Atomic-Physics Tests of QED & the Standard Model

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Abstract

A brief review of tests of strong-field QED in many-electron atoms and of atomic parity-nonconservation (PNC) induced by $Z_0$ exchange is given, with emphasis on the role of precise atomic many-body calculations.
(1970’s) “Strong-Field” Lamb-shift corrections for K-shell electrons in heavy atoms were tested to about 2%.

(Dirac Hartree-Fock level atomic theory)

(1980’s) QED corrections for valence electrons in highly-charged Li-like, Na-like, and Cu-like ions were tested to about 5%.

(Third-order relativistic MBPT)

(1990’s) Amplitudes for parity-forbidden transitions in Cs, Tl, Bi, and Pb were measured to better than 1%. Comparisons with precise theory in Cs gave a test of the standard-model at the 1% level and the first measurement of a nuclear anapole moment.

(All-order relativistic MBPT)
**K-shell Binding Energies**

![Graph showing K-shell binding energies vs nuclear charge]

- Experiment (Ry)
- Dirac-Coulomb (Ry)

**Dirac-Coulomb 1s energy**

\[
\epsilon_{1s} = \frac{2Z^2}{1 + \sqrt{1 - \alpha^2 Z^2}} \quad \text{Ry}
\]
Electron-Electron Interaction

Dirac Hartree-Fock Approximation:

One-electron orbitals $\phi_a$ satisfy Dirac equations

\[
(H_{\text{Dirac}} + V_{\text{HF}})\phi_a = \epsilon_a \phi_a
\]

\[
V_{\text{HF}} \phi_a(r) = \sum_b \int \frac{d^3r'}{|r - r'|} \phi_b^\dagger(r') \phi_b(r') \phi_a(r)
\]

\[
- \sum_b \int \frac{d^3r'}{|r - r'|} \phi_b^\dagger(r') \phi_a(r') \phi_b(r)
\]

Equations solved self-consistently.

$-\epsilon_a$ = “frozen-core” binding energy.
**Smaller Corrections**

**Relaxation Energy:**

\[ E_{1s} = E_{\text{ion}} - E_{\text{atom}} = -\epsilon_{1s} + \Delta_{1s} \]

**Breit Interaction:**

\[ b_{12} = \frac{\alpha_1 \cdot \alpha_2 + \alpha_1 \cdot \hat{r}_{12} \alpha_2 \cdot \hat{r}_{12}}{2r_{12}} \]

<table>
<thead>
<tr>
<th>Hg 1s binding energy (Ry)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coulomb</td>
</tr>
<tr>
<td>HF correction</td>
</tr>
<tr>
<td>Relaxation</td>
</tr>
<tr>
<td>Relaxed Breit</td>
</tr>
<tr>
<td>Breit Retardation</td>
</tr>
<tr>
<td>Nuclear size</td>
</tr>
<tr>
<td>Structure</td>
</tr>
<tr>
<td>Expt.</td>
</tr>
</tbody>
</table>

Estimated accuracy \( \approx 0.1 \text{ Ry} \approx 1 \text{ eV} \)
Figure 1: Difference between experimental and theoretical K-shell energies as a function of $Z$. 
K-electron Self-Energy

\[ \Delta E_{1s} = \frac{\alpha}{\pi} (\alpha Z)^4 F(Z\alpha) mc^2 \]

\[ F(Z\alpha) = A_4 + \alpha Z A_5 + (\alpha Z)^2 A_6 + \cdots \]

where,

\[ A_4 = \frac{4}{3} \left[ \ln(Z\alpha)^{-2} + \frac{5}{6} - 2.9841285 \right] \]

\[ A_5 = 3\pi \left[ 1 + \frac{11}{128} - \frac{1}{2} \ln 2 \right] \]

\[ A_6 = \left[ 7 \ln 2 - \frac{63}{80} \right] \ln(Z\alpha)^{-2} - \frac{3}{4} \ln^2(Z\alpha)^{-2} - 33.2 \]
Non-Perturbative 1s Self-Energy

- Brown et al. 1959 – $Z = 80$ (incorrect numerics)
- Desiderio 1971 – Coulomb (70 – 90)
- Mohr 1972 – Coulomb (10 – 100)
- Cheng 1976 – Nuclear Size
- Soff et al. 1982 – Nuclear Size
- Blundell & Snyderman 1991
- Lindgren et al. 1993
Test of Lowest-Order QED

QED = Self-Energy + Vacuum Polarization
Vacuum-Polarization ≈ Uehling Potential

“Experimental” vs. Theoretical QED

<table>
<thead>
<tr>
<th>Atom</th>
<th>Expt.</th>
<th>Theory</th>
<th>Diff.</th>
<th>QED</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xe</td>
<td>2540.28(8)</td>
<td>2543.43</td>
<td>-3.15(8)</td>
<td>-3.22</td>
</tr>
<tr>
<td>W</td>
<td>5110.42(2)</td>
<td>5119.03</td>
<td>-8.61(2)</td>
<td>-8.73</td>
</tr>
<tr>
<td>Hg</td>
<td>6108.44(6)</td>
<td>6119.59</td>
<td>-11.15(6)</td>
<td>-11.36</td>
</tr>
<tr>
<td>Pb</td>
<td>6468.71(5)</td>
<td>6480.83</td>
<td>-12.11(5)</td>
<td>-12.35</td>
</tr>
<tr>
<td>Rn</td>
<td>7232.73(89)</td>
<td>7247.07</td>
<td>-14.34(89)</td>
<td>-14.51</td>
</tr>
</tbody>
</table>

“Experimental” and Theoretical radiative corrections agree at 2% level of accuracy.
Many-Electron Hamiltonian

The no-pair Hamiltonian\(^1\) is a many-electron generalization of the Dirac Hamiltonian derived from the field-theoretic Hamiltonian of QED:

\[
H = H_0 + V
\]

\[
H_0 = \sum_i \epsilon_i a_i^\dagger a_i
\]

\[
V = \frac{1}{2} \sum_{ijkl} v_{ijkl} a_i^\dagger a_j^\dagger a_l a_k - \sum_{ij} U_{ij} a_i^\dagger a_j
\]

1. the sums are restricted to electron states only,

2. \(v_{ijkl}\) is a two-particle matrix element of the sum of the Coulomb and Breit interactions,

3. \(U_{ij}\) compensates for including \(U(r)\) in \(h_0\).

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QED Tests for Copper-like Ions

Largest atom with a “complete” isoelectronic sequence.

Structure:
\[(1s)^2(2s)^2(2p)^6(3s)^2(3p)^6(3d)^{10} + \text{valence}\]

Outline of MBPT calculations:

\[E^{(0)} = \sum_a \epsilon_a + \epsilon_v\]
\[E^{(1)} = E_c^{(1)} = -1/2 \sum_a (V_{HF})_{aa}\]
\[E^{(2)} = E_c^{(2)} + E_v^{(2)}\]
\[E^{(3)} = E_c^{(3)} + E_v^{(3)} \ldots\]

\[E_v^{(2)} = \sum_{amn} \frac{\tilde{v}_{mnab}v_{avmn}}{\epsilon_m + \epsilon_n - \epsilon_v - \epsilon_a} - \sum_{abm} \frac{\tilde{v}_{abmv}v_{mvab}}{\epsilon_m + \epsilon_v - \epsilon_a - \epsilon_b}\]

\[E_v^{(3)} = \sum_{mabcd} \frac{\tilde{v}_{abvm}v_{cdab}v_{mvcd}}{(\epsilon_{ab} - \epsilon_{vm})(\epsilon_{cd} - \epsilon_{mv})} + 17 \text{ more terms}\]
### Energies of Cu-like Sn (Z=50) (a.u.)

<table>
<thead>
<tr>
<th>Term</th>
<th>$4s_{1/2}$</th>
<th>$4p_{1/2}$</th>
<th>$4p_{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^{(0)}$</td>
<td>-23.56930</td>
<td>-21.91682</td>
<td>-21.48453</td>
</tr>
<tr>
<td>$E^{(2)}$</td>
<td>-0.06009</td>
<td>-0.06000</td>
<td>-0.05583</td>
</tr>
<tr>
<td>$E^{(3)}$</td>
<td>0.00406</td>
<td>0.00383</td>
<td>0.00355</td>
</tr>
<tr>
<td>$B^{(1)}$</td>
<td>0.01493</td>
<td>0.02117</td>
<td>0.01478</td>
</tr>
<tr>
<td>$B^{(2)}$</td>
<td>-0.00433</td>
<td>-0.00451</td>
<td>-0.00449</td>
</tr>
<tr>
<td>$B^{(3)}$</td>
<td>0.00017</td>
<td>0.00027</td>
<td>0.00019</td>
</tr>
<tr>
<td>Recoil</td>
<td>0.00009</td>
<td>0.00005</td>
<td>0.00005</td>
</tr>
<tr>
<td>$E_{Tot}$</td>
<td>-23.61488</td>
<td>-21.95601</td>
<td>-21.52627</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$p_{3/2} - p_{1/2}$</th>
<th>$p_{3/2} - s_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{Tot}$</td>
<td>0.4297(2)</td>
<td>2.0886(3)</td>
</tr>
<tr>
<td>$E_{Expt}$</td>
<td>0.4303(3)</td>
<td>2.0807(1)</td>
</tr>
</tbody>
</table>
Tests of QED in Cu-like Ions

Figure 2: Comparison of “experimental” and theoretical values of the $4p_{3/2} - 4s_{1/2}$ Lamb-shift in copper-like ions.

**Conclusion:** Calculations of the self-energy and vacuum-polarization corrections carried out in a HF potential agree with ”experimental” Lamb-shift to $\sim 5\%$ for high-$Z$ Cu-like ions.
Atomic Tests of the Standard Model

\[ H^{(1)} = \frac{G}{\sqrt{2}} (\bar{\psi} e\gamma_\mu \gamma_5 \psi) \sum_i \left[ c_{1p} (\bar{\psi}_i \gamma^\mu \psi_i) + c_{1n} (\bar{\psi}_n \gamma^\mu \psi_n) \right] \]

\[ H^{(2)} = \frac{G}{\sqrt{2}} (\bar{\psi} e\gamma_\mu \psi) \sum_i \left[ c_{2p} (\bar{\psi}_i \gamma^\mu \gamma_5 \psi_i) + c_{2n} (\bar{\psi}_n \gamma^\mu \gamma_5 \psi_n) \right] \]

\[ c_{1p} = \frac{1}{2} \left( 1 - 4 \sin^2 \theta_W \right) \approx 0.038, \]
\[ c_{1n} = -\frac{1}{2}, \]
\[ c_{2p} = \frac{1}{2} g_A \left( 1 - 4 \sin^2 \theta_W \right) \approx 0.047, \]
\[ c_{2n} = -\frac{1}{2} g_A \left( 1 - 4 \sin^2 \theta_W \right) \approx -0.047. \]

\[ g_A \approx 1.25 \] for the partially conserved axial current
\[ \sin^2 \theta_W = 0.23124(24) \] is Weinberg’s angle.
W.R. Johnson

Nonrelativistic Nucleon Reduction

\[ H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 Q_w \rho(r) \]

\[ H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} \frac{\kappa - 1/2}{I(I+1)} \alpha \cdot I \left[ c_{2p} \rho_{pv}(r) + c_{2n} \rho_{nv}(r) \right] \]

\[ H_{\text{eff}}^{(a)} = \frac{G}{\sqrt{2}} K_a \frac{\kappa}{I(I+1)} \alpha \cdot I \rho_v(r) \]

\[ Q_w = \left[ 2 \, Z \, c_{1p} + 2 \, N \, c_{1n} \right] = -N + Z \left( 1 - 4 \sin^2 \theta_W \right) \]

Nuclear Anapole
Single-Double Equations

\[ \Psi_v = \Psi_{\text{DHF}} + \delta \Psi \]

\[ \delta \Psi = \left\{ \sum_{am} \rho_m a_m^\dagger a_a + \frac{1}{2} \sum_{abmn} \rho_{mnab} a_m^\dagger a_n^\dagger a_b a_a + \sum_{m \neq v} \rho_{mv} a_m^\dagger a_v + \sum_{bmn} \rho_{mnvb} a_m^\dagger a_n^\dagger a_b a_v \right\} \Psi_{\text{DHF}} \]

\[ E_C = E_{C}^{\text{DHF}} + \delta E_C \]

\[ E_v = E_{v}^{\text{DHF}} + \delta E_v \]

Later, we will discuss triples
Core Excitation Equations

\[(\epsilon_a - \epsilon_m)\rho_{ma} = \sum_{bn} \tilde{v}_{mban}\rho_{nb} \]
\[+ \sum_{bnr} v_{mbnr}\tilde{\rho}_{nrab} - \sum_{bcn} v_{bcan}\tilde{\rho}_{mnbc} \]

\[(\epsilon_a + \epsilon_b - \epsilon_m - \epsilon_n)\rho_{mnab} = v_{mnab} \]
\[+ \sum_{cd} v_{cdab}\rho_{mncd} + \sum_{rs} v_{mnrs}\rho_{rsab} \]
\[+ \left[ \sum_r v_{mnr}\rho_{ra} - \sum_c v_{cnab}\rho_{mc} + \sum_{rc} \tilde{v}_{cnrb}\tilde{\rho}_{mrac} \right] \]
\[+ \left[ a \leftrightarrow b \quad m \leftrightarrow n \right] \]

\[\delta E_C = \frac{1}{2} \sum_{abmn} v_{abmn}\tilde{\rho}_{mnab} \]

\[\approx 15,000,000 \quad \rho_{mnab} \text{ coefficients for Cs } (\ell = 6).\]
Figure 3: Many-body diagrams for the core SD equations.
Valence Equations

\[(\epsilon_v - \epsilon_m + \delta E_v)\rho_{mv} = \sum_{bn} \tilde{v}_{mbvn}\rho_{nb} \]

\[+ \sum_{bnr} v_{mbnr}\tilde{\rho}_{nrvb} - \sum_{bcn} v_{bcvn}\tilde{\rho}_{mnbv} \]

\[(\epsilon_v + \epsilon_b - \epsilon_m - \epsilon_n + \delta E_v)\rho_{mnvb} = v_{mnvb} \]

\[+ \sum_{cd} v_{cdvb}\rho_{mncd} + \sum_{rs} v_{mnrs}\rho_{rsvb} \]

\[+ \left[ \sum_{r} v_{mnrb}\rho_{rv} - \sum_{c} v_{cnvb}\rho_{mc} + \sum_{rc} \tilde{v}_{cnrb}\tilde{\rho}_{mrvc} \right] \]

\[+ \left[ v \leftrightarrow b \quad m \leftrightarrow n \right] \]

\[
\delta E_v = \sum_{ma} \tilde{v}_{vavm}\rho_{ma} + \sum_{mab} v_{abvm}\tilde{\rho}_{mvab} \]

\[+ \sum_{mna} v_{vbmn}\tilde{\rho}_{mnbv} \]

\[\approx 1,000,000 \quad \rho_{mnvb} \text{ coefficients for each state (Cs)} \]
Perturbation Expansion of the Energy

- $\delta E_C$ agrees with MBPT through third order.
- $\delta E_v$ agrees with MBPT through second order.
- $\delta E_v^{(3)}$ disagrees with $E_v^{(3)}$ from MBPT.

Add triple excitations to wave function

$$
\frac{1}{6} \sum_{abmnr} \rho_{mnrvab} a^\dagger_m a^\dagger_n a^\dagger_r a_v a_b a_a \Psi_{HF}
$$

$$
E_v^{\text{extra}} = \frac{1}{2} \sum_{mnab} \tilde{\nu}_{abmn} \rho_{mnvvab}
$$

- $\delta E_v^{(3)} + E_v^{(3)}$ extra gives the entire third-order valence correlation energy.
\[ E_v^{(3)} = \delta E_v^{(3)} + E_v^{(3)}_{\text{extra}} \]

**Table 1: Na 3s and 3p states (a.u.)**

<table>
<thead>
<tr>
<th>Term</th>
<th>3s(^{1/2})</th>
<th>3p(^{1/2})</th>
<th>3p(^{3/2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_v^{(3)}_{\text{extra}})</td>
<td>-0.418[-4]</td>
<td>-0.070[-4]</td>
<td>-0.072[-4]</td>
</tr>
</tbody>
</table>
Figure 4: Energy comparisons for alkali-metal atoms.
Higher-order terms

Figure 5: Contributions beyond 3rd-order for Na-like ions.

- $E^{(4)+} < 1\%$ of correlation energy for $Z > 20$
## All-order energies

Table 2: Summary for sodium $3s$ and $3p$ states ($\text{cm}^{-1}$)

<table>
<thead>
<tr>
<th>Term</th>
<th>$3s$</th>
<th>$3p_{1/2}$</th>
<th>$3p_{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DHF</td>
<td>-39951.6</td>
<td>-24030.4</td>
<td>-24014.1</td>
</tr>
<tr>
<td>$\delta E$</td>
<td>-1488.8(4)</td>
<td>-463.9</td>
<td>-461.6</td>
</tr>
<tr>
<td>$E^{(3)}_{\text{extra}}$</td>
<td>-9.2</td>
<td>-1.5</td>
<td>-1.6</td>
</tr>
<tr>
<td>Breit</td>
<td>1.2</td>
<td>1.4</td>
<td>0.1</td>
</tr>
<tr>
<td>RM+MP</td>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Theory</td>
<td>-41447.3(4)</td>
<td>-24493.9</td>
<td>-24476.7</td>
</tr>
<tr>
<td>Expt.</td>
<td>-41449.4</td>
<td>-24493.3</td>
<td>-24476.1</td>
</tr>
</tbody>
</table>
Results for Heavy Atoms

Table 3: Energies for cesium and francium cm$^{-1}$

<table>
<thead>
<tr>
<th></th>
<th>6s</th>
<th>7s</th>
<th>8s</th>
<th>9s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td>31262</td>
<td>12801</td>
<td>7060</td>
<td>4479</td>
</tr>
<tr>
<td>Expt.</td>
<td>31407</td>
<td>12871</td>
<td>7089</td>
<td>4496</td>
</tr>
<tr>
<td>Cs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td>6p$_{1/2}$</td>
<td>7p$_{1/2}$</td>
<td>8p$_{1/2}$</td>
<td>9p$_{1/2}$</td>
</tr>
<tr>
<td>Expt.</td>
<td>20204</td>
<td>9621</td>
<td>5687</td>
<td>3760</td>
</tr>
<tr>
<td>Cs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td>7s</td>
<td>8s</td>
<td>9s</td>
<td>10s</td>
</tr>
<tr>
<td>Expt.</td>
<td>32735</td>
<td>13051</td>
<td>7148</td>
<td>4522</td>
</tr>
<tr>
<td>Fr</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Theory</td>
<td>7p$_{1/2}$</td>
<td>8p$_{1/2}$</td>
<td>9p$_{1/2}$</td>
<td>10p$_{1/2}$</td>
</tr>
<tr>
<td>Expt.</td>
<td>20583</td>
<td>9712</td>
<td>5724</td>
<td>3782</td>
</tr>
</tbody>
</table>

Notes: Theoretical and experimental data for cesium (Cs) and francium (Fr) are presented for various energy levels. The table includes data for $6s$, $7s$, $8s$, and $9s$ orbitals, as well as $6p_{1/2}$, $7p_{1/2}$, $8p_{1/2}$, and $9p_{1/2}$ orbitals for cesium, and $7s$, $8s$, and $9s$ orbitals for francium. The data is given in units of cm$^{-1}$. The experimental values are shown in red for clarity.
### Fine-Structure Intervals

**Table 4: Fine-structure (cm$^{-1}$).**

<table>
<thead>
<tr>
<th></th>
<th>This work</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4p_{3/2} - 4p_{1/2}$</td>
<td>57.3</td>
<td>57.7</td>
</tr>
<tr>
<td>$5p_{3/2} - 5p_{1/2}$</td>
<td>18.5</td>
<td>18.8</td>
</tr>
<tr>
<td>$6p_{3/2} - 6p_{1/2}$</td>
<td>8.5</td>
<td>8.4</td>
</tr>
<tr>
<td>$7p_{3/2} - 7p_{1/2}$</td>
<td>4.4</td>
<td>4.5</td>
</tr>
<tr>
<td><strong>Rb</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5p_{3/2} - 5p_{1/2}$</td>
<td>236.5</td>
<td>237.6</td>
</tr>
<tr>
<td>$6p_{3/2} - 6p_{1/2}$</td>
<td>76.5</td>
<td>77.5</td>
</tr>
<tr>
<td>$7p_{3/2} - 7p_{1/2}$</td>
<td>34.8</td>
<td>35.1</td>
</tr>
<tr>
<td>$8p_{3/2} - 8p_{1/2}$</td>
<td>18.6</td>
<td>18.9</td>
</tr>
<tr>
<td><strong>Cs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6p_{3/2} - 6p_{1/2}$</td>
<td>552.2</td>
<td>554.1</td>
</tr>
<tr>
<td>$7p_{3/2} - 7p_{1/2}$</td>
<td>178.6</td>
<td>181.0</td>
</tr>
<tr>
<td>$8p_{3/2} - 8p_{1/2}$</td>
<td>81.4</td>
<td>82.6</td>
</tr>
<tr>
<td>$9p_{3/2} - 9p_{1/2}$</td>
<td>43.9</td>
<td>44.7</td>
</tr>
<tr>
<td><strong>Fr</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$7p_{3/2} - 7p_{1/2}$</td>
<td>1676</td>
<td>1687</td>
</tr>
<tr>
<td>$8p_{3/2} - 8p_{1/2}$</td>
<td>536</td>
<td>545</td>
</tr>
<tr>
<td>$9p_{3/2} - 9p_{1/2}$</td>
<td>244</td>
<td>250</td>
</tr>
<tr>
<td>$10p_{3/2} - 10p_{1/2}$</td>
<td>132</td>
<td>136</td>
</tr>
</tbody>
</table>
Table 5: Matrix Elements for heavy alkali metals (a.u.)

<table>
<thead>
<tr>
<th></th>
<th>K</th>
<th>Rb</th>
<th>Cs</th>
<th>Fr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{1/2}-s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>4.098</td>
<td>4.221</td>
<td>4.478</td>
<td>4.256</td>
</tr>
<tr>
<td>Expt.</td>
<td>4.102(5)</td>
<td>4.231(3)</td>
<td>4.489(7)</td>
<td>4.277(8)</td>
</tr>
<tr>
<td>$p_{3/2}-s$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>5.794</td>
<td>5.956</td>
<td>6.298</td>
<td>5.851</td>
</tr>
<tr>
<td>Expt.</td>
<td>5.800(8)</td>
<td>5.977(4)</td>
<td>6.324(7)</td>
<td>5.898(15)</td>
</tr>
</tbody>
</table>
Application to PNC

\[
\langle F|e_z|I \rangle = \sum_n \frac{\langle F|e_z|n \rangle \langle n|H_W|I \rangle}{E_n - E_I} + \sum_n \frac{\langle F|H_W|n \rangle \langle n|e_z|I \rangle}{E_n - E_F},
\]

where \( H_W = H_{\text{eff}}^{(1)} + H_{\text{eff}}^{(2)} + H_{\text{eff}}^{(a)} \)

\[
\langle F|e_z|I \rangle = E_{\text{PNC}} [1 + A(F_F, F_I)]
\]

For \(^{133}\text{Cs}\) (\(I = 7/2\)), we obtain:

\[
E_{\text{PNC}} = 0.906(9) \times 10^{-11} \ i |e| a_0 Q_w / N
\]

\[
A(F_F, F_I) = \begin{bmatrix} 0.028 & 0.057 \\ -0.051 & -0.022 \end{bmatrix}
\]
Status of PNC Experiments

(a) Optical rotation: \( n_+ \neq n_- \quad \phi = \frac{E_{\text{PNC}}}{M_1} \)

\[
\begin{array}{lll}
\text{Element} & \text{Group} & 10^8 \times \phi \\
\hline
\text{Thallium} & \text{Oxford} & -15.7(5) \\
\text{Thallium} & \text{Seattle} & -14.7(2) \\
\text{Lead} & \text{Oxford} & -9.8(1) \\
\text{Lead} & \text{Seattle} & -9.9(1) \\
\text{Bismuth} & \text{Oxford} & -10.1(20) \\
\end{array}
\]

(b) Stark interference: Add \( E(t) = A \cos \omega t \) and detect the heterodyning signal \( R = \frac{E_{\text{PNC}}}{\beta} \)

\[
\begin{array}{llll}
\text{Element} & \text{Group} & R_{4-3} & R_{3-4} \\
\hline
\text{Cesium} & \text{Paris (1984)} & -1.5(2) & -1.5(2) \\
\text{Cesium} & \text{Boulder (1988)} & -1.64(5) & -1.51(5) \\
\text{Cesium} & \text{Boulder (1997)} & -1.635(8) & -1.558(8) \\
\end{array}
\]
Cesium: Theory vs. Experiment

\[ \beta = 27.02(8) a_0^3 \quad \text{(1999)} \]

(eliminating axial vector + anapole contribution)

\[ \mathcal{S}(E_{\text{PNC}}) = -0.836(4) \times 10^{-11} |e|a_0 \quad \text{(1997)} \]

(dividing by theoretical matrix element)

\[ Q_w = -71.9(4) \quad \text{(Expt.+ Atomic Theory)} \]

\[ K = 0.50(7) \quad \text{(Expt.+ Atomic Theory)} \]

Marciano & Rosner (with radiative corrections)

\[ Q_w = -73.20(13) \quad \text{(Standard Model)} \]

\[
\text{Expt. - Theory} = 1.3(4)
\]

Principal source of error is atomic theory.
Conclusions

I Strong-field QED corrections inner-shell electrons in heavy atoms are tested to 2%.

II Comparing theoretical and “experimental” values of the Lamb-shift for valence electrons in highly-charged ions provide stringent tests of higher-order MBPT calculations.

III Tests of the standard-model by comparing experiment and theory show a $\sim 3\sigma$ effect in $Q_w$.

IV Modern atomic PNC experiments are underway at Stonybrook, Berkeley, and Seattle. Calculations are needed to aid in extracting quantitative information from such measurements.