Stark interference experiment\(^{34}\) to detect the PNC mixing between \(a\) and \(b\) gave \(|H_W| = |2.3 \pm 2.9 \pm 0.7|\) Hz,
- A multi-configuration Dirac-Fock calculation\(^{35}\) gave \(H_W = 70(40)\) Hz — correlation dependent matrix element!

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Samarium \((Z=62)\)

The optical rotation parameter \(R_\phi\) was measured\(^{36}\) for five \(M1\) transitions in the ground-state multiplet of atomic samarium.

- Lower state: \((4f^66s^2)^7F\)
- Upper state: \((4f^66s^2)^5D\)

- The upper state levels are nearly degenerate with levels of opposite parity from the \((4f^66s6p)\) configuration. (expect enhancement)

- \(|H_W| = 1\text{–}30 \text{ kHz for the 5 levels.}\)

- Result is 1 - 2 orders of magnitude smaller than expected from semi-empirical calculations!

Nuclear spin dependent terms

For a nucleus with a single unpaired nucleon, the axial current gives:

$$H^{(2)} = -\frac{G}{\sqrt{2}} c_{2N} \frac{\kappa - 1/2}{I(I + 1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

where $c_{2N} = c_{2p}$ or $c_{2n}$, $\kappa = \pm(I + 1/2)$ for $I = L \pm 1/2$. This term is $\sim 1/A$ as large as $H^{(1)}$.

Anapole Moment\textsuperscript{37}

PNC in nucleus $\Rightarrow$ nuclear anapole:

$$H^{(a)} = \frac{G}{\sqrt{2}} K_a \frac{\kappa}{I(I + 1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

Spin-Dependent Interference Term

Interference between the hyperfine interaction $H_{hf}$ and $H^{(1)}$ gives yet another nuclear spin-dependent correction

$$H^{(\text{int})} = \frac{G}{\sqrt{2}} K_{\text{int}} \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

- $c_{2p} \approx 0.047$ and $c_{2n} \approx -0.047$ arise from weak coupling of the electron vector current to the nucleon axial vector current.

- $K_a$ is the nuclear anapole coupling constant – value obtained from nuclear PNC calculations. ($K_a \gg c_{2p}$)

- $K_{\text{int}}$ from hyperfine structure measurements and atomic structure calculations.
Results on anapole moment

\[ \kappa_{\text{exp}}(^{133}\text{Cs}) = 0.112(16) \]
\[ \kappa_{\text{exp}}(^{205}\text{Tl}) = 0.29(40) \]

The axial current term gives

\[ \kappa_{\text{SM}}(^{133}\text{Cs}) = 0.0140 \]
\[ \kappa_{\text{SM}}(^{205}\text{Tl}) = -0.127 \]

The interference term gives\(^{38}\)

\[ \kappa_{\text{int}}(^{133}\text{Cs}) = 0.0078 \]
\[ \kappa_{\text{int}}(^{205}\text{Tl}) = 0.044 \]

Residual anapole contribution\(^{39}\)

\[ \kappa_{a}(^{133}\text{Cs}) = 0.090(16) \]
\[ \kappa_{a}(^{205}\text{Tl}) = 0.38(40) \]


Conclusions for Anapole Moment

1. Measured anapole moments are not consistent with most general theory constraints on nuclear weak coupling constants.

2. Nuclear theory favors a negative anapole moment for Tl; experiment gives a positive value.

3. Nuclear theory predicts a value for Cs larger than observed.

What would be useful?

1. Improved thallium measurement

2. Moments from odd-neutron nuclei (Yb, Dy, Ba$^+$)