Status Report on Atomic PNC Experiments and Calculations

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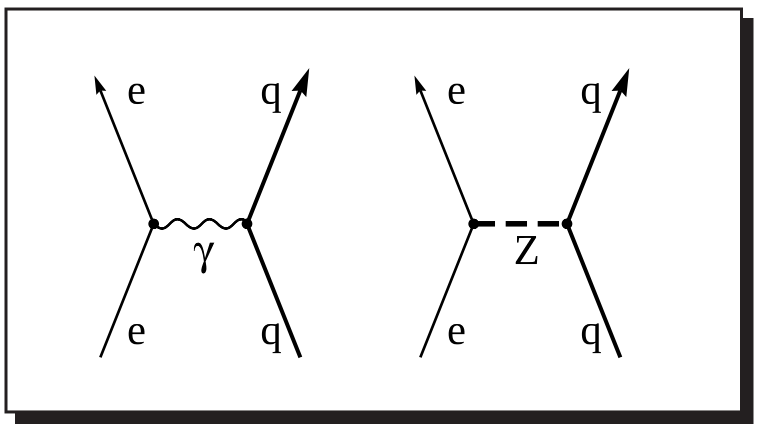
Mechanism: Photon – Z interference



Consequence: Laporte's rule[1] is violated!

$$\phi_{lj} \longrightarrow \phi_{lj} + \epsilon\, \phi_{l\pm 1 j}$$

[1] O. Laporte, Z. Physik **23** 135 (1924).

Background

Standard electroweak model[2]

$$H_{\mathsf{PV}} = \frac{G}{\sqrt{2}}\Big[\bar{e}\gamma_\mu\gamma_5 e\left(c_{1u}\,\bar{u}\gamma_\mu u + c_{1d}\,\bar{d}\gamma_\mu d + \cdots\right) + \bar{e}\gamma_\mu e\left(c_{2u}\,\bar{u}\gamma_\mu\gamma_5 u + c_{2d}\,\bar{d}\gamma_\mu\gamma_5 d + \cdots\right)\Big]$$

where $\cdots$ corresponds to $q = s, t, b, c$

$$c_{1u} = -1/2 + 4/3s^2$$
$$c_{1d} = 1/2 - 2/3s^2$$
$$c_{2u} = -1/2\left(1 - 4s^2\right)$$
$$c_{2d} = 1/2\left(1 - 4s^2\right)$$

$s^2 = \sin^2\theta_W$

[2] W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 170

Nonrelativistic Quarks

Contribution of vector nucleon current:

$$(\bar{u}\gamma^\mu u) \to \phi_u^\dagger \phi_u \, \delta_{\mu 0} \qquad \left(\bar{d}\gamma^\mu d\right) \to \phi_d^\dagger \phi_d \, \delta_{\mu 0}$$

where ϕ_u and ϕ_d are nonrelativistic field operators. From this we extract an "effective" Hamiltonian to be used in the electron sector

$$\boxed{H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 \, Q_W \, \rho(r)}$$

where $\rho(r)$ is a nuclear density ($\sim$ neutron density) normalized to 1, and (omitting radiative corrections)

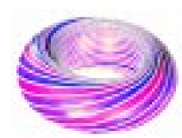
$$\begin{aligned} Q_W &= 2[(2Z+N)c_{1u} + (Z+2N)c_{1d}] \\ &= -N + Z\,(1 - 4\,s^2) \approx -N \end{aligned}$$

Nuclear Spin Dependent Terms

For a nucleus with a single unpaired nucleon, the axial current gives:

$$H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} c_{2N} \frac{\kappa - 1/2}{I(I+1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

where $c_{2N} = c_{2p}$ or c_{2n}, $\kappa = \mp(I + 1/2)$ for $I = L \pm 1/2$. This term is $\sim 1/A$ as large as $H_{\text{eff}}^{(1)}$.

PNC in nucleus $\Rightarrow$ nuclear anapole: 

$$H_{\text{eff}}^{(a)} = \frac{G}{\sqrt{2}} K_a \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

Spin-Dependent Interference Term

The action of $H_{\text{hyperfine}} \times H^{(1)}_{\text{eff}}$ gives yet another nuclear spin-dependent correction

$$H^{(Q_W)}_{\text{eff}} = \frac{G}{\sqrt{2}} K_{Q_W} \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I}\, \rho_N(r)$$

For ^{133}Cs, this gives[3]

$$K_{Q_W}(^{133}\text{Cs}) \approx 0.0307$$

[3]C. Bouchiat and C. A. Piketty, Z. Phys. C 49, 91 (1991); Phys. Lett. B 269, 195 (1991).

Experiments

An electric-dipole transition matrix element between states of the same parity is measured. In cesium, for example, the matrix element

$$E_{\mathsf{PNC}} = \langle 7s|ez|6s\rangle \propto Q_W \times \text{“Structure Factor”}$$

is measured.

- Interference with a Stark-induced PV matrix element:

  $$\boxed{R_{\mathsf{Stark}} = \mathrm{Im}\left(E_{\mathsf{PNC}}\right)/\beta}$$

  where β is the vector polarizability of the transition.

- Optical Rotation: $n_- \neq n_+$

  $$\boxed{R_{\mathsf{OR}} = \mathrm{Im}\left(E_{\mathsf{PNC}}\right)/M1}$$

  where M1 is the magnetic-dipole transition matrix element.

Optical Rotation Experiments

Experimental values of R_{OR} over past decade			
Element	Transition	Group	$10^8 \times R_{\mathrm{OR}}$
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Oxford† (95)	-15.33(45)
^{205}Tl	$^2P_{1/2} - {}^2P_{3/2}$	Seattle (95)	-14.68(20)
^{208}Pb	$^3P_0 - {}^3P_1$	Oxford (94)	-9.80(33)
^{208}Pb	$^3P_0 - {}^3P_1$	Seattle (95)	-9.86(12)
^{209}Bi	$^4S_{3/2} - {}^2D_{3/2}$	Oxford (91)	-10.12(20)

† using a remeasured E2/M1 ratio

Stark-Induced Transition Experiments

Evolving values of $R = \text{Im}\left(E_{\text{PNC}}\right)/\beta$ (mV/cm) for cesium

Element	Transition	Group	R_{4-3}	R_{3-4}
^{133}Cs	$6s_{1/2} - 7s_{1/2}$	Paris (1984)	-1.5(2)	-1.5(2)
^{133}Cs	$6s_{1/2} - 7s_{1/2}$	Boulder (1988)	-1.64(5)	-1.51(5)
^{133}Cs	$6s_{1/2} - 7s_{1/2}$	Boulder (1997)	-1.635(8)	-1.558(8)

Other Cases

Many other cases have also been considered and studied over the past decade. Here is a (partial) list:

Atom	Transition	Group
Tl	$6P_{1/2} \rightarrow 7P_{1/2}$	Berkeley
Ba^{+}	$6S_{1/2} \rightarrow 5D_{3/2}$	Seattle
Fr	$7S_{1/2} \rightarrow 8S_{1/2}$	Stony Brook
Yb	$(6s^2)\,{}^1S_0 \rightarrow (6s5d)\,{}^3D_1$	Berkeley
Dy	$(4f^{10}5d6s)[10] \rightarrow (4f^9 5d^2 6s)[10]$	Berkeley
Sm	$(4f^6 6s^2)\,{}^7F_J \rightarrow (4f^6 6s^2)\,{}^5D_{J'}$	Oxford

Calculations

Reliable calculations of the PNC matrix element are based on "all-order" methods in MBPT. These include PTSCI calculations[4] in which dominant MBPT diagrams are summed to all orders and SD Coupled-Cluster[5] calculations.

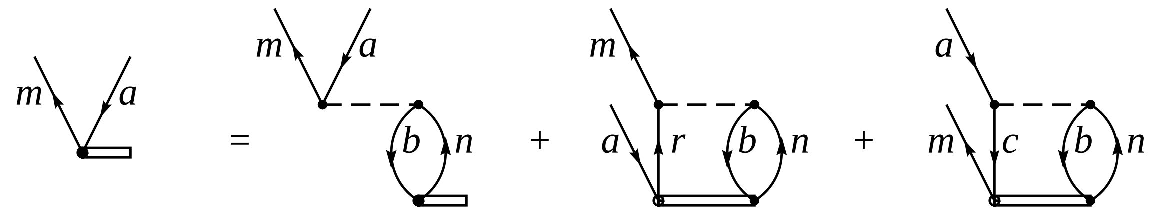
For the $6s \rightarrow 7s$ transition in atomic cesium:

$$\langle 7s|ez|6s\rangle = \sum_n \left\{ \frac{\langle 7s|ez|np_{1/2}\rangle\langle np_{1/2}|H^{(1)}|6s\rangle}{E_{np} - E_{6s}} + \frac{\langle 7s|H^{(1)}|np_{1/2}\rangle\langle np_{1/2}|ez|6s\rangle}{E_{np} - E_{7s}} \right\}$$

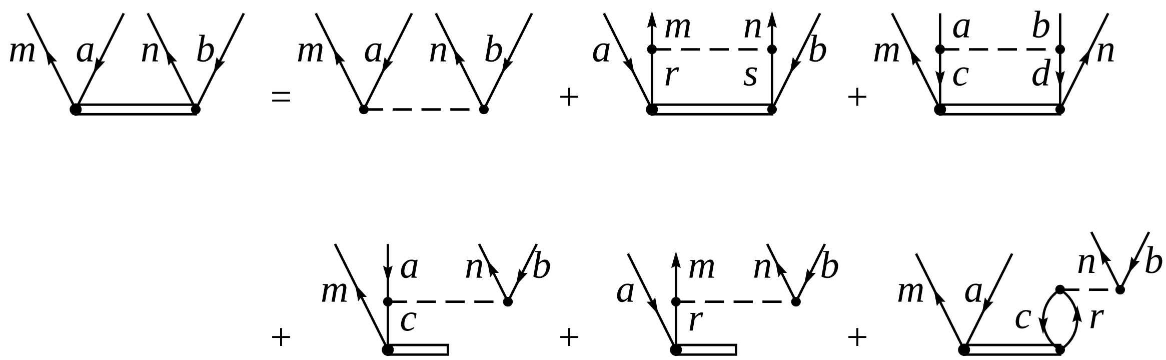
- $\sum_{n=6}^{9}$ using SDCC (>95%)
- $\sum_{n=10}^{\infty}$ weak RPA level (<5%)

[4] V. A. Dzuba et al., arXiv:hep-ph/0204134 (2002)
[5] S. A. Blundell et al., Phys. Rev. D**45**, 1602 (1992)

Brueckner-Goldstone Diagrams for the SDCC Equations



+ *exchange terms*



+ *exchange terms*

Breakdown of Dominant Contribution: ^{133}Cs

n	$\langle 7s \vert ez \vert np \rangle$	$\langle np \vert H^{(1)} \vert 6s \rangle$	$E_{6s} - E_{np}$	Term
6	1.7291	-0.0562	-0.05093	1.908
7	4.2003	0.0319	-0.09917	-1.352
8	0.3815	0.0215	-0.11714	-0.070
9	0.1532	0.0162	-0.12592	-0.020
n	$\langle 7s \vert H^{(1)} \vert np \rangle$	$\langle np \vert ez \vert 6s \rangle$	$E_{7s} - E_{np}$	Term
6	-1.8411	0.0272	0.03352	-1.493
7	0.1143	-0.0154	-0.01472	0.120
8	0.0319	-0.0104	-0.03269	0.010
9	0.0171	-0.0078	-0.04147	0.003
Total				-0.893

Units of $H^{(1)}$: $i(-Q_W/N) \times 10^{-11}$

Residuals

$\sum_{n=6}^{9}$	All-Order	-0.893(7)
$\sum_{n=10}^{\infty}$	RPA	-0.018(5)
Autoionizing	HF	0.002(2)
Total		-0.909(9)

n.b. Most recent PTSCI value[6] is -0.908(5)

What's missing in this calculation?

1. Breit Interaction
2. Higher-Order in αZ QED corrections
3. Nuclear "Skin" Effects

[6] V. A. Dzuba et al., arXiv:hep-ph/0204134 (2002)

Breit Correction[7]

Mixed-Parity Calculation:

$$\left(h_0 + V^{\mathrm{HF}} + \hat{\Sigma} - \epsilon_v\right) \tilde{\psi}_v = -H^{(1)}\psi_v - \delta V_W^{\mathrm{HF}} \psi_v$$

$$E_{\mathrm{PNC}} = \langle \psi_{7s} | ez + \delta V_{\mathrm{z}}^{\mathrm{HF}} | \tilde{\psi}_{6s} \rangle + \langle \tilde{\psi}_{7s} | ez + \delta V_{\mathrm{z}}^{\mathrm{HF}} | \psi_{6s} \rangle$$

Type	$\langle 7s \vert ez + \delta V_{\mathrm{z}}^{\mathrm{HF}} \vert \widetilde{6s} \rangle$	$\langle \widetilde{7s} \vert ez + \delta V_{\mathrm{z}}^{\mathrm{HF}} \vert 6s \rangle$	Sum
Coul	0.43942	-1.33397	-0.89456
+ Breit	0.43680	-1.32609	-0.88929
Δ%	-0.60%	-0.59%	-0.59%

[7] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).

Electroweak Radiative Corrections: ^{133}Cs

Using $s^2 = 0.23114(17)$, one finds

$Q_W^{\mathsf{SM}} = -73.86 \pm 0.03$ (without radiative corrections)

Including electroweak radiative corrections,[8] the coupling constants can be written

$$c_{1u} = \rho'_{\mathrm{PV}} \left(-\frac{1}{2} + \frac{4}{3} \kappa'_{\mathrm{PV}} \, s^2 \right) + \lambda_{1u}$$

$$c_{1d} = \rho'_{\mathrm{PV}} \left(\frac{1}{2} - \frac{2}{3} \kappa'_{\mathrm{PV}} \, s^2 \right) + \lambda_{1d}$$

For Atomic PNC:

$$\begin{aligned} \rho'_{\mathrm{PV}} &= 0.9878 \\ \kappa'_{\mathrm{PV}} &= 1.0026 \\ \lambda_{1u} &= -1.9 \times 10^{-5} \\ \lambda_{1d} &= 3.7 \times 10^{-5} \end{aligned}$$

$Q_W^{\mathsf{SM}} = -73.09 \pm 0.03$ (with radiative corrections)

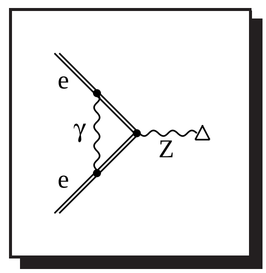
[8] D. E. Groom et al., Euro. Phys. J. C **15**, 1 (2000)

αZ-dependent terms

Perturbation Theory:[9]

$$\rho'_{\rm PV} = 1 - \frac{\alpha}{2\pi} + \frac{\alpha(m_Z)}{2\pi}\left\{\frac{3}{8s^2}\frac{m_t^2}{m_W^2} + \frac{3}{8s^4}\ln c^2 - \frac{7}{8s^2} + \frac{3\xi}{8s^2}\left(\frac{\ln(c^2/\xi)}{c^2-\xi} + \frac{1}{c^2}\frac{\ln\xi}{1-\xi}\right) - \frac{1}{s^2} + \cdots\right\}$$

The term $-\frac{\alpha}{2\pi}$ is an electromagnetic vertex correction that should be evaluated using bound-state electron propagators.



Assume that binding corrections modify

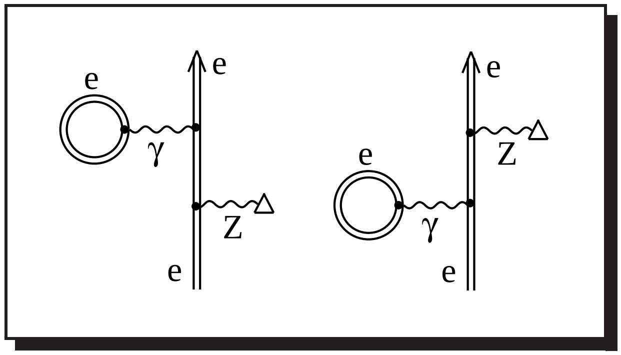
$$-\frac{\alpha}{2\pi} \rightarrow -\frac{\alpha}{2\pi} \pm \frac{\alpha}{2\pi},$$

and we find

$$Q_W^{\rm SM} = -73.09 \rightarrow -73.09 \pm 0.08$$

[9] W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 182

Vacuum-Polarization Corrections[10]



The double line represents the electron in the field of the nucleus. To leading order in powers of $Z\alpha$

$$-\frac{Z}{r} \Rightarrow -\frac{Z}{r} + \delta V$$

where δV given by the Uehling potential[11]:

$$\delta V(r) = -\frac{2\alpha Z}{3\pi r}\int_1^\infty dt\sqrt{t^2-1}\left(\frac{1}{t^2}+\frac{1}{2t^4}\right)e^{-2ctr}$$

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[10] A. I. Milstein and O. P. Sushkov, arXiv:hep-ph/0109257

[11] E. A. Uehling, Phys. Rev. **48**, 55 (1935), E. H. Wichmann and N. H. Kroll, Phys. Rev. **101**, 843 (1956)

Vacuum-Polarization Corrections: ^{133}Cs

Mixed-Parity RPA Calculation:

$$\left(h_0 + V^{\rm HF} - \epsilon_v^{\rm HF}\right) \delta\phi_v^{\rm RPA} = -\left[h_{\rm PNC} + V_{\rm PNC}^{\rm HF}\right] \phi_v^{\rm HF}$$

$$E_{\rm PNC} = \langle \phi_{7s}^{\rm HF} | ez | \delta\phi_{6s}^{\rm RPA} \rangle + \langle \delta\phi_{7s}^{\rm RPA} | ez | \phi_{6s}^{\rm HF} \rangle$$

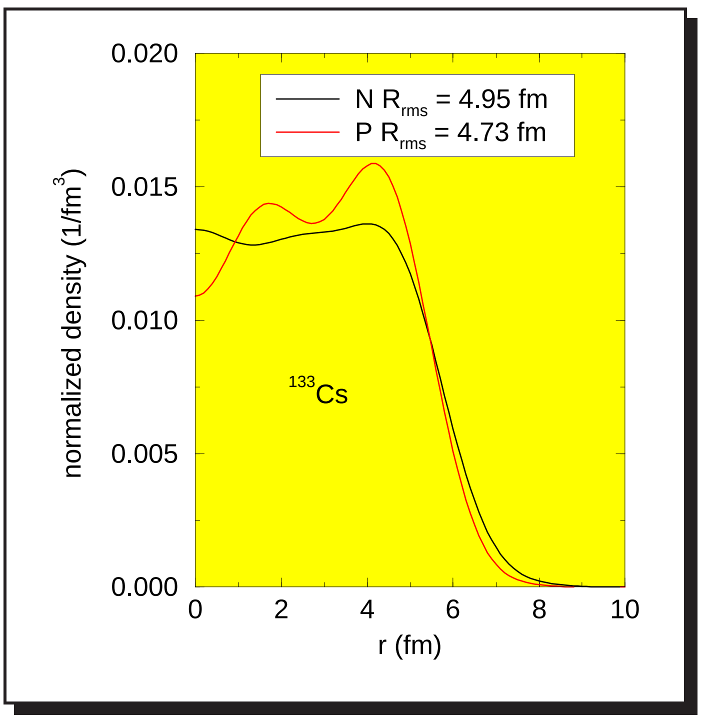
Type	$\langle \phi_{7s} \| ez \| \delta\phi_{6s} \rangle$	$\langle \delta\phi_{7s} \| ez \| \phi_{6s} \rangle$	$E_{\rm PNC}$
RPA	-3.4570	1.2726	-0.9269
RPA+ δV	-3.4712	1.2778	-0.9307
Δ (%)	0.41	0.41	0.41

VP corrections increase the PNC amplitude by 0.41%.

Nuclear "skin" correction

Neutrons are primarily the source of the vector atomic PNC interaction, but proton densities are used in calculations of atomic PNC.

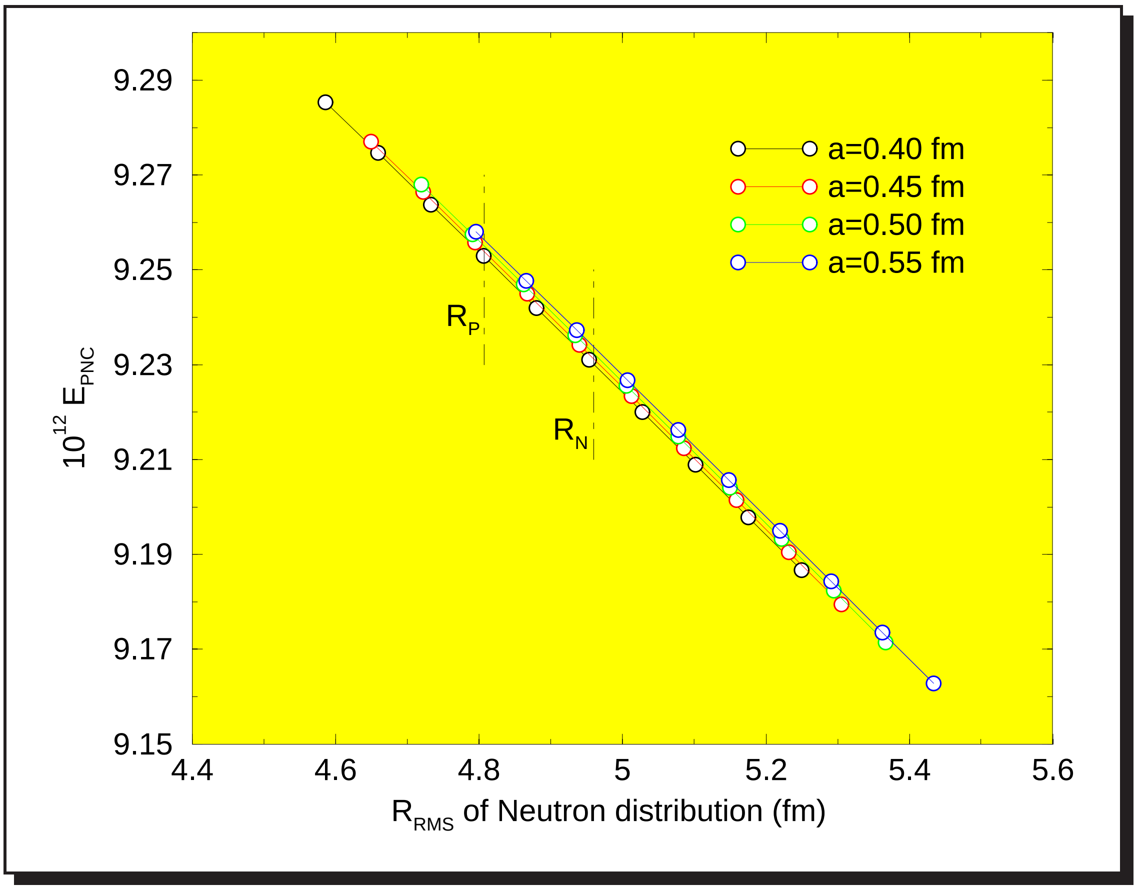
Replacing proton densities by neutron densities leads to "skin" corrections proportional to $\delta\rho = \rho_n - \rho_p$.



Proton and Neutron distributions[12] for ^{133}Cs.

[12] D. Vretenar et al., Phys. Rev. C **62**, 045502 (2000).

RPA Calculations



Conclusion:[13] [14] [15] The "skin" effect decreases the size of E_{PNC} by 0.23%

[13] A. Dervianko, arXiv:physics/0108033 (2001)
[14] S. J. Pollock and M. C. Welliver, Phys. Lett. B **464**, 177 (1999)
[15] J. James and P. G. H. Sandars, J. Phys. B**32**, 3295 (1999)

Conclusion (for ^{133}Cs)

Including the (Br+VP+Skin) corrections,

$$E_{\mathrm{PNC}} = -0.9053 \ \pm 0.0037 \ iea_0 \times 10^{-11}(-Q_W/N)$$

where we take the 0.4% error estimate and the value of β from Bennett and Wieman.[16] Combining these with the experimental value [17] of E_{PNC}/β, leads to an experimental value for the weak charge

$$Q_W^{\mathrm{expt}}(^{133}\mathrm{Cs}) = -72.15 \pm (0.26)_{\mathrm{expt}} \pm (0.34)_{\mathrm{theor}}$$

This differs by 2.2 σ from the standard-model value[18]

$$Q_W^{\mathrm{SM}}(^{133}\mathrm{Cs}) = -73.09 \pm 0.03$$

[16] S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999); **82**, 4153 (1999); **83**, 889 (1999)

[17] C. S. Wood *et al.*, Science **275**, 1759 (1997)

[18] D. E. Groom et al., Euro. Phys. J. C **15**, 1 (2000).