Thomson Scattering from WDM

Average-Atom Approximation

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The cross section for Thomson scattering of x-rays by warm dense matter is studied within the framework of the average-atom model.

Outline

Average-Atom Model

- Divide plasma into WS cells with a nucleus and Z electrons
- $\left[\frac{p^2}{2} - \frac{Z}{r} + V\right] \psi_a(\boldsymbol{r}) = \epsilon_a \psi_a(\boldsymbol{r})$
- $V(r) = V_{\mathrm{KS}}(n(r), r)$
- $n(r) = n_b(r) + n_c(r)$
- $4\pi r^2 n_b(r) = \sum_{nl} \frac{2(2l+1)}{1+\exp[(\epsilon_{nl}-\mu)/k_B T]} P_{nl}(r)^2$
- $Z = \int_{r<R_{\mathrm{WS}}} n(r)\, d^3 r$
- Number of equations = $N_b + N_l \times N_\epsilon \sim 500$
- Equations are solved self-consistently

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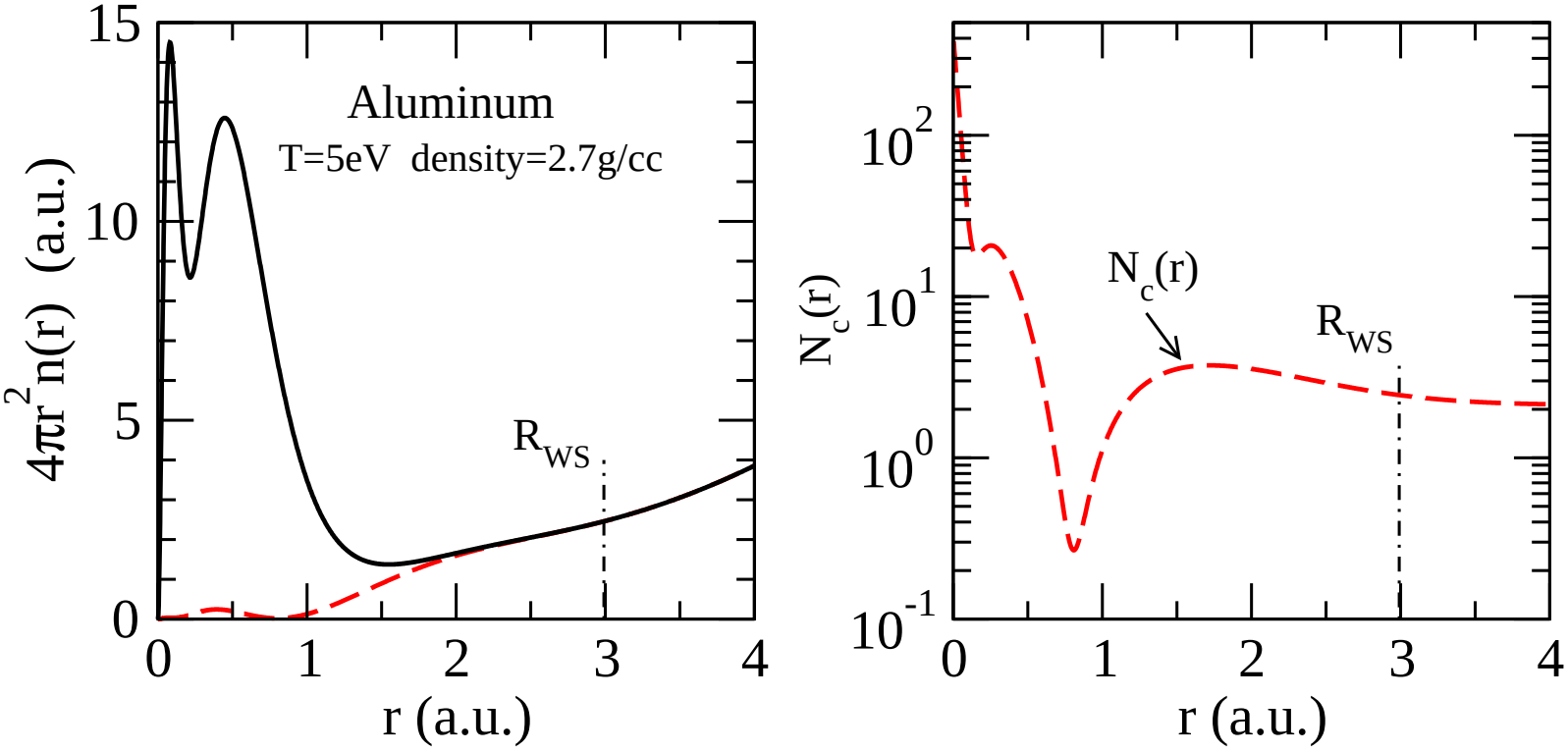
Aluminum Metal



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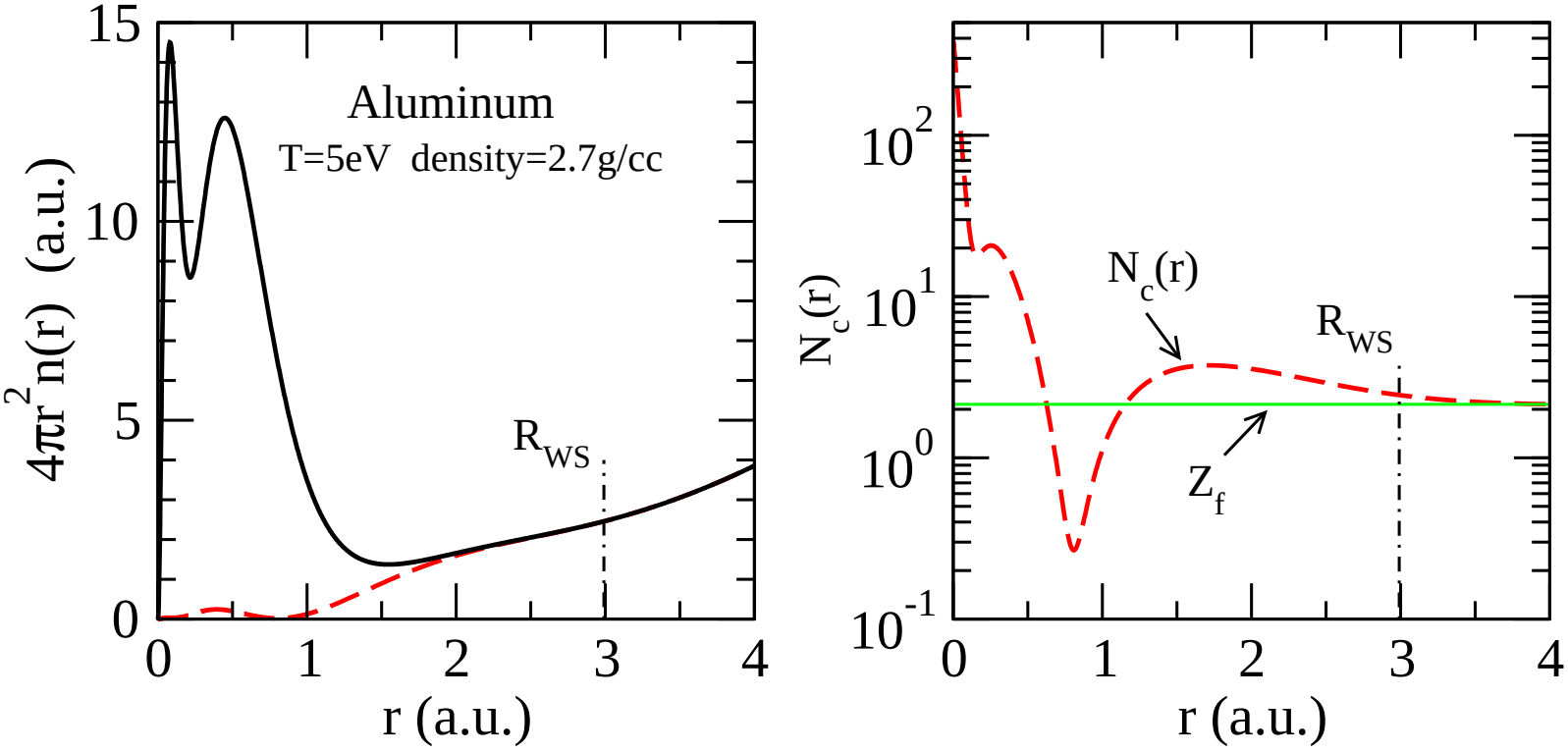
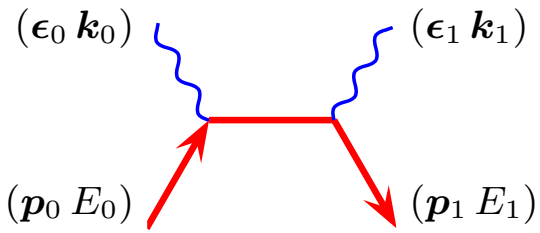


Thomson Scattering



+ Exchange of photons

In nonrelativistic limit, this leads to

$$\frac{d\sigma}{d\omega_1 d\Omega} = |\epsilon_0 \cdot \epsilon_1|^2\, r_0^2\, \frac{\omega_1}{\omega_0}\, S(k,\omega)$$

with $k = |\boldsymbol{k}_0 - \boldsymbol{k}_1|$, $\omega = \omega_0 - \omega_1$, where $S(k,\omega)$ is the *dynamic structure function* of the plasma.

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- Modified by Gregori et al. (2006) to include $T_i \neq T_e$

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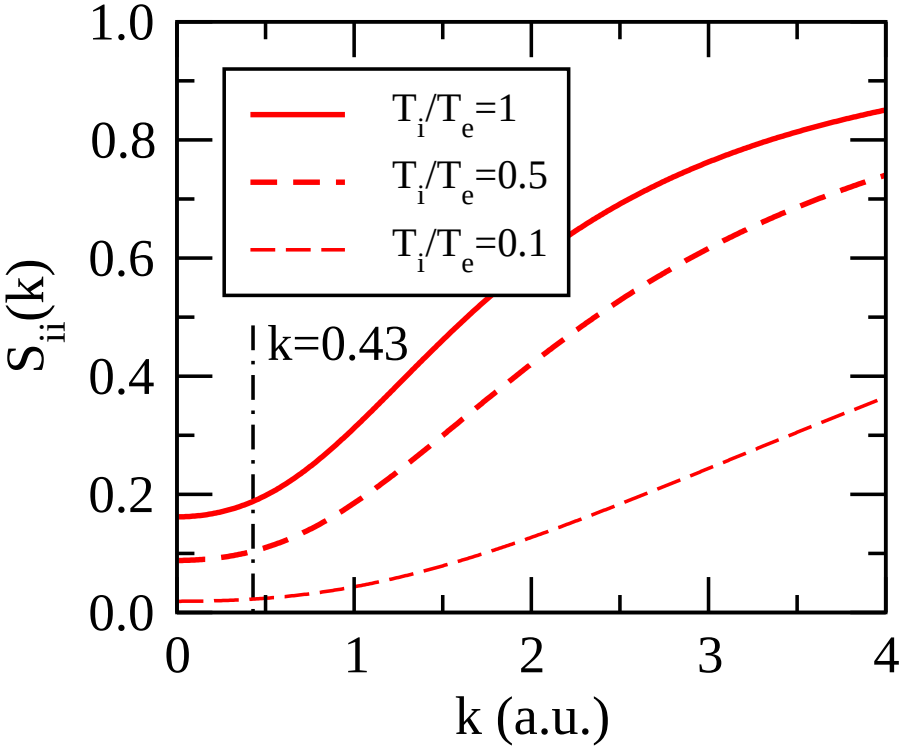
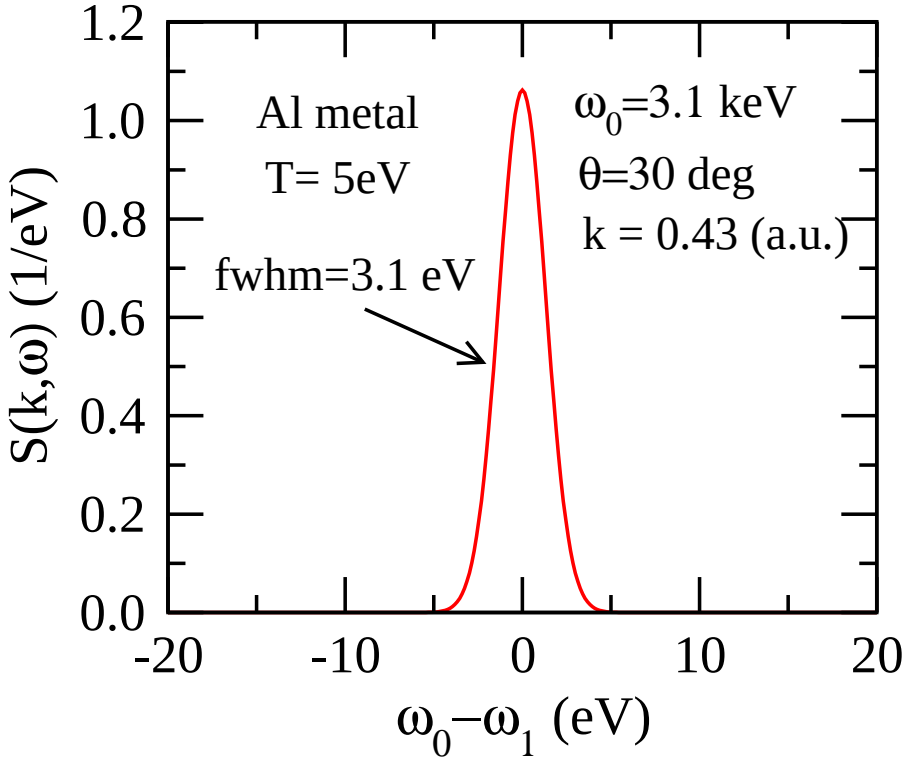




Inelastic Scattering by Free Electrons

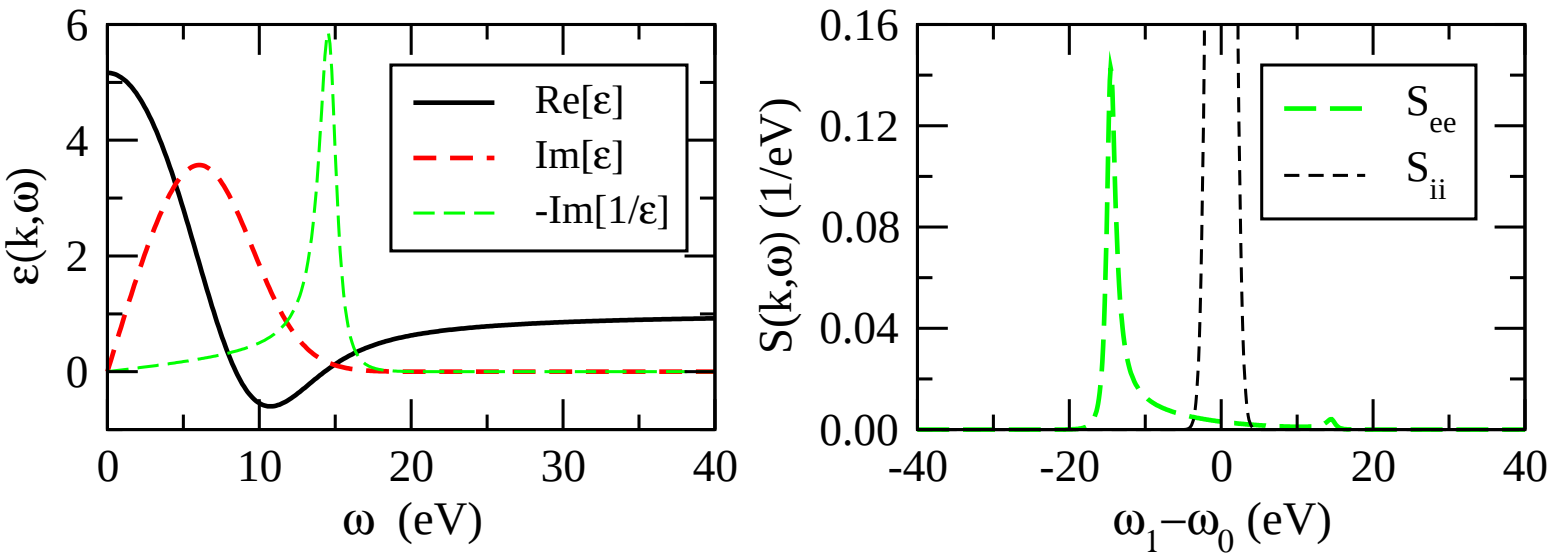
From Gregori et al. (2003):

$$S_{ee}(k,\omega) = -\frac{1}{1-\exp(-\omega/k_B T)}\frac{k^2}{4\pi^2 n_e}\mathrm{Im}\left[\frac{1}{\varepsilon(k,\omega)}\right]$$

Random-phase approximation for dielectric function $\varepsilon(k,\omega)$:

$$\varepsilon(k,\omega) = 1 + \frac{4}{\pi k^2}\int_0^\infty \frac{p^2}{1+\exp[(p^2/2-\mu)/k_B T]}dp$$
$$\times \int_{-1}^{1} d\eta \left[\frac{1}{k^2 - 2pk\eta + 2\omega + i\nu} + \frac{1}{k^2 + 2pk\eta - 2\omega - i\nu}\right].$$

Dielectric Function



Scattering from Al metal, $k_B T = 5$ eV, $\omega_0 = 3.1$ keV, $\theta = 30°$

Inelastic Scattering by Bound Electrons

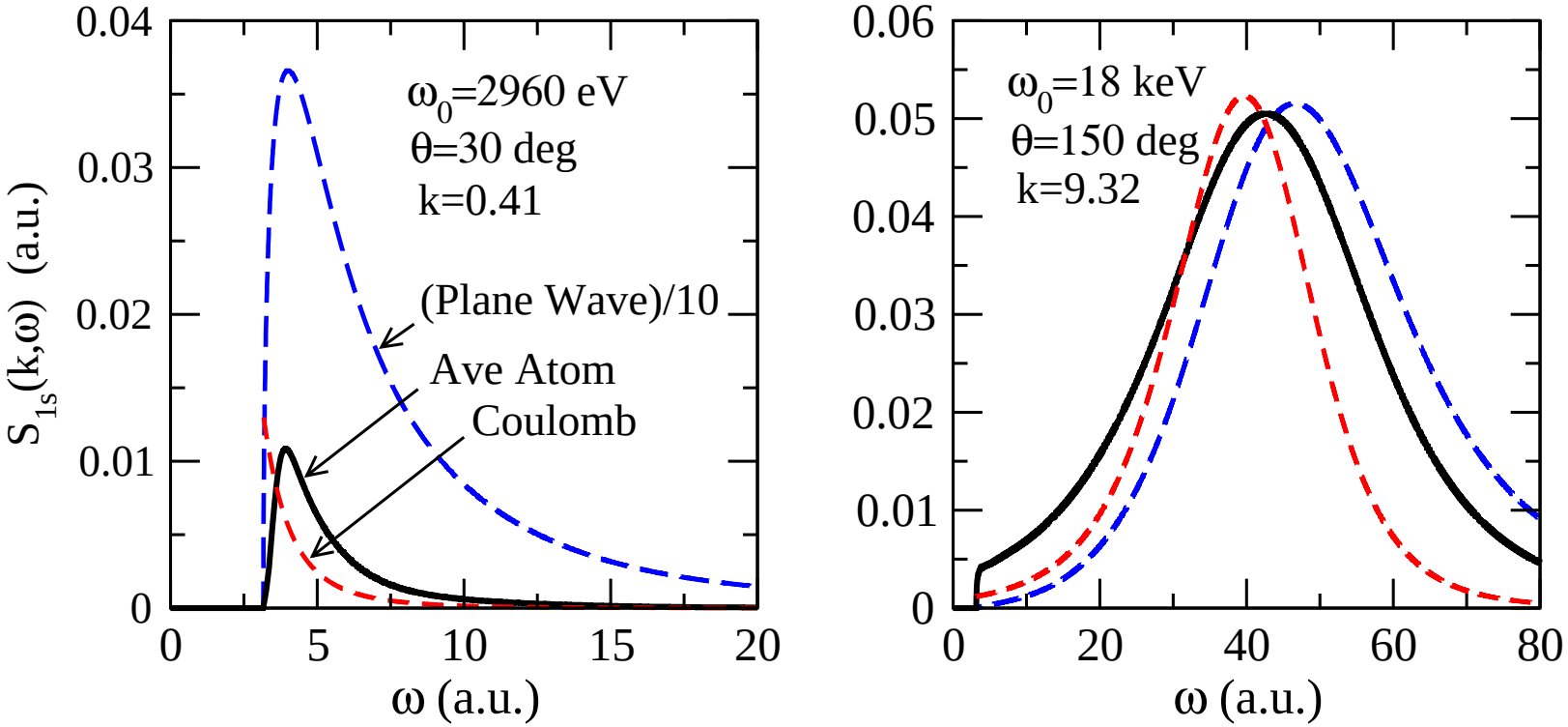
$$S_B(k,\omega) = \sum_{nl} S_{nl}(k,\omega)$$

$$S_{nl}(k,\omega) = \frac{o_{nl}}{2l+1} \sum_m \int \frac{p\, d\Omega_p}{(2\pi)^3} \left| \int d^3r\, \psi^\dagger_{\boldsymbol{p}}(\boldsymbol{r}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\, \psi_{nlm}(\boldsymbol{r}) \right|^2_{E_p=\omega+E_{nl}} .$$

Plane-wave Approximation: $\psi_{\boldsymbol{p}}(\boldsymbol{r}) \approx e^{i\boldsymbol{p}\cdot\boldsymbol{r}}$

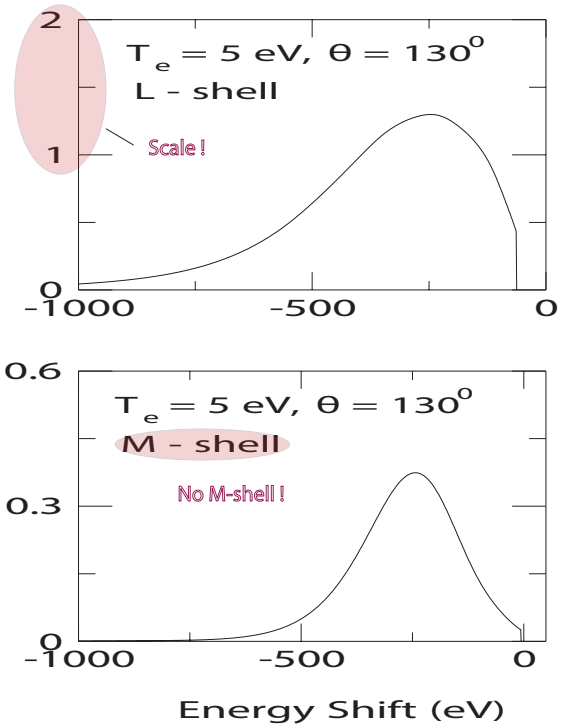
$$S_{nl}(k,\omega) = \frac{o_{nl}}{\pi k} \int_{|p-k|}^{p+k} q\, dq\, |K_{nl}(q)|^2, \quad \text{where}$$

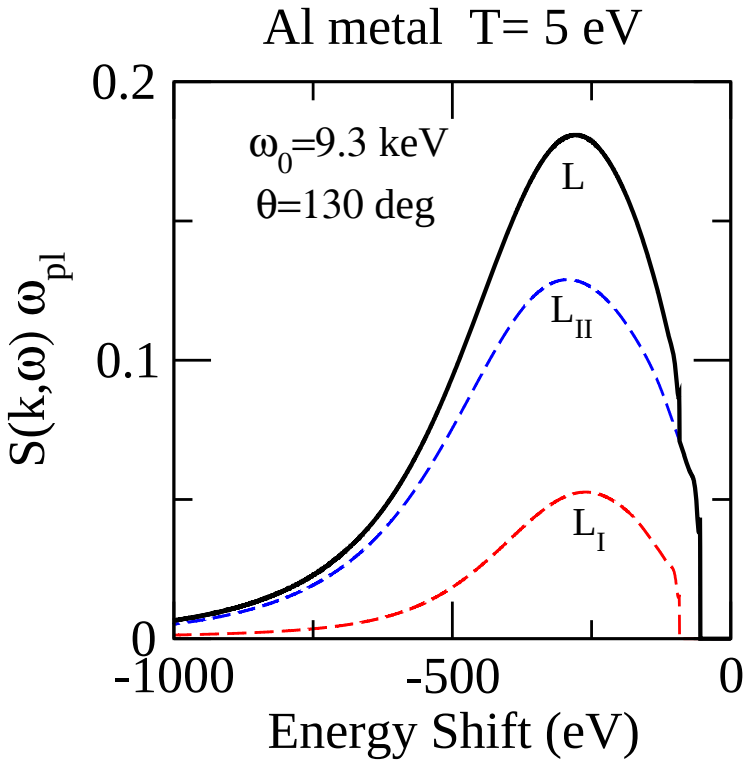
$$K_{nl}(q) = \int_0^\infty dr\, r\, j_l(qr) P_{nl}(r).$$



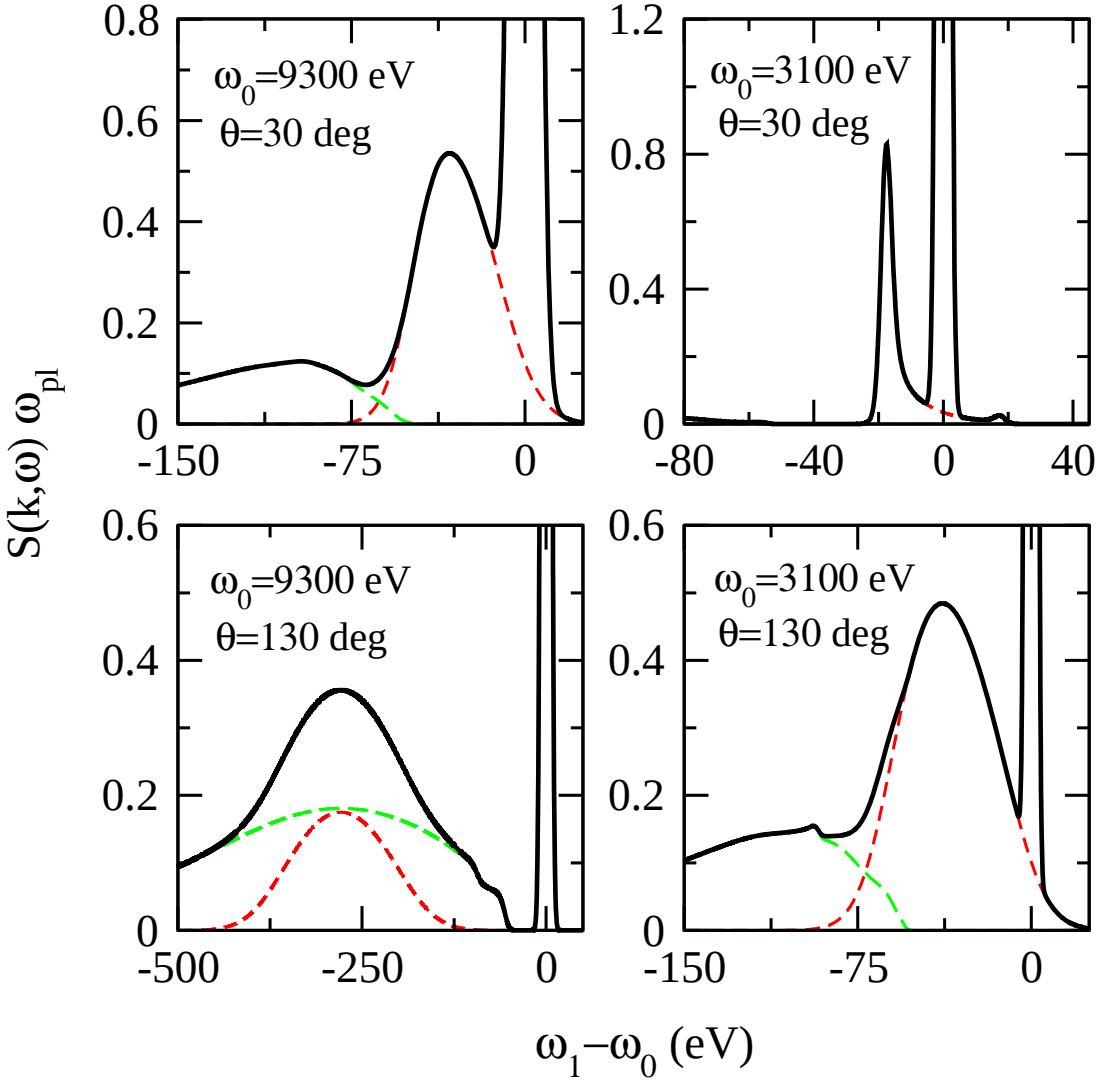
Bound-state contribution to $S(k,\omega)$ for x-ray scattering from the K-shell of Be metal at $k_B T$=18 eV.

Comparison with Sahoo et al. Phys. Rev. E 77, 046402 (2008)





Application to Aluminum



Summary:

- AA model is used to study x-ray scattering from WDM.
- Scattering from bound-states easily accommodated.
- Present results disagree with earlier AA results by Sahoo et al. for Al.

To be done:

- Understand why $T_i/T_e \neq 1$ is needed in $S_{ii}(k,\omega)$ in some equilibrium cases but not in others?
- Go beyond RPA for $S_{ee}(k,\omega)$ and include correlation effects.

References

- G. Gregori et al., Phys. Rev. E 67, 026412 (2003)
- S. H. Glenzer & R. Redmer, Rev. Mod. Phys. 81, 1625 (2009)
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- Y. Arkhipov & A. Davletov, Phys. Letts. A 227, 339 (1998)
- Johnson, Nilsen & Cheng, Phys. Rev. E 86, 036410 (2012)
- S. Sahoo et al., Phys. Rev. E 77, 046402 (2008)