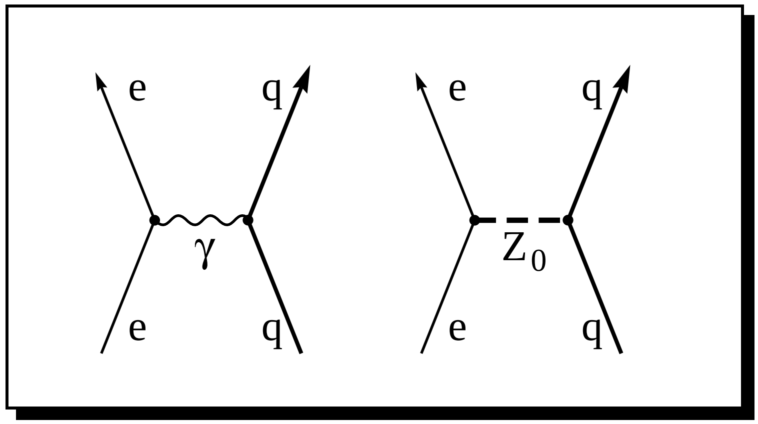
Nuclear spin-dependent contributions to atomic PNC: combined effect of coherent Z_0 exchange and the hyperfine interaction.

November 20, 2002

Abstract

Third-order many-body perturbation theory is applied to a problem addressed more than a decade ago by Bouchiat and Piketty. A substantially smaller value is obtained for the interference term in ^{133}Cs, leading to a revised *experimental* anapole moment for this nucleus.

Atomic Parity Nonconservation



Consequence of Z_0 exchange: Laporte's[1] rule is violated!

[1]Otto Laporte (1902-1971) is the discoverer of parity conservation in physics. He divided states of the iron spectrum into two classes, even and odd, and found that no radiative transitions occurred between like states: O. Laporte, Z. Physik **23** 135 (1924).

Background

Standard Model prediction for parity violation by Z_0 exchange[2]

$$H_{\mathsf{PV}} = \frac{G}{\sqrt{2}} \Big[\bar{e}\gamma_\mu\gamma_5 e \left(c_{1u}\, \bar{u}\gamma_\mu u + c_{1d}\, \bar{d}\gamma_\mu d + \cdots \right) + \bar{e}\gamma_\mu e \left(c_{2u}\, \bar{u}\gamma_\mu\gamma_5 u + c_{2d}\, \bar{d}\gamma_\mu\gamma_5 d + \cdots \right) \Big]$$

where

$$c_{1u} = -\,\frac{1}{2} + \frac{4}{3}\; s^2$$
$$c_{1d} = \frac{1}{2} - \frac{2}{3}\; s^2$$
$$c_{2u} = -\,\frac{1}{2}\left(1 - 4\; s^2\right)$$
$$c_{2d} = \frac{1}{2}\left(1 - 4\; s^2\right)$$

$s^2 = \sin^2\theta_W$

[2] W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 170.

Effective Interaction

Contribution of vector nucleon current:

$$H^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 \, Q_W \, \rho(r)$$

where $\rho(r)$ is a nuclear density ($\sim$ neutron density) and

$$\begin{aligned} Q_W &= 2[(2Z+N)c_{1u} + (Z+2N)c_{1d}] \\ &= -N + Z\,(1-4\,s^2) \approx -N \end{aligned}$$

Contribution of axial-vector current:

$$H^{(2)} = -\frac{G}{\sqrt{2}} \boldsymbol{\alpha} \cdot \left[c_{2p} \left\langle \phi_p^\dagger \boldsymbol{\sigma} \phi_p \right\rangle + c_{2n} \left\langle \phi_n^\dagger \boldsymbol{\sigma} \phi_n \right\rangle \right]$$

where $\langle \cdots \rangle$ designates nuclear matrix elements.
$c_{2p} \sim 1.25 \times c_{2u} = -0.068$, $c_{2n} \sim 1.25 \times c_{2d} = 0.068$

Shell Model Estimates

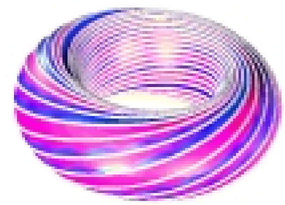
$$H^{(2)} = \frac{G}{\sqrt{2}}\,\eta^{(2)}\;\alpha\cdot\mathbf{I}\,\rho(r)$$

Values of $\eta^{(2)}$ obtained in the "extreme" shell model approximation and values from recent nuclear physics calculations.[3]

Element	A	State	$\eta^{(2)}$	Ref. [3]
K	39	$1d_{3/2}\ (p)$	0.0272	
Cs	133	$1g_{7/2}\ (p)$	0.0151	0.0140
Ba	135	$2d_{3/2}\ (n)$	-0.0272	
Tl	205	$3s_{1/2}\ (p)$	-0.136	-0.127
Fr	209	$1h_{9/2}\ (p)$	0.0124	

[3] W. C. Haxton, C.-P. Liu, and M. J. Ramsey-Musolf, Phys. Rev. Lett. **86**, 5247 (2001).

Nuclear Anapole Moment

PNC in nucleus ⇒ nuclear anapole: 

$$\boldsymbol{A} = \boldsymbol{a}\,\delta(\boldsymbol{r})$$

$$\boldsymbol{a} = -\pi \int d^3r\, r^2\, \boldsymbol{j}(\boldsymbol{r}) = \frac{1}{e}\,\frac{G}{\sqrt{2}}\,\eta^{(a)}\boldsymbol{I}$$

$$\boxed{H^{(a)} = e\,\boldsymbol{\alpha}\cdot\boldsymbol{A} \rightarrow \frac{G}{\sqrt{2}}\eta^{(a)}\,\alpha\cdot\mathbf{I}\,\rho(r)}$$

Early estimates[4] for ^{133}Cs gave $\eta^{(a)} = 0.063 - 0.084$. More recent studies[5] give larger values:

$$\boxed{\eta^{(a)} \gg \eta^{(2)}}$$

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[4] V. V. Flambaum, I. B. Khriplovich, O. P. Sushkov Phys. Letts. B **146** 367-369 (1984).

[5] V. V. Flambaum and D. W. Murray, Phys. Rev. C**56**, 1641 (1997); W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. **51**, 261 (2001)

Spin-Dependent Interference Term

According to Bouchiat and Piketty[6], interference between the hyperfine interaction H_{hf} and $H^{(1)}$ gives another nuclear spin-dependent correction of the form

$$\boxed{H^{(\mathsf{int})} = \frac{G}{\sqrt{2}} \eta^{(\mathsf{int})} \alpha \cdot \mathbf{I} \rho(r)}$$

$$^{133}\mathsf{Cs}: \quad \eta^{(\mathsf{int})} = 0.0078$$
$$^{205}\mathsf{Tl}: \quad \eta^{(\mathsf{int})} = 0.044$$

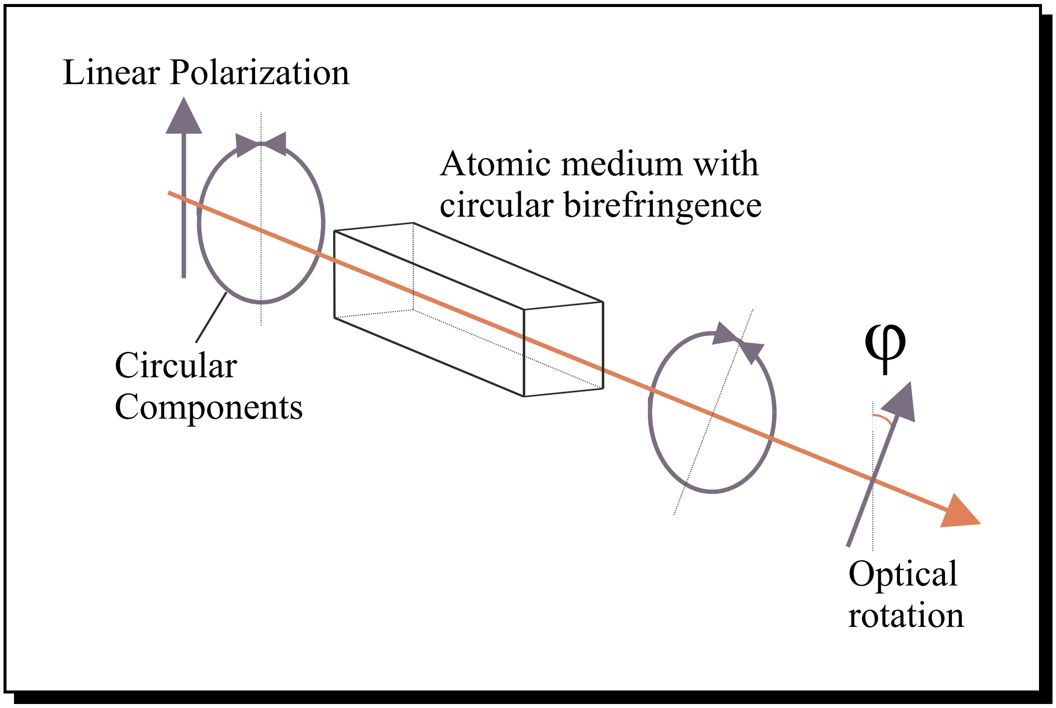
$$\boxed{\eta^{(\mathsf{int})} \sim \tfrac{1}{2}\, \eta^{(2)}}$$

Our aim is to analyze this contribution in detail.

[6] C. Bouchiat and C. A. Piketty, Z. Phys. C 49, 91 (1991); Phys. Lett. B 269, 195 (1991).

Optical Rotation Experiments

Aim is to measure $E_{\mathsf{PNC}} = \langle f|z|i\rangle \propto Q_W$:



The plane of polarization of a linearly polarized laser beam passing through a medium with $n_+ \neq n_-$ is rotated. The rotation angle $\phi \propto R_\phi = \mathrm{Im}\left(E_{\mathsf{PNC}}\right)/M1$.

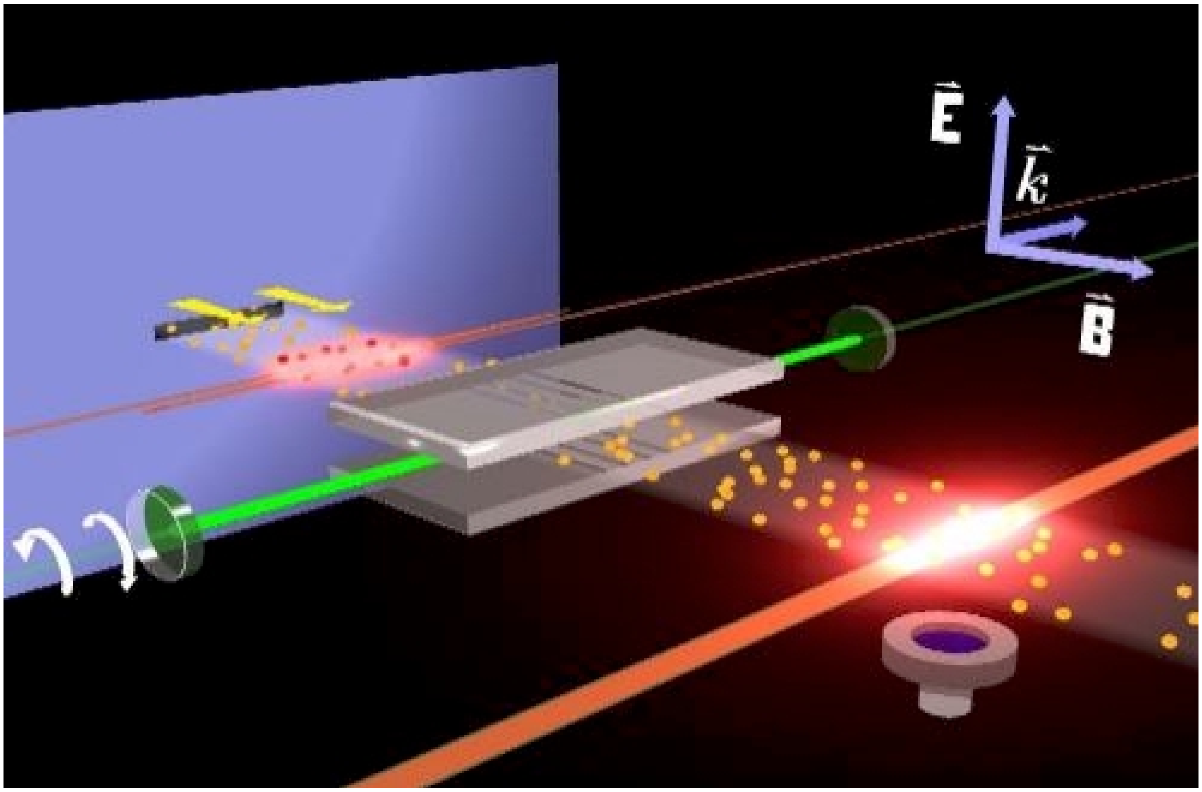
Optical Rotation Experiments

$$R_\phi = \mathrm{Im}\left(E_{\mathrm{PNC}}\right)/M1$$

Measured values of R_ϕ

Element	Transition	Group	$10^8 \times R_\phi$
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Oxford (95)	-15.33(45)
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Seattle (95)	-14.68(20)
^{208}Pb	$^3P_0 - {}^3P_1$	Oxford (94)	-9.80(33)
^{208}Pb	$^3P_0 - {}^3P_1$	Seattle (95)	-9.86(12)
^{209}Bi	$^4S_{3/2} - {}^2D_{3/2}$	Oxford (91)	-10.12(20)

A Stark-Interference Experiment



Schematic of the Boulder PNC apparatus. A beam of cesium atoms is optically pumped by diode laser beams, then passes through a region of perpendicular electric and magnetic fields where a green laser excites the transition from the 6S to the 7S state. Finally the excitations are detected by observing the florescence (induced by another laser beam) with a photo-diode.

Stark-Induced Transition Experiments

Evolving values of $R = \mathrm{Im}\,(E_{\mathrm{PNC}})\,/\beta$ (mV/cm) for ^{133}Cs			
Transition	Group	R_{4-3}	R_{3-4}
$6s_{1/2} - 7s_{1/2}$	Paris (1984)	-1.5(2)	-1.5(2)
$6s_{1/2} - 7s_{1/2}$	Boulder (1988)	-1.64(5)	-1.51(5)
$6s_{1/2} - 7s_{1/2}$	Boulder (1997)	-1.635(8)	-1.558(8)

The vector current contribution from the last row is

$$R_{\mathrm{Stark}} = -1.593 \pm 0.006$$

$$\mathrm{Im}\left[E_{\mathrm{PNC}}(6s \to 7s) \times 10^{11}\right] = -0.8376 \pm (0.0031)_{\mathrm{exp}} \pm (0.0021)_{\mathrm{th}}$$

Calculations of the $6s \rightarrow 7s$ amplitude

The most recent many-body calculation[7] uses a method referred to as "perturbation theory in the screened Coulomb interaction" (PTSCI) in which important classes of many-body diagrams are summed to all orders. This gives results consistent with the SD Coupled-Cluster (SDCC) calculations.[8] Also, unpublished nonlinear Coupled-Cluster calculations were presented at this workshop by B. Das.[9]

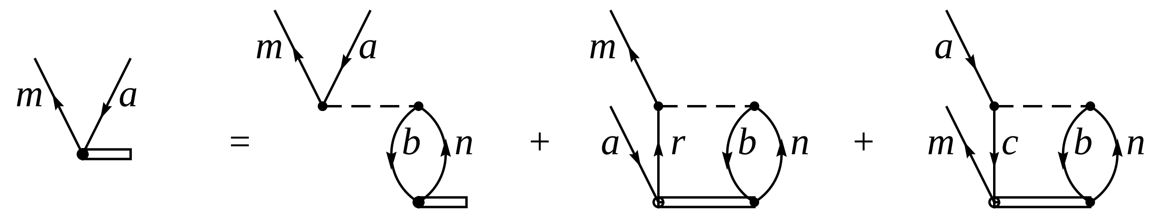
Theoretical values for E_{PNC}

PTSCI	-0.908 (5)
SDCC	-0.909 (4)
B. Das	-0.911

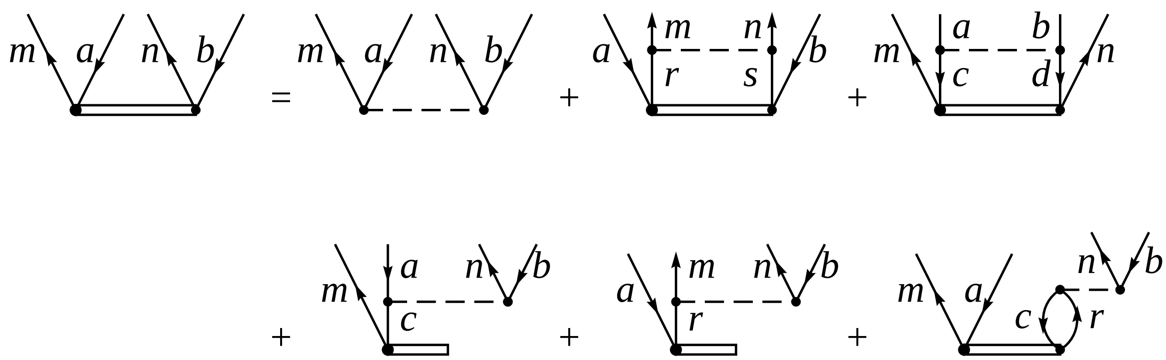
Units: $i(-Q_W/N) \times 10^{-11} e a_0$

[7] V. A. Dzuba et al., arXiv:hep-ph/0204134 (2002).
[8] S. A. Blundell et al., Phys. Rev. D**45**, 1602 (1992).
[9] B. Das, INT website

Brueckner-Goldstone Diagrams for the SDCC Equations



+ *exchange terms*



+ *exchange terms*

Analysis of $6s \to 7s$ amplitude in ^{133}Cs

Combining the calculations and the measurements

$$Q_W^{\text{exp}}(^{133}\text{Cs}) = -71.91(46) \Rightarrow -72.73(46)$$

differs by 2.5 σ $\Rightarrow$ 0.8 σ from the standard model value

$$Q_W^{\text{SM}}(^{133}\text{Cs}) = -73.09(3).$$

- Breit interaction[10] -0.6%
- Vacuum Polarization[11] +0.4%
- αZ Vertex Corrections[12] -0.7%
- Nuclear Skin Effect[13] -0.2%

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[10] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).

[11] W. R. Johnson, I. Bednyakov, and G. Soff, Phys. Rev. Lett. **87**, 233001 (2001).

[12] Kuchiev, Flambaum, Milstein, Sushkov, Terekov (2002).

[13] S. J. Pollock and M. C. Welliver, Phys. Lett. B **464**, 177 (1999); J. James and P. G. H. Sandars, J. Phys. B**32**, 3295 (1999)

Other Experiments

Atom	Transition	Group
Cs	$6S_{1/2} \rightarrow 7S_{1/2}$	Paris
Fr	$7S_{1/2} \rightarrow 8S_{1/2}$	Stony Brook
Fr	$7S_{1/2}[F=4] \rightarrow 7S_{1/2}[F=5]$	Stony Brook
Yb	$(6s^2)\,^1S_0 \rightarrow (6s5d)\,^3D_1$	Berkeley
Yb	$(6s6p)\,^3P_0 \rightarrow (6s6p)\,^3P_1$	Berkeley
Ba^+	$6S_{1/2} \rightarrow 5D_{3/2}$	Seattle
Dy	$(4f^{10}5d6s)[10] \rightarrow (4f^9 5d^2 6s)[10]$	Berkeley
Sm	$(4f^6 6s^2)\,^7F_J \rightarrow (4f^6 6s^2)\,^5D_{J'}$	Oxford

Angular Momentum Considerations

$$\langle F\|z\|I\rangle^{(1)} = (-1)^{j_F+F_I+I+1}\sqrt{[F_I][F_F]}\left\{\begin{array}{ccc} F_F & F_I & 1 \\ j_I & j_F & I \end{array}\right\}$$

$$\times \sum_{nj_n}\left[\frac{\langle j_F\|z\|n\, j_n\rangle\langle n\, j_n\|H^{(1)}\|j_I\rangle}{E_I - E_n} + \frac{\langle j_F\|H^{(1)}\|n\, j_n\rangle\langle n\, j_n\|z\|j_I\rangle}{E_F - E_n}\right]$$

$$\langle F\|z\|I\rangle^{(2)} = \sqrt{I(I+1)}\sqrt{[I][F_I][F_F]}\times$$

$$\sum_{nj_n}\left[(-1)^{j_I-j_F+1}\left\{\begin{array}{ccc} F_F & F_I & 1 \\ j_n & j_F & I \end{array}\right\}\left\{\begin{array}{ccc} I & I & 1 \\ j_n & j_I & F_I \end{array}\right\}\right.$$

$$\times \frac{\langle j_F\|z\|n\, j_n\rangle\langle n\, j_n\|H^{(2)}\|j_I\rangle}{E_I - E_n}$$

$$+ (-1)^{F_I-F_F+1}\left\{\begin{array}{ccc} F_F & F_I & 1 \\ j_I & j_n & I \end{array}\right\}\left\{\begin{array}{ccc} I & I & 1 \\ j_n & j_F & F_F \end{array}\right\}$$

$$\left.\times \frac{\langle j_F\|H^{(2)}\|n\, j_n\rangle\langle n\, j_n\|z\|j_I\rangle}{E_F - E_n}\right]$$

Data Analysis for ^{133}Cs

Mat. El. (10^{-11})	$H^{(1)}$	$H^{(2)}$	$\epsilon_{F'F}$
$\langle 7s\ [3]\ \|\| \ z\ \|\| 6s\ [3] \rangle$	-2.037	-0.2250	0.1105
$\langle 7s\ [3]\ \|\| \ z\ \|\| 6s\ [4] \rangle$	-3.528	-0.7296	0.2068
$\langle 7s\ [4]\ \|\| \ z\ \|\| 6s\ [3] \rangle$	3.328	-0.6430	-0.1823
$\langle 7s\ [4]\ \|\| \ z\ \|\| 6s\ [4] \rangle$	2.981	-0.2562	-0.0859

$$E_{\mathsf{PNC}}^{\mathsf{exp}} = E_{\mathsf{PNC}}^{(1)} \left[\frac{Q_W}{-N} + \eta\ \epsilon_{F'F} \right]$$

β (a_0^3)	27.024(80)
E_{34}/β (mV/cm)	-1.6349(80)
E_{43}/β (mV/cm)	-1.5576(77)
E_{34} (10^{-11})	-0.8592(49)
E_{43} (10^{-11})	-0.8186(47)
$E_{\mathsf{V}}^{\mathsf{exp}}$ (10^{-11})	-0.8376(34)
E_{V} $[10^{-11}\left(\frac{Q_W}{-N}\right)]$	-0.9085(45)
Q_W	-71.91(46)
η^{exp}	0.115(15)

Weak-Hyperfine Interference

$$Z_{wv}^{(\mathsf{int})} = \sum_{\substack{i\neq w \\ j\neq v}} \left[\frac{(H^{(1)})_{wi}\ z_{ij}\ (H_{\mathsf{hf}})_{jv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_v)} + \frac{(H_{\mathsf{hf}})_{wi}\ z_{ij}\ (H^{(1)})_{jv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_v)} \right]$$

$$+ \sum_{\substack{i\neq v \\ j\neq v}} \left[\frac{z_{wi}\ (H^{(1)})_{ij}\ (H_{\mathsf{hf}})_{jv}}{(\epsilon_i - \epsilon_v)(\epsilon_j - \epsilon_v)} + \frac{z_{wi}\ (H_{\mathsf{hf}})_{ij}\ (H^{(1)})_{jv}}{(\epsilon_i - \epsilon_v)(\epsilon_j - \epsilon_v)} \right]$$

$$+ \sum_{\substack{i\neq w \\ j\neq w}} \left[\frac{(H^{(1)})_{wj}\ (H_{\mathsf{hf}})_{ji}\ z_{iv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_w)} + \frac{(H_{\mathsf{hf}})_{wj}\ (H^{(1)})_{ji}\ z_{iv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_w)} \right]$$

$$- \sum_{i\neq v} \frac{z_{wi}\ (H^{(1)})_{iv}}{(\epsilon_i - \epsilon_v)^2}\ (H_{\mathsf{hf}})_{vv} - (H_{\mathsf{hf}})_{ww} \sum_{i\neq w} \frac{(H^{(1)})_{wi}\ z_{iv}}{(\epsilon_i - \epsilon_w)^2}$$

Sums over Magnetic Substates

$$\langle wIF_w \| z \| vIF_v \rangle^{(\mathrm{int})} = g_I \sqrt{I(I+1)(2I+1)[F_v][F_w]} \times$$

$$\Bigg\{ \sum_{j \neq v} (-1)^{j_v - j_w + 1} \left\{ \begin{array}{ccc} F_w & F_v & 1 \\ j_j & j_w & I \end{array} \right\} \left\{ \begin{array}{ccc} I & I & 1 \\ j_j & j_v & F_v \end{array} \right\}$$

$$\Bigg(\sum_i \left[\frac{\langle w \| H^{(1)} \| i \rangle \, \langle i \| z \| j \rangle}{(\epsilon_i - \epsilon_w)} + \frac{\langle w \| z \| i \rangle \, \langle i \| H^{(1)} \| j \rangle}{(\epsilon_i - \epsilon_v)} \right] \frac{\langle j \| t \| v \rangle}{(\epsilon_j - \epsilon_v)}$$

$$+ \frac{\langle w \| z \| j \rangle}{(\epsilon_j - \epsilon_v)} \left[\sum_i \frac{\langle j \| t \| i \rangle \, \langle i \| H^{(1)} \| v \rangle}{(\epsilon_i - \epsilon_v)} - \frac{\langle j \| H^{(1)} \| v \rangle}{(\epsilon_j - \epsilon_v)} \frac{\langle v \| t \| v \rangle}{[j_v]} \right] \Bigg)$$

$$+ \sum_{j \neq w} (-1)^{F_v - F_w + 1} \left\{ \begin{array}{ccc} F_w & F_v & 1 \\ j_v & j_j & I \end{array} \right\} \left\{ \begin{array}{ccc} I & I & 1 \\ j_j & j_w & F_w \end{array} \right\}$$

$$\Bigg(\frac{\langle w \| t \| j \rangle}{(\epsilon_j - \epsilon_w)} \sum_i \left[\frac{\langle j \| z \| i \rangle \, \langle i \| H^{(1)} \| v \rangle}{(\epsilon_i - \epsilon_v)} + \frac{\langle j \| H^{(1)} \| i \rangle \, \langle i \| z \| v \rangle}{(\epsilon_i - \epsilon_w)} \right]$$

$$+ \left[\sum_i \frac{\langle w \| H^{(1)} \| i \rangle \, \langle i \| t \| j \rangle}{(\epsilon_i - \epsilon_w)} - \frac{\langle w \| t \| w \rangle}{[j_w]} \frac{\langle w \| H^{(1)} \| j \rangle}{(\epsilon_j - \epsilon_w)} \right] \frac{\langle j \| z \| v \rangle}{(\epsilon_j - \epsilon_w)} \Bigg) \Bigg\}$$

Analysis for ^{133}Cs

Mat. El. (10^{-11})	$H^{(2)}$	Inter	$\sim \eta^{(\text{int})}$
$\langle 7s\ [3]\ \|\ z\ \|6s\ [3]\rangle$	-0.2250	-0.000848	0.00377
$\langle 7s\ [3]\ \|\ z\ \|6s\ [4]\rangle$	-0.7296	-0.002778	0.00381
$\langle 7s\ [4]\ \|\ z\ \|6s\ [3]\rangle$	-0.6430	-0.002451	0.00381
$\langle 7s\ [4]\ \|\ z\ \|6s\ [4]\rangle$	-0.2562	-0.000965	0.00377

Thus, for the $7s - 6s$ transition in ^{133}Cs, we can describe the interference term approximately as

$$H^{(\text{int})} = \eta^{(\text{int})}\ \boldsymbol{\alpha} \cdot \boldsymbol{I} \rho(\boldsymbol{r})$$

with $\eta^{(\text{int})} = 0.0038$.
Compare with Bouchiat and Piketty: $\eta^{(\text{int})} = 0.0078$
Combine these results

$$\eta^{(a)} = \eta^{\text{exp}} - \eta^{(2)} - \eta^{(\text{int})} = 0.097(15)$$

Other[14] $\eta^{(a)} = 0.090(16)$ $\quad [\eta^{\text{exp}} = 0.112(16)]$

[14] W. C. Haxton and C. E. Wieman, Ann. Rev. Nucl. Part. Sci. **51**, 261 (2001)

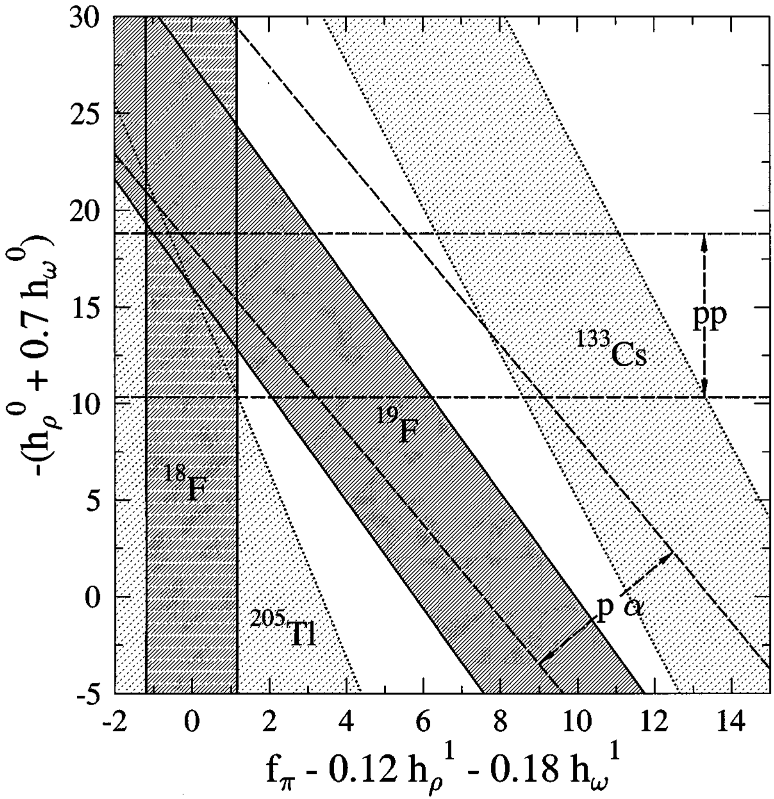

Figure 8 Constraints on the PNC meson couplings ($\times 10^7$) that follow from the results in Table 4. The error bands are one standard deviation. The illustrated region contains all of the DDH reasonable ranges for the indicated parameters.

Microwave Experiments

The nucleon vector current does not contribute to transitions such as $|(6s\,I)F\rangle \rightarrow |(6s\,I)F'\rangle$ between different hyperfine components of an atomic level. Therefore, measurements of PNC between such levels directly measure the spin-dependent PNC amplitude.[15]

$D = \langle (jI)F' \| ez \| (jI)F \rangle$ ($i\eta\ 10^{-12} ea_0$)

Element	A	nl_j	I	D
K	39	$4s_{1/2}$	3/2	-0.178
Rb	87	$5s_{1/2}$	3/2	-1.083
Cs	133	$6s_{1/2}$	7/2	-13.90
Ba^+	135	$6s_{1/2}$	3/2	-5.425
Tl	205	$6p_{1/2}$	1/2	-8.099
Fr	209	$7s_{1/2}$	9/2	-205.2

[15] S. Aubin et al. 16th Int. Conf. on Laser Spect. (2001); S. G. Porsev and M. G. Kozlov, Phys. Rev. A **64**, 064101, (2001).

Conclusions

1. Z_0-hyperfine interference for the 4-3 hyperfine transition in ^{133}Cs can be described approximately by
$$H^{(\text{int})} = \frac{G}{\sqrt{2}}\,\eta^{(\text{int})}\boldsymbol{\alpha}\cdot I$$
with $\eta^{(\text{int})} = 0.0038$, a factor of 2 smaller than obtained by Bouchiat and Piketty.

2. The resulting experimental value of the anapole moment of ^{133}Cs obtained from the Boulder PNC measurements is about 7% larger than previously determined, increasing differences with other constraints on the parity violating nuclear coupling constants.

3. Microwave experiments, some presently underway, will provide more direct measurements of nuclear anapole moments.