

Atomic PNC and the Weak Charge of ^{133}Cs

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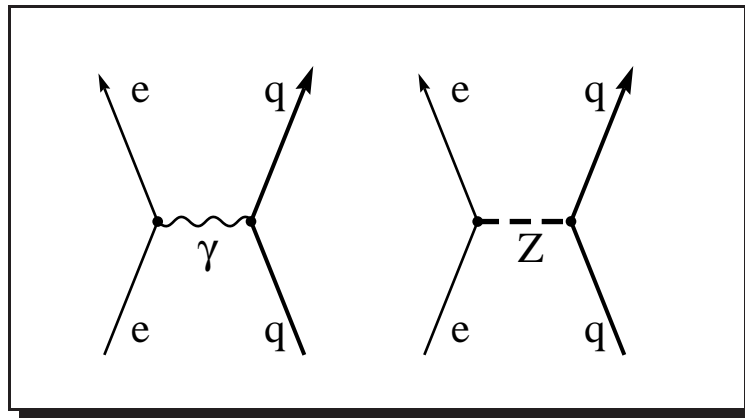
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Abstract

Atomic PNC measurements and calculations are reviewed with emphasis on the $6s \rightarrow 7s$ transition in cesium and the corresponding value of the weak charge $Q_W(^{133}\text{Cs})$.

Review

Mechanism: Photon – Z interference



Consequence: Laporte's rule¹ is violated!

$$\phi_{lj} \rightarrow \phi_{lj} + \epsilon \phi_{l\pm 1j}$$

¹O. Laporte, Z. Physik **23** 135 (1924).

Background

Standard electroweak model²

$$H_{PV} = \frac{G}{\sqrt{2}} \left[\bar{e} \gamma_{\mu} \gamma_5 e \left(c_{1u} \bar{u} \gamma_{\mu} u + c_{1d} \bar{d} \gamma_{\mu} d + \dots \right) + \bar{e} \gamma_{\mu} e \left(c_{2u} \bar{u} \gamma_{\mu} \gamma_5 u + c_{2d} \bar{d} \gamma_{\mu} \gamma_5 d + \dots \right) \right]$$

where \dots corresponds to $q = s, t, b, c$

$$c_{1u} = -1/2 + 4/3s^2$$

$$c_{1d} = 1/2 - 2/3s^2$$

$$c_{2u} = -1/2 (1 - 4s^2)$$

$$c_{2d} = 1/2 (1 - 4s^2)$$

$$s^2 = \sin^2 \theta_W$$

²W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 170

Nonrelativistic Quarks

Contribution of vector nucleon current:

$$(\bar{u}\gamma^\mu u) \rightarrow \phi_u^\dagger \phi_u \delta_{\mu 0} \quad (\bar{d}\gamma^\mu d) \rightarrow \phi_d^\dagger \phi_d \delta_{\mu 0}$$

where ϕ_u and ϕ_d are nonrelativistic field operators. From this we extract an “effective” Hamiltonian to be used in the electron sector

$$H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 Q_W \rho(r)$$

where $\rho(r)$ is a nuclear density (\sim neutron density) normalized to 1, and (omitting radiative corrections)

$$\begin{aligned} Q_W &= 2[(2Z + N)c_{1u} + (Z + 2N)c_{1d}] \\ &= -N + Z(1 - 4s^2) \approx -N \end{aligned}$$

Nuclear Spin Dependent Terms

For a nucleus with a single unpaired nucleon, the axial current gives:

$$H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} c_{2N} \frac{\kappa - 1/2}{I(I + 1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

where $c_{2N} = c_{2p}$ or c_{2n} , $\kappa = \mp(I + 1/2)$ for $I = L \pm 1/2$. This term is $\sim 1/A$ as large as $H_{\text{eff}}^{(1)}$.

PNC in nucleus \Rightarrow nuclear anapole:



$$H_{\text{eff}}^{(a)} = \frac{G}{\sqrt{2}} K_a \frac{\kappa}{I(I + 1)} \alpha \cdot \mathbf{I} \rho_N(r)$$

for ^{133}Cs , $c_{2N} = 0.094$ and $K_a = 0.25 - 0.4^3$

³V. V. Flambaum, I. B. Khriplovich, O. P. Sushkov Phys. Letts. B **146** 367-369 (1984).

Spin-Dependent Interference Term

Interference between the hyperfine interaction H_{hf} and $H^{(1)}$ gives yet another nuclear spin-dependent correction

$$H_{\text{eff}}^{(\text{int})} = \frac{G}{\sqrt{2}} K_{\text{int}} \frac{\kappa}{I(I+1)} \boldsymbol{\alpha} \cdot \mathbf{I} \rho_N(r)$$

For ^{133}Cs , $K_{\text{int}} = 0.031^4$

⁴C. Bouchiat and C. A. Piketty, Z. Phys. C 49, 91 (1991); Phys. Lett. B 269, 195 (1991).

Experiments

An electric-dipole transition matrix element between states of the same nominal parity is measured. In cesium, for example, the matrix element

$$E_{\text{PNC}} = \langle 7s | ez | 6s \rangle \propto Q_W \times \text{“Structure Factor”}$$

is measured.

- Interference with a Stark-induced PV matrix element:

$$R_{\text{Stark}} = \text{Im}(E_{\text{PNC}}) / \beta$$

where β is the vector polarizability of the transition.

- Optical Rotation: $n_- \neq n_+$

$$R_{\text{OR}} = \text{Im}(E_{\text{PNC}}) / M1$$

where M1 is the magnetic-dipole transition matrix element.

Optical Rotation Experiments

Experimental values of R_{OR} over past decade			
Element	Transition	Group	$10^8 \times R_{\text{OR}}$
^{205}Tl	$^2P_{1/2} - ^2P_{3/2}$	Oxford [†] (95)	-15.33(45)
^{205}Tl	$^2P_{1/2} - ^2P_{3/2}$	Seattle (95)	-14.68(20)
^{208}Pb	$^3P_0 - ^3P_1$	Oxford (94)	-9.80(33)
^{208}Pb	$^3P_0 - ^3P_1$	Seattle (95)	-9.86(12)
^{209}Bi	$^4S_{3/2} - ^2D_{3/2}$	Oxford (91)	-10.12(20)

[†] using a remeasured E2/M1 ratio

Stark-Induced Transition Experiments

Evolving values of $R = \text{Im}(E_{\text{PNC}}) / \beta$ (mV/cm) for cesium				
Element	Transition	Group	R_{4-3}	R_{3-4}
^{133}Cs	$6s_{1/2} - 7s_{1/2}$	Paris (1984)	-1.5(2)	-1.5(2)
^{133}Cs	$6s_{1/2} - 7s_{1/2}$	Boulder (1988)	-1.64(5)	-1.51(5)
^{133}Cs	$6s_{1/2} - 7s_{1/2}$	Boulder (1997)	-1.635(8)	-1.558(8)

Other Cases

Other cases have also been considered and studied over the past decade. Here is a (partial) list:

Atom	Transition	Group
Ba ⁺	$6S_{1/2} \rightarrow 5D_{3/2}$	Seattle
Dy	$(4f^{10}5d6s)[10] \rightarrow (4f^95d^26s)[10]$	Berkeley
Fr	$7S_{1/2} \rightarrow 8S_{1/2}$	Stony Brook
Sm	$(4f^66s^2) {}^7F_J \rightarrow (4f^66s^2) {}^5D_{J'}$	Oxford
Tl	$6P_{1/2} \rightarrow 7P_{1/2}$	Berkeley
Yb	$(6s^2) {}^1S_0 \rightarrow (6s5d) {}^3D_1$	Berkeley
Yb	$(6s6p) {}^3P_0 \rightarrow (6s6p) {}^3P_1$	Berkeley

Calculations

Reliable calculations of the PNC matrix element are based on “all-order” methods in MBPT. These include PTSCI calculations⁵ in which dominant MBPT diagrams are summed to all orders and SD Coupled-Cluster calculations.⁶

For the $6s \rightarrow 7s$ transition in atomic cesium:

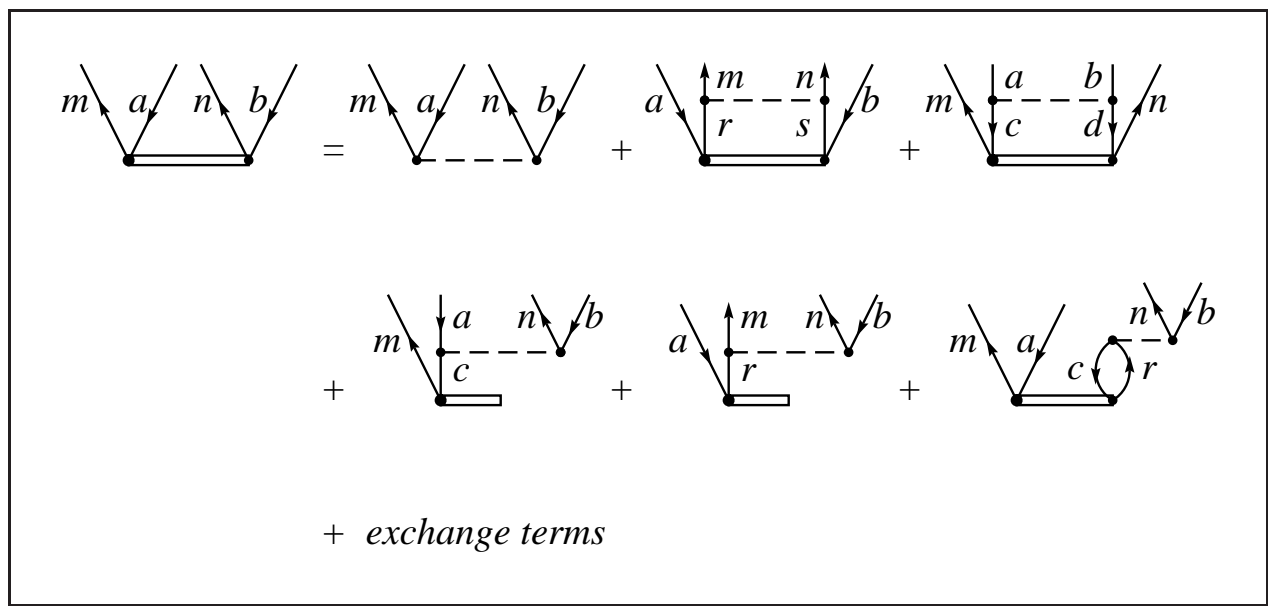
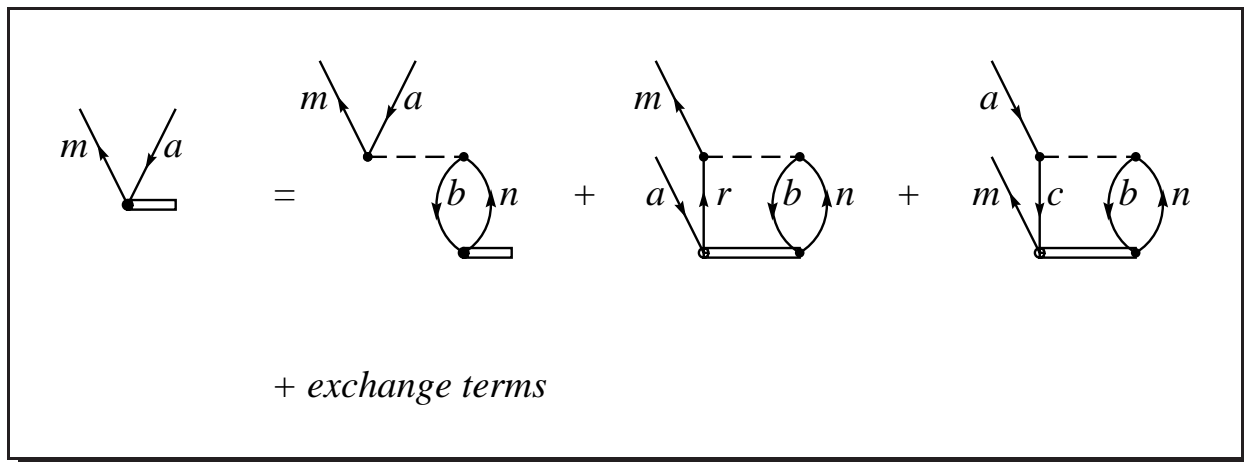
$$\langle 7s|ez|6s\rangle = \sum_n \left\{ \frac{\langle 7s|ez|np_{1/2}\rangle \langle np_{1/2}|H^{(1)}|6s\rangle}{E_{np} - E_{6s}} + \frac{\langle 7s|H^{(1)}|np_{1/2}\rangle \langle np_{1/2}|ez|6s\rangle}{E_{np} - E_{7s}} \right\}$$

- $\sum_{n=6}^9$ using SDCC (>95%)
- $\sum_{n=10}^{\infty}$ weak RPA level (<5%)

⁵V. A. Dzuba et al., arXiv:hep-ph/0204134 (2002)

⁶S. A. Blundell et al., Phys. Rev. D**45**, 1602 (1992)

Brueckner-Goldstone Diagrams for the SDCC Equations



Breakdown of Dominant Contribution: ^{133}Cs

n	$\langle 7s ez np\rangle$	$\langle np H^{(1)} 6s\rangle$	$E_{6s} - E_{np}$	Term
6	1.7291	-0.0562	-0.05093	1.908
7	4.2003	0.0319	-0.09917	-1.352
8	0.3815	0.0215	-0.11714	-0.070
9	0.1532	0.0162	-0.12592	-0.020
n	$\langle 7s H^{(1)} np\rangle$	$\langle np ez 6s\rangle$	$E_{7s} - E_{np}$	Term
6	-1.8411	0.0272	0.03352	-1.493
7	0.1143	-0.0154	-0.01472	0.120
8	0.0319	-0.0104	-0.03269	0.010
9	0.0171	-0.0078	-0.04147	0.003
Total				-0.893

Units of $H^{(1)}$: $i(-Q_W/N) \times 10^{-11}$

Residuals

$\sum_{n=6}^9$	All-Order	-0.893(4)
$\sum_{n=10}^{\infty}$	RPA	-0.018(1)
Autoionizing	HF	0.002(2)
Total		-0.909(4)

n.b. Most recent PTSCI value⁷ is **-0.908(5)**

What's missing in this calculation?

1. Breit Interaction
2. Higher-Order in αZ QED corrections
3. Nuclear "Skin" Effects

⁷V. A. Dzuba et al., arXiv:hep-ph/0204134 (2002)

Breit Correction⁸

Mixed-Parity Calculation:

$$\left(h_0 + V^{\text{HF}} + \hat{\Sigma} - \epsilon_v\right) \tilde{\psi}_v = -H^{(1)}\psi_v - \delta V_W^{\text{HF}}\psi_v$$

$$E_{\text{PNC}} = \langle 7s|ez + \delta V_z^{\text{HF}}|\tilde{6}s\rangle + \langle \tilde{7}s|ez + \delta V_z^{\text{HF}}|6s\rangle$$

Type	$\langle 7s ez + \delta V_z^{\text{HF}} \tilde{6}s\rangle$	$\langle \tilde{7}s ez + \delta V_z^{\text{HF}} 6s\rangle$	E_{PNC}
Coul	0.43942	-1.33397	-0.89456
+ Breit	0.43680	-1.32609	-0.88929
$\Delta\%$	-0.60%	-0.59%	-0.59%

⁸A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).

Electroweak Radiative Corrections: ^{133}Cs

Using $s^2 = 0.23114(17)$, one finds

$$Q_W^{\text{SM}} = -73.86 \pm 0.03 \text{ (without radiative corrections)}$$

Including electroweak radiative corrections calculated using **free-particle propagators**,⁹ the coupling constants become

$$c_{1u} = \rho'_{\text{PV}} \left(-\frac{1}{2} + \frac{4}{3} \kappa'_{\text{PV}} s^2 \right) + \lambda_{1u}$$

$$c_{1d} = \rho'_{\text{PV}} \left(\frac{1}{2} - \frac{2}{3} \kappa'_{\text{PV}} s^2 \right) + \lambda_{1d}$$

For Atomic PNC:

$$\begin{aligned} \rho'_{\text{PV}} &= 0.9878 \\ \kappa'_{\text{PV}} &= 1.0026 \\ \lambda_{1u} &= -1.9 \times 10^{-5} \\ \lambda_{1d} &= 3.7 \times 10^{-5} \end{aligned}$$

$$Q_W^{\text{SM}} = -73.09 \pm 0.03 \text{ (with radiative corrections)}$$

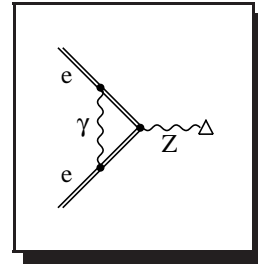
⁹D. E. Groom et al., Euro. Phys. J. C **15**, 1 (2000)

αZ -dependent terms

Perturbation Theory:¹⁰

$$\rho'_{\text{PV}} = 1 - \frac{\alpha}{2\pi} + \frac{\alpha(m_Z)}{2\pi} \left\{ \frac{3}{8s^2} \frac{m_t^2}{m_W^2} + \frac{3}{8s^4} \ln c^2 - \frac{7}{8s^2} + \frac{3\xi}{8s^2} \left(\frac{\ln(c^2/\xi)}{c^2 - \xi} + \frac{1}{c^2} \frac{\ln \xi}{1 - \xi} \right) - \frac{1}{s^2} + \dots \right\}$$

The term $-\frac{\alpha}{2\pi}$ is an electromagnetic vertex correction that should be evaluated using bound-state electron propagators.



Assume that binding corrections modify

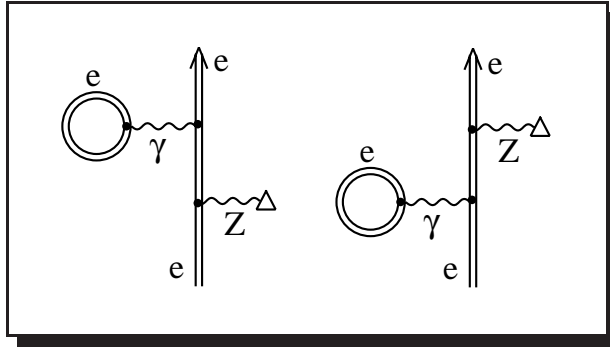
$$-\frac{\alpha}{2\pi} \rightarrow -\frac{\alpha}{2\pi} \pm \frac{\alpha}{2\pi},$$

and we find

$$Q_W^{\text{SM}} = -73.09 \rightarrow -73.09 \pm 0.08$$

¹⁰W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 182

Vacuum-Polarization^{11 12}



The double line represents the electron in the field of the nucleus. To leading order in powers of $Z\alpha$

$$-\frac{Z}{r} \Rightarrow -\frac{Z}{r} + \delta V$$

where δV given by the Uehling potential¹³:

$$\delta V(r) = -\frac{2\alpha Z}{3\pi r} \int_1^\infty dt \sqrt{t^2 - 1} \left(\frac{1}{t^2} + \frac{1}{2t^4} \right) e^{-2ctr}$$

¹¹W. R. Johnson, I. Bednyakov, and G. Soff, Phys. Rev. Lett. **87**, 233001 (2001)

¹²A. I. Milstein and O. P. Sushkov, arXiv:hep-ph/0109257

¹³E. A. Uehling, Phys. Rev. **48**, 55 (1935), E. H. Wichmann and N. H. Kroll, Phys. Rev. **101**, 843 (1956)

Vacuum-Polarization Corrections: ^{133}Cs

Mixed-Parity RPA Calculation:

$$(h_0 + V^{\text{HF}} - \epsilon_v) \tilde{\phi}_v = -H^{(1)}\phi_v - \delta V_W^{\text{HF}}\phi_v$$

$$E_{\text{PNC}} = \langle 7s | ez + \delta V_z^{\text{HF}} | \tilde{6s} \rangle + \langle \tilde{7s} | ez + \delta V_z^{\text{HF}} | 6s \rangle$$

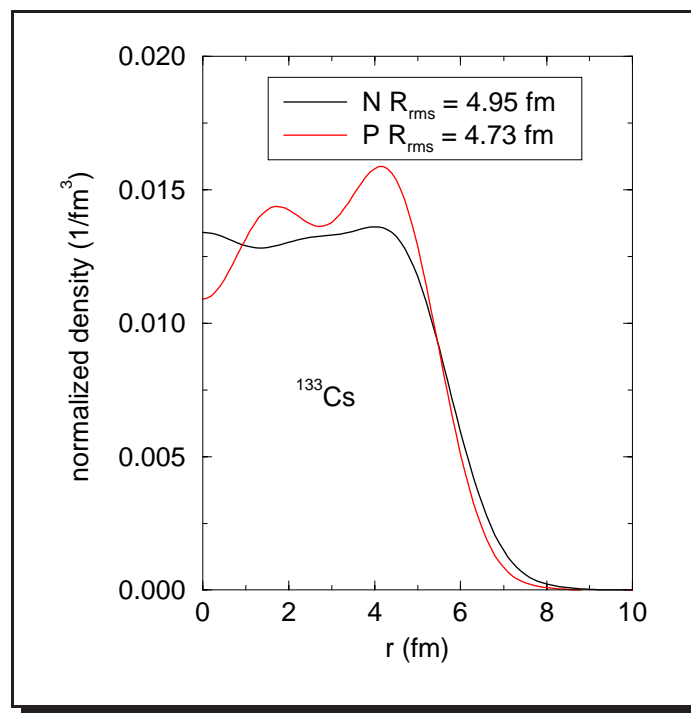
Type	$\langle 7s ez + \delta V_z^{\text{HF}} \tilde{6s} \rangle$	$\langle \tilde{7s} ez + \delta V_z^{\text{HF}} 6s \rangle$	E_{PNC}
RPA	-3.4570	1.2726	-0.9269
RPA + δV	-3.4712	1.2778	-0.9307
Δ (%)	0.41	0.41	0.41

VP corrections increase the PNC amplitude by **0.41%**.

Nuclear “skin” correction

Neutrons are primarily the source of the vector atomic PNC interaction, but proton densities are used in calculations of atomic PNC.

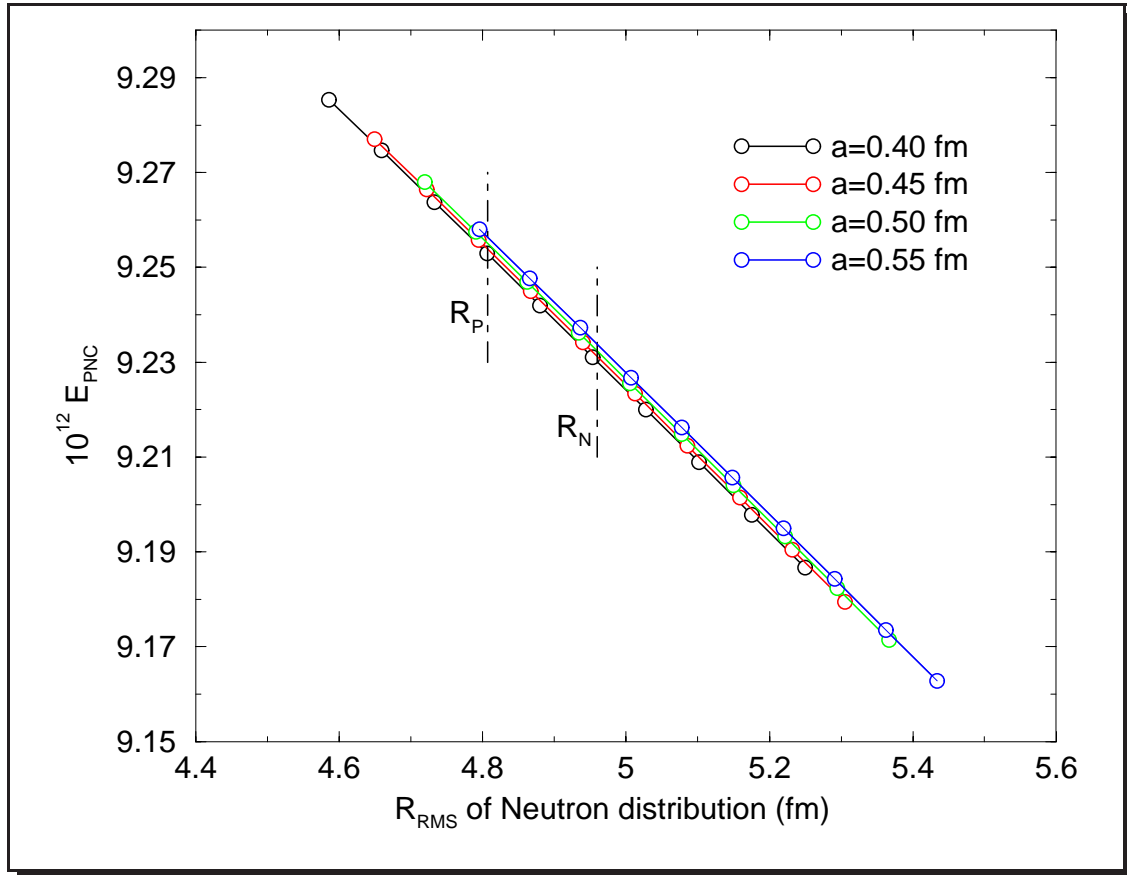
Replacing proton densities by neutron densities leads to “skin” corrections proportional to $\delta\rho = \rho_n - \rho_p$.



Proton and Neutron distributions¹⁴ for ¹³³Cs.

¹⁴D. Vretenar et al., Phys. Rev. C **62**, 045502 (2000).

RPA Calculations



Conclusion:^{15 16 17} The “skin” effect decreases the size of E_{PNC} by **0.23%**

¹⁵A. Dervianko, arXiv:physics/0108033 (2001)

¹⁶S. J. Pollock and M. C. Welliver, Phys. Lett. B **464**, 177 (1999)

¹⁷J. James and P. G. H. Sandars, J. Phys. B**32**, 3295 (1999)

Conclusion (for ^{133}Cs)

Including the (Br+VP+Skin) corrections,

$$E_{\text{PNC}} = -0.9053 \pm 0.0037 \text{ } iea_0 \times 10^{-11} (-Q_W/N)$$

where we take the 0.4% error estimate and the value of β from Bennett and Wieman.¹⁸ Combining these with the experimental value¹⁹ of E_{PNC}/β , leads to an experimental value for the weak charge

$$Q_W^{\text{expt}}(^{133}\text{Cs}) = -72.15 \pm (0.26)_{\text{expt}} \pm (0.34)_{\text{theor}}$$

This differs by 2.2σ from the standard-model value²⁰

$$Q_W^{\text{SM}}(^{133}\text{Cs}) = -73.09 \pm 0.03$$

¹⁸S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999); **82**, 4153 (1999); **83**, 889 (1999)

¹⁹C. S. Wood *et al.*, Science **275**, 1759 (1997)

²⁰D. E. Groom *et al.*, Euro. Phys. J. C **15**, 1 (2000).

Notes

- ★ The vertex QED correction is controversial!!

Milstein and Sushkov:²¹ -0.1%

Kuchiev and Flambaum:²² -0.7%

- ★ The value of β has been remeasured.²³

$$\beta = 27.22(11) a_0^3 \quad \beta_{\text{BW}} = 27.024(60)$$

$$\beta_{\text{ave}} = 27.06(6)$$

$$Q_W \Rightarrow -72.25 \pm (0.26)_{\text{expt}} \pm (0.34)_{\text{theor}}$$

$$\Delta Q_W \Rightarrow 2.0 \sigma$$

²¹A. I. Milstein and O. P. Sushkov, arXiv:hep-ph/0109257

²²M. Yu. Kuchiev, V. V. Flambaum, arXiv:hep-ph/0206124

²³A. A. Vasilyev et al., arXiv:physics/0112071