Many-Body Methods Applied to Parity Nonconserving Transitions in Atoms: The Weak Charge and Anapole Moment of $^{133}\text{Cs}$

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1) Weak charge $Q_W$ of $^{133}\text{Cs}$ provides a test of the Standard Model.

2) First (only) observation of an anapole moment was in $^{133}\text{Cs}$.

3) Accurate atomic many-body calculations required.

Collaborators: M. S. Safronova, (Delaware) and U. I. Safronova (Nevada)
A consequence of Z exchange is that Laporte’s rule “Electric dipole transitions take place only between states of opposite parity” is violated.
Otto Laporte (1902-1971) discovered the law of parity conservation in physics. He divided states of the iron spectrum into two classes, even and odd, and found that no radiative transitions occurred between like states.\footnote{O. Laporte, Z. Physik 23 135 (1924).}
Z Exchange in the Standard Model

\[ H_{\text{PV}} = \frac{G}{\sqrt{2}} \left[ \bar{e} \gamma_\mu \gamma_5 e \left( c_{1u} \bar{u} \gamma_\mu u + c_{1d} \bar{d} \gamma_\mu d + \cdots \right) + \bar{e} \gamma_\mu e \left( c_{2u} \bar{u} \gamma_\mu \gamma_5 u + c_{2d} \bar{d} \gamma_\mu \gamma_5 d + \cdots \right) \right] \]

where \( \cdots = t, b, s, c \)

\[
\begin{align*}
c_{1u} &= -\frac{1}{2} + \frac{4}{3} \sin^2 \theta_W \\
c_{2u} &= -\frac{1}{2} \left( 1 - 4 \sin^2 \theta_W \right) \\
c_{1d} &= \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \\
c_{2d} &= \frac{1}{2} \left( 1 - 4 \sin^2 \theta_W \right)
\end{align*}
\]

Electron Axial-Vector – Nucleon Vector

Contribution of *coherent* vector nucleon current:

\[ H^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 Q_W \rho(r) \]

where \( \rho(r) \) is a nuclear density (\( \sim \) neutron density) and

\[
Q_W = 2[(2Z + N)c_{1u} + (Z + 2N)c_{1d}]
\]

\[
= -N + Z (1 - 4 \sin^2 \theta_W)
\]

\[ \sim -N \]
Electron Vector – Nucleon Axial-Vector

Contribution of vector axial-vector nucleon current:

\[
H^{(2)} = -\frac{G}{\sqrt{2}} \alpha \cdot \left[ c_{2p} \langle \phi_p^\dagger \sigma \phi_p \rangle + c_{2n} \langle \phi_n^\dagger \sigma \phi_n \rangle \right]
\]

where \(\langle \cdots \rangle\) designates nuclear matrix elements.

\[c_{2p} \sim 1.25 \times c_{2u} = -0.068\]
\[c_{2n} \sim 1.25 \times c_{2d} = 0.068\]
Shell Model Estimates

\[ H^{(2)} = \frac{G}{\sqrt{2}} \kappa_2 \alpha \cdot I \rho(r) \]

\( \kappa_2 \) from “Extreme” Shell Model and from Recent Calculations.\(^3\)

<table>
<thead>
<tr>
<th>Element</th>
<th>A</th>
<th>State</th>
<th>( \kappa_2 )</th>
<th>Ref. [3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>39</td>
<td>1(d_{3/2} ) (p)</td>
<td>0.0272</td>
<td></td>
</tr>
<tr>
<td>Cs</td>
<td>133</td>
<td>1(g_{7/2} ) (p)</td>
<td>0.0151</td>
<td>0.0140</td>
</tr>
<tr>
<td>Ba</td>
<td>135</td>
<td>2(d_{3/2} ) (n)</td>
<td>-0.0272</td>
<td></td>
</tr>
<tr>
<td>Tl</td>
<td>205</td>
<td>3(s_{1/2} ) (p)</td>
<td>-0.136</td>
<td>-0.127</td>
</tr>
<tr>
<td>Fr</td>
<td>209</td>
<td>1(h_{9/2} ) (p)</td>
<td>0.0124</td>
<td></td>
</tr>
</tbody>
</table>

Nuclear Anapole Moment

PNC in nucleus $\Rightarrow$ nuclear anapole:

$$A = a \delta(r)$$

$$a = -\pi \int d^3r \, r^2 \, j(r) = \frac{1}{e} \frac{G}{\sqrt{2}} \kappa_a I$$

$$H^{(a)} = e \alpha \cdot A \rightarrow \frac{G}{\sqrt{2}} \kappa_a \alpha \cdot I \rho(r)$$

Early estimates$^4$ for $^{133}$Cs gave $\kappa_a = 0.063 - 0.084$. Recent estimates given in$^5$


Spin-Dependent Interference Term

According to Flambaum and Khriplovich\cite{Flambaum1985} and Bronchiat and Piketty\cite{Bronchiat1991}, interference between the hyperfine interaction $H_{hf}$ and $H^{(1)}$ gives another nuclear spin-dependent correction of the form

$$H^{(hf)} = \frac{G}{\sqrt{2}} \kappa_{hf} \alpha \cdot I \rho(r)$$

For $^{133}$Cs: $\kappa_{hf} = 0.0078$
For $^{205}$Tl: $\kappa_{hf} = 0.044$

$$\kappa_{hf} \sim \frac{1}{2} \kappa_2$$

\footnote{V. V. Flambaum and I. B. Khriplovich, Sov. Phys. JETP 62, 872 (1985).}
Optical Rotation Experiments

Aim is to measure $E_{\text{PNC}} = \langle f | z | i \rangle \propto Q_W$:

The plane of polarization of a linearly polarized laser beam passing through a medium with $n_+ \neq n_-$ is rotated. The rotation angle $\phi \propto R_\phi = \text{Im} (E_{\text{PNC}}) / M_1$. 
Optical Rotation Experiments-II

$$R_\phi = \text{Im} (E_{\text{PNC}}) / M1$$

Measured values of $R_\phi$

<table>
<thead>
<tr>
<th>Element</th>
<th>Transition</th>
<th>Group</th>
<th>$10^8 \times R_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{205}\text{Tl}$</td>
<td>$^2P_{1/2} - ^2P_{3/2}$</td>
<td>Oxford (95)</td>
<td>-15.33(45)</td>
</tr>
<tr>
<td>$^{205}\text{Tl}$</td>
<td>$^2P_{1/2} - ^2P_{3/2}$</td>
<td>Seattle (95)</td>
<td>-14.68(20)</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>$^3P_0 - ^3P_1$</td>
<td>Oxford (94)</td>
<td>-9.80(33)</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>$^3P_0 - ^3P_1$</td>
<td>Seattle (95)</td>
<td>-9.86(12)</td>
</tr>
<tr>
<td>$^{209}\text{Bi}$</td>
<td>$^4S_{3/2} - ^2D_{3/2}$</td>
<td>Oxford (91)</td>
<td>-10.12(20)</td>
</tr>
</tbody>
</table>
Stark-Interference Experiment

Boulder PNC apparatus: A beam of cesium atoms is optically pumped by diode laser beams, then passes through a region of perpendicular electric and magnetic fields where a green laser excites the transition from the 6S to the 7S state. The excitations are detected by observing the florescence (induced by another laser beam) with a photo-diode.
### Stark-Interference Experiments II

<table>
<thead>
<tr>
<th>Transition</th>
<th>Group</th>
<th>$R_{4\rightarrow3}$</th>
<th>$R_{3\rightarrow4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6s_{1/2} \rightarrow 7s_{1/2}$</td>
<td>Paris (1984)</td>
<td>-1.5(2)</td>
<td>-1.5(2)</td>
</tr>
<tr>
<td>$6s_{1/2} \rightarrow 7s_{1/2}$</td>
<td>Boulder (1988)</td>
<td>-1.64(5)</td>
<td>-1.51(5)</td>
</tr>
<tr>
<td>$6s_{1/2} \rightarrow 7s_{1/2}$</td>
<td>Boulder (1997)</td>
<td>-1.635(8)</td>
<td>-1.558(8)</td>
</tr>
</tbody>
</table>

The vector current contribution from the last row is

$$R_{\text{Stark}} = -1.593 \pm 0.006$$

$$\text{Im}[E_{\text{PNC}}(6s \rightarrow 7s) \times 10^{11}] = -0.8376 \pm (0.0031)_{\text{exp}} \pm (0.0021)_{\text{th}}$$
Other Experiments

<table>
<thead>
<tr>
<th>Element</th>
<th>Transition</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fr</td>
<td>$7S_{1/2} \rightarrow 8S_{1/2}$</td>
<td>Stony Brook</td>
</tr>
<tr>
<td>Fr</td>
<td>$7S_{1/2}[F = 4] \rightarrow 7S_{1/2}[F = 5]$</td>
<td>Stony Brook</td>
</tr>
<tr>
<td>Yb</td>
<td>$(6s^2)^1S_0 \rightarrow (6s5d)^3D_1$</td>
<td>Berkeley</td>
</tr>
<tr>
<td>Yb</td>
<td>$(6s6p)^3P_0 \rightarrow (6s6p)^3P_1$</td>
<td>Berkeley</td>
</tr>
<tr>
<td>Ba$^+$</td>
<td>$6S_{1/2} \rightarrow 5D_{3/2}$</td>
<td>Seattle</td>
</tr>
<tr>
<td>Dy</td>
<td>$(4f^{10}5d6s)[10] \rightarrow (4f^95d^26s)[10]$</td>
<td>Berkeley</td>
</tr>
<tr>
<td>Sm</td>
<td>$(4f^66s^2)^7F_J \rightarrow (4f^66s^2)^5D_{J'}$</td>
<td>Oxford</td>
</tr>
</tbody>
</table>
Calculations of the $6s \rightarrow 7s$ Amplitude in Cs

Units: $i(-Q_W/N) \times 10^{-11} e a_0$

- SD$^8$ -0.909 (4)
- CI+MBPT$^9$ -0.905
- PTSCI$^{10}$ -0.908 (5)
- PNC-CI$^{11}$ -0.904
- SDCC (preliminary)$^{12}$ -0.907

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$^8$S. A. Blundell et al., PRD 45, 1602 (1992).
$^{11}$V. M. Shabaev et al., PRA 72 (2005)
$^{12}$B. P. Das et al., THEOCHEM 768, 141 (2006)
SD Calculation of PNC amplitude

\[ E_{\text{PNC}} = \sum \frac{\langle 7s | D | np \rangle \langle np | H^{(1)} | 6s \rangle}{E_{6s} - E_{np}} + \sum \frac{\langle 7s | H^{(1)} | np \rangle \langle np | D | 6s \rangle}{E_{7s} - E_{np}} \]

“Weak” RPA gives \( E_{\text{PNC}} \) accurate to about 3%. Therefore, we organize calculation as follows:

- \( n = 6 - 9 \) valence states: evaluate matrix elements using SD wave functions (99%)
- \( n = 1 - 5 \) core states and \( n > 10 \): evaluate using “weak” RPA amplitudes (1%)
## Contributions to PNC Amplitude

Contributions to $E_{\text{PNC}}$ in units $-iea_0Q_\text{W}/N$.

| n   | $\langle 7s | D | np \rangle$ | $\langle np | H^{(1)} | 6s \rangle$ | $E_{6s} - E_{np}$ | Contrib. |
|-----|-----------------|-----------------|-----------------|----------|
| 6   | 1.7291          | -0.0562         | -0.05093        | 1.908    |
| 7   | 4.2003          | 0.0319          | -0.09917        | -1.352   |
| 8   | 0.3815          | 0.0215          | -0.11714        | -0.070   |
| 9   | 0.1532          | 0.0162          | -0.12592        | -0.020   |

| n   | $\langle 7s | H^{(1)} | np \rangle$ | $\langle np | D | 6s \rangle$ | $E_{7s} - E_{np}$ | Contrib. |
|-----|-----------------|-----------------|-----------------|----------|
| 6   | -1.8411         | 0.0272          | 0.03352         | -1.493   |
| 7   | 0.1143          | -0.0154         | -0.01472        | 0.120    |
| 8   | 0.0319          | -0.0104         | -0.03269        | 0.010    |
| 9   | 0.0171          | -0.0078         | -0.04147        | 0.003    |

$n = 6 - 9$ \hspace{1cm} -0.894(4)

RPA part \hspace{1cm} -0.015(1)

Total \hspace{1cm} -0.909(4)
Brueckner-Goldstone Diagrams for the SDCC Equations

\[ m \rightarrow a = \begin{array}{c} \text{exchange terms} \\ \end{array} + \begin{array}{c} m \rightarrow b \rightarrow n + a \rightarrow r \rightarrow b \rightarrow n + m \rightarrow c \rightarrow b \rightarrow n \end{array} \]

\[ m \rightarrow a \rightarrow n \rightarrow b = \begin{array}{c} \text{exchange terms} \\ \end{array} + \begin{array}{c} m \rightarrow a \rightarrow n \rightarrow b + a \rightarrow m \rightarrow n \rightarrow s \rightarrow b + m \rightarrow a \rightarrow c \rightarrow b \rightarrow d \rightarrow n \end{array} \]

+ exchange terms
Analysis of $6s \rightarrow 7s$ Amplitude in $^{133}$Cs

Combining the calculations and the measurements

$$Q_{W}^{\text{exp}}(^{133}\text{Cs}) = -71.91(46)$$

differs with the standard model value

$$Q_{W}^{\text{SM}}(^{133}\text{Cs}) = -73.09(3)$$

by $2.5 \, \sigma$

- Breit interaction -0.6%
- Vacuum Polarization +0.4%
- $\alpha Z$ Vertex Corrections -0.7%
- Nuclear Skin Effect -0.2%

Additional Corrections:
Analysis of $6s \rightarrow 7s$ Amplitude in $^{133}\text{Cs}$

Combining the calculations and the measurements

$$Q_{W}^{\exp}(^{133}\text{Cs}) = -71.91(46) \Rightarrow -72.73(46)$$

differs with the standard model value

$$Q_{W}^{\text{SM}}(^{133}\text{Cs}) = -73.09(3)$$

by $2.5 \sigma \Rightarrow 0.8 \sigma$

Additional Corrections:

- Breit interaction -0.6%
- Vacuum Polarization +0.4%
- $\alpha Z$ Vertex Corrections -0.7%
- Nuclear Skin Effect -0.2%
Angular Momentum Considerations

\[ \langle F \| z \| I \rangle^{(1)} = (-1)^{j_{F} + F_{I} + I + 1} \sqrt{[F_{I}][F_{F}]} \left\{ \begin{array}{ccc} F_{F} & F_{I} & 1 \\ j_{I} & j_{F} & I \end{array} \right\} \]

\[ \times \sum_{n,j_{n}} \left[ \frac{\langle j_{F} \| z \| n \cdot j_{n} \rangle \langle n \cdot j_{n} \| H^{(1)} \| j_{I} \rangle}{E_{I} - E_{n}} + \frac{\langle j_{F} \| H^{(1)} \| n \cdot j_{n} \rangle \langle n \cdot j_{n} \| z \| j_{I} \rangle}{E_{F} - E_{n}} \right] \]

\[ \langle F \| z \| I \rangle^{(2)} = \sqrt{I(I + 1)} \sqrt{[I][F_{I}][F_{F}]} \times \sum_{n,j_{n}} \left[ (-1)^{j_{I} - j_{F} + 1} \left\{ \begin{array}{ccc} F_{F} & F_{I} & 1 \\ j_{n} & j_{F} & I \end{array} \right\} \left\{ \begin{array}{ccc} I & I & 1 \\ j_{n} & j_{I} & F_{I} \end{array} \right\} \right. \]

\[ \times \frac{\langle j_{F} \| z \| n \cdot j_{n} \rangle \langle n \cdot j_{n} \| H^{(2)} \| j_{I} \rangle}{E_{I} - E_{n}} \]

\[ + (-1)^{F_{I} - F_{F} + 1} \left\{ \begin{array}{ccc} F_{F} & F_{I} & 1 \\ j_{I} & j_{n} & I \end{array} \right\} \left\{ \begin{array}{ccc} I & I & 1 \\ j_{n} & j_{F} & F_{F} \end{array} \right\} \]

\[ \times \frac{\langle j_{F} \| H^{(2)} \| n \cdot j_{n} \rangle \langle n \cdot j_{n} \| z \| j_{I} \rangle}{E_{F} - E_{n}} \]
Matrix Element \( (10^{-11}) \) | \( H^{(1)} \) | \( H^{(2)} \) | \( \epsilon_{F'F} \)  \\
\( \langle 7s\ [3]\ |\ |z\ |\ 6s\ [3]\rangle \) | -2.037 | -0.2250 | 0.1105  \\
\( \langle 7s\ [3]\ |\ |z\ |\ 6s\ [4]\rangle \) | -3.528 | -0.7296 | 0.2068  \\
\( \langle 7s\ [4]\ |\ |z\ |\ 6s\ [3]\rangle \) | 3.328 | -0.6430 | -0.1823  \\
\( \langle 7s\ [4]\ |\ |z\ |\ 6s\ [4]\rangle \) | 2.981 | -0.2562 | -0.0859  

\[ E_{\text{exp}}^{\text{PNC}} = E_{\text{PNC}}^{(1)} \left[ \frac{Q_W}{-N} + \kappa \epsilon_{F'F} \right] \]

| \( \beta \left( a_0^3 \right) \) | 27.024(80)  \\
| \( E_{34}^{\text{exp}} / \beta \) (mV/cm) | -1.6349(80)  \\
| \( E_{43}^{\text{exp}} / \beta \) (mV/cm) | -1.5576(77)  \\
| \( E_{34}^{\text{exp}} \) \( (10^{-11}) \) | -0.8592(49)  \\
| \( E_{43}^{\text{exp}} \) \( (10^{-11}) \) | -0.8186(47)  \\
| \( E_V^{\text{exp}} \) \( (10^{-11}) \) | -0.8376(37)  \\
| \( E_{\text{PNC}}^{(1)} \) \( (10^{-11}) \) | -0.9085(45)  \\
| \( Q_W^{\text{exp}} \) | -71.91(46)  \\
| \( \kappa^{\text{exp}} \) | 0.117(16)  

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Weak-Hyperfine Interference

\[ Z_{wv}^{(hf)} = \sum_{i \neq w, j \neq v} \left[ \frac{(H^{(1)})_{wi} z_{ij} (H_{hf})_{jv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_v)} + \frac{(H_{hf})_{wi} z_{ij} (H^{(1)})_{jv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_v)} \right] \]

\[ + \sum_{i \neq v, j \neq v} \left[ \frac{z_{wi} (H^{(1)})_{ij} (H_{hf})_{jv}}{(\epsilon_i - \epsilon_v)(\epsilon_j - \epsilon_v)} + \frac{z_{wi} (H_{hf})_{ij} (H^{(1)})_{jv}}{(\epsilon_i - \epsilon_v)(\epsilon_j - \epsilon_v)} \right] \]

\[ + \sum_{i \neq w, j \neq w} \left[ \frac{(H^{(1)})_{wj} (H_{hf})_{ji} z_{iv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_w)} + \frac{(H_{hf})_{wj} (H^{(1)})_{ji} z_{iv}}{(\epsilon_i - \epsilon_w)(\epsilon_j - \epsilon_w)} \right] \]

\[ - \sum_{i \neq v} \frac{z_{wi} (H^{(1)})_{iv}}{(\epsilon_i - \epsilon_v)^2} (H_{hf})_{vv} - (H_{hf})_{ww} \sum_{i \neq w} \frac{(H^{(1)})_{wi} z_{iv}}{(\epsilon_i - \epsilon_w)^2} \]
**Analysis of $\kappa_{hf}$ for $^{133}\text{Cs}$**

<table>
<thead>
<tr>
<th>Dipole Matrix Element</th>
<th>$H^{(2)}$</th>
<th>$H^{(1)} \times H_{hf}$</th>
<th>$\sim \kappa_{hf}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 7s\ [3] \parallel z \parallel 6s\ [3] \rangle$</td>
<td>2.249[-12]</td>
<td>1.141[-14]</td>
<td>5.076[-03]</td>
</tr>
<tr>
<td>$\langle 7s\ [3] \parallel z \parallel 6s\ [4] \rangle$</td>
<td>7.299[-12]</td>
<td>3.579[-14]</td>
<td>4.903[-03]</td>
</tr>
<tr>
<td>$\langle 7s\ [4] \parallel z \parallel 6s\ [3] \rangle$</td>
<td>6.432[-12]</td>
<td>3.139[-14]</td>
<td>4.880[-03]</td>
</tr>
<tr>
<td>$\langle 7s\ [4] \parallel z \parallel 6s\ [4] \rangle$</td>
<td>2.560[-12]</td>
<td>1.300[-14]</td>
<td>5.076[-03]</td>
</tr>
</tbody>
</table>

Thus, for the $7s - 6s$ transition in $^{133}\text{Cs}$, we can describe the interference term approximately as $H^{hf} = \kappa_{hf} \alpha \cdot I \rho(r)$ with $\kappa_{hf} = 0.0049$.

- $H^{(2)}$ is sensitive to correlations, Hyperfine term is not.
- Hyperfine term is sensitive to negative-energy states, $H^{(2)}$ is not.
Anapole Moment of $^{133}$Cs

<table>
<thead>
<tr>
<th>Group</th>
<th>$\kappa$</th>
<th>$\kappa_2$</th>
<th>$\kappa_{hf}$</th>
<th>$\kappa_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present</td>
<td>0.117(16)</td>
<td>0.0140$^1$</td>
<td>0.0049</td>
<td>0.098(16)</td>
</tr>
<tr>
<td>Haxton et al.</td>
<td>0.112(16)$^2$</td>
<td>0.0140</td>
<td>0.0078$^3$</td>
<td>0.090(16)</td>
</tr>
<tr>
<td>Flambaum and Murray</td>
<td>0.112(16)$^4$</td>
<td>0.0111$^5$</td>
<td>0.0071$^6$</td>
<td>0.092(16)$^7$</td>
</tr>
<tr>
<td>Bouchiat and Piketty</td>
<td>0.0084</td>
<td>0.0078</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^1$from Haxton et al.

$^2$from Flambaum and Murray

$^3$from Bouchiat and Piketty

$^4$The spin-dependent matrix elements from Kraftmakher are used.

$^5$Shell-model value with $\sin^2\theta_W = 0.23$.

$^6$This value was obtained by scaling the analytical result from Flambaum and Khriplovich ($\kappa_{hf} = 0.0049$) by a factor 1.5.

$^7$Contains a 1.6% correction for finite nuclear size; the raw value is 0.094(16).
Evaluation of the Anapole Moment

The (low-energy) parity nonconserving nucleon-nucleon interaction is conventionally described by a one-meson exchange potential having one strong-interaction vertex \( \{ g_{\pi NN}, g_{\rho}, g_{\omega} \} \) and one weak vertex \( \{ f_{\pi}, h_{\rho}^0, h_{\rho}^1, h_{\rho}^2, h_{\omega}^0, h_{\omega}^1 \} \)\(^{13}\)

Constraints on Weak Coupling Constants

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Microwave Experiments

The nucleon vector current does not contribute to transitions such as $|(6s\ I)\ F\rangle \rightarrow |(6s\ I)\ F'\rangle$ between different hyperfine components of an atomic level. Therefore, measurements of PNC between such levels directly measure the spin-dependent PNC amplitude.$^{14}$

$$D = \langle (jI)F'\|ez\|(jI)F\rangle (i\kappa\ 10^{-12}ea_0)$$

<table>
<thead>
<tr>
<th>Element</th>
<th>A</th>
<th>$nl_j$</th>
<th>$I$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>39</td>
<td>$4s_{1/2}$</td>
<td>3/2</td>
<td>-0.222</td>
</tr>
<tr>
<td>Rb</td>
<td>87</td>
<td>$5s_{1/2}$</td>
<td>3/2</td>
<td>-1.363</td>
</tr>
<tr>
<td>Cs</td>
<td>133</td>
<td>$6s_{1/2}$</td>
<td>7/2</td>
<td>-17.24</td>
</tr>
<tr>
<td>Ba$^+$</td>
<td>135</td>
<td>$6s_{1/2}$</td>
<td>3/2</td>
<td>-6.169</td>
</tr>
<tr>
<td>Tl</td>
<td>205</td>
<td>$6p_{1/2}$</td>
<td>1/2</td>
<td>-30.00</td>
</tr>
<tr>
<td>Fr</td>
<td>211</td>
<td>$7s_{1/2}$</td>
<td>9/2</td>
<td>-237.9</td>
</tr>
</tbody>
</table>

Conclusions

• Measurements of the weak charge in heavy atoms provide important tests of the validity of the electroweak standard model and provide limits on possible extensions.

• Measurements of the nuclear anapole moment provide constraints on nucleon-nucleon weak coupling constants.

• The measurements above must be combined with precise atomic many-body calculations to provide useful new information concerning weak interaction physics.