Vacuum-polarization corrections to the PNC amplitude in ^{133}Cs*

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Abstract

One-loop radiative corrections to atomic wave functions associated with vacuum polarization in the nuclear Coulomb field are evaluated for the $6s - 7s$ PNC amplitude in ^{133}Cs. These corrections increase the size of the PNC amplitude by 0.4%.

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Background - ^{133}Cs

1. $E_{\rm PNC} = F \times \frac{Q_W}{-N}$ evaluated (1%) by two groups.[1]

2. $E_{\rm PNC}/\beta$ precisely measured.[2]

3. β measured and theory error reviewed (0.4%):[3]
   $\Delta Q_W = Q_W^{\rm expt} - Q_W^{\rm SM} = 2.5\,\sigma$

4. Breit interaction reconsidered:[4] $2.5\,\sigma \rightarrow 1\,\sigma$

5. Fact that bound-state radiative corrections are comparable to Breit corrections pointed out.[5]

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[1] V. A. Dzuba *et al.*, Phys. Lett. A **141**, 147 (1989); S. A. Blundell *et al.*, Phys. Rev. D **45**, 1602 (1992).

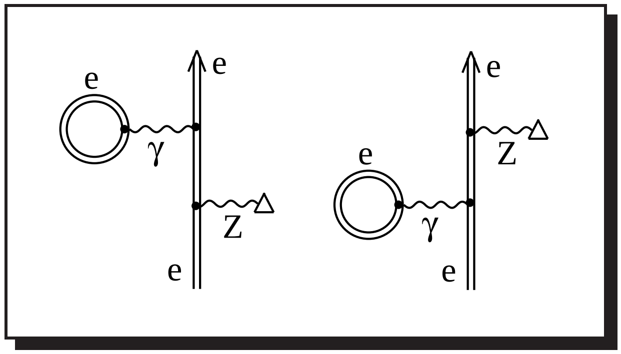
[2] C. S. Wood *et al.*, Science **275**, 1759 (1997).

[3] S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999).

[4] A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000).

[5] O. P. Sushkov, Phys. Rev. A **63**, 042504 (2001).

Which Corrections?



The double line represents the electron in the field of the nucleus. To leading order in powers of $Z\alpha$,[6]

$$-\frac{Z}{r} \Rightarrow -\frac{Z}{r} + \delta V$$

where δV given by the Uehling potential[7]:

$$\delta V(r) = -\frac{2\alpha Z}{3\pi r} \int_1^\infty dt \sqrt{t^2 - 1} \left(\frac{1}{t^2} + \frac{1}{2t^4} \right) e^{-2ctr}$$

[6] E. H. Wichmann and N. H. Kroll, Phys. Rev. **101**, 843 (1956)
[7] E. A. Uehling, Phys. Rev. **48**, 55 (1935)

Distributed Nuclear Charge

The nuclear charge distribution for ^{133}Cs is

$$\rho(r) = \frac{\rho_0}{1 + \exp\left[(r - c_{\rm nuc})/a_{\rm nuc}\right]}$$

with $c_{\rm nuc} = 5.675$ fm and 10%–90% fall-off distance is $t_{\rm nuc} = 2.3$ fm, corresponding to $a_{\rm nuc} = 0.523$ fm and to a root-mean-square radius of the nuclear charge distribution $R_{\rm rms} = 4.807$ fm.

For a distributed charge:[8]

$$\delta V(r) = -\frac{2\alpha^2}{3r} \int_0^\infty dx\, x\, \rho(x) \int_1^\infty dt \sqrt{t^2 - 1} \times \left(\frac{1}{t^3} + \frac{1}{2t^5}\right) \left(e^{-2ct|r-x|} - e^{-2ct(r+x)}\right).$$

[8] L. W. Fullerton and G. A. Rinker, Jr., Phys. Rev. A**13**, 1283 (1976)

Test of Method

Comparison of corrections to inner-shell Dirac-Coulomb energies δE on replacing $V \rightarrow V + \delta V$ with results from QED perturbation theory[9].

State	Present δE	Perturbation Theory Uehl	Uehl-FS	FS	Total
$1s_{1/2}$	-0.1419	-0.2584	0.0010(1)	0.1159	-0.1415(1)
$2s_{1/2}$	-0.1554	-0.2901	0.0011(1)	0.1339(1)	-0.1551(2)
$2p_{1/2}$	-0.0100	-0.0145	0.0000(1)	0.0045	-0.0100(1)
$2p_{3/2}$	-0.0016	-0.0016	0.0000	0.0000	-0.0016

Units: $\alpha mc^2(\alpha Z)^4/(\pi n^3)$

[9] W. R. Johnson and G. Soff, At. Data Nucl. Data Tables **33**, 405 (1985)

Calculation #1 - Weak Hartree Fock

No Correlation Corrections

Effect of PNC: $\phi_v^{\rm HF} \rightarrow \phi_v^{\rm HF} + \delta\phi_v^{\rm HF}$

$$\left(h_0 + V^{\rm HF} - \epsilon_v^{\rm HF}\right) \delta\phi_v^{\rm HF} = -h_{\rm PNC}\, \phi_v^{\rm HF}$$

$$E_{\rm PNC} = \langle \phi_{7s}^{\rm HF} | D | \delta\phi_{6s}^{\rm HF} \rangle + \langle \delta\phi_{7s}^{\rm HF} | D | \phi_{6s}^{\rm HF} \rangle$$

Corrections to $E_{\rm PNC}$ amplitude for ^{133}Cs
Units: $iea_0 \times 10^{-11}(-Q_W/N)$

Type	$\langle \phi_{7s} \vert D \vert \delta\phi_{6s} \rangle$	$\langle \delta\phi_{7s} \vert D \vert \phi_{6s} \rangle$	$E_{\rm PNC}$
HF	-2.7492	1.0144	-0.7395
HF + δV	-2.7604	1.0186	-0.7425
Δ (%)	0.41	0.41	0.41

(This calculation gives a value of the PNC amplitude that is 20% smaller than the fully correlated value.)

Calculation #2 - Weak RPA

With Correlation Corrections

$$\left(h_0 + V^{\rm HF} - \epsilon_v^{\rm HF}\right) \delta\phi_v^{\rm RPA} = -\left[h_{\rm PNC} + V_{\rm PNC}^{\rm HF}\right] \phi_v^{\rm HF},$$

$$E_{\rm PNC} = \langle \phi_{7s}^{\rm HF} | D | \delta\phi_{6s}^{\rm RPA} \rangle + \langle \delta\phi_{7s}^{\rm RPA} | D | \phi_{6s}^{\rm HF} \rangle$$

Corrections to $E_{\rm PNC}$ amplitude for ^{133}Cs, with RPA.
Units: $iea_0 \times 10^{-11}(-Q_W/N)$

Type	$\langle \phi_{7s} \| D \| \delta\phi_{6s} \rangle$	$\langle \delta\phi_{7s} \| D \| \phi_{6s} \rangle$	$E_{\rm PNC}$
RPA	-3.4570	1.2726	-0.9269
RPA+ δV	-3.4712	1.2778	-0.9307
Δ (%)	0.41	0.41	0.41

(This calculation gives a value of the PNC amplitude that is 3% larger than the fully correlated value.)

Conclusion:
VP corrections increase the PNC amplitude by 0.41%.

Comments on the PV Hamiltonian[10]

$$H_{\rm PV} = \frac{G_\mu}{\sqrt{2}} \Big[\bar{e}\gamma_\mu\gamma_5 e \left(C_{1u}\, \bar{u}\gamma_\mu u + C_{1d}\, \bar{d}\gamma_\mu d + \cdots \right) + \bar{e}\gamma_\mu e \left(C_{2u}\, \bar{u}\gamma_\mu\gamma_5 u + C_{2d}\, \bar{d}\gamma_\mu\gamma_5 d + \cdots \right) \Big]$$

where $\cdots$ corresponds to $q = s, t, b, c$

$$C_{1u} = \left(-\frac{1}{2} + \frac{4}{3} s^2 \right)$$
$$C_{1d} = \left(\frac{1}{2} - \frac{2}{3} s^2 \right)$$
$$C_{2u} = -\frac{1}{2} \left(1 - 4s^2 \right)$$
$$C_{2d} = \frac{1}{2} \left(1 - 4s^2 \right)$$

$s^2 = \sin^2\theta_W$

[10] W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 170

Electroweak Radiative Corrections

For atomic PNC with nonrelativistic nucleons, the Hamiltonian (in the electron sector) reduces to

$$h_{\mathrm{PNC}} = \frac{G_\mu}{\sqrt{2}} Q_W \, \gamma_5 \, \rho_{\mathrm{nuc}}(r)$$

with

$$Q_W = 2[(2Z + N)C_{1u} + (Z + 2N)C_{1d}]$$

Using $s^2 = 0.23114(17)$, one finds

$Q_W^{\mathsf{SM}} = -73.86 \pm 0.03$ (no radiative corrections)

Electroweak Radiative Corrections

With electroweak radiative corrections,[11] the coupling constants can be written

$$C_{1u} = \rho'_{\rm PV}\left(-\frac{1}{2}+\frac{4}{3}\,\kappa'_{\rm PV}\,s^2\right)+\lambda_{1u}$$

$$C_{1d} = \rho'_{\rm PV}\left(\frac{1}{2}-\frac{2}{3}\,\kappa'_{\rm PV}\,s^2\right)+\lambda_{1d}$$

For Atomic PNC:

$$\rho'_{\rm PV} = 0.9878$$
$$\kappa'_{\rm PV} = 1.0026$$
$$\lambda_{1u} = -1.9\times 10^{-5}$$
$$\lambda_{1d} = 3.7\times 10^{-5}$$

$Q_W^{\rm SM} = -73.09 \pm 0.03$ (with radiative corrections)

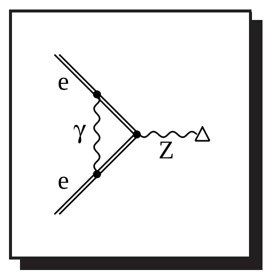
[11] D. E. Groom et al., Euro. Phys. J. C **15**, 1 (2000)

αZ-dependent terms

Perturbation Theory:[12]

$$\rho'_{\mathrm{PV}} = 1 - \frac{\alpha}{2\pi} + \frac{\alpha(m_Z)}{2\pi}\left\{\frac{3}{8s^2}\frac{m_t^2}{m_W^2} + \frac{3}{8s^4}\ln c^2 - \frac{7}{8s^2} + \frac{3\xi}{8s^2}\left(\frac{\ln(c^2/\xi)}{c^2-\xi} + \frac{1}{c^2}\frac{\ln\xi}{1-\xi}\right) - \frac{1}{s^2} + \cdots\right\}$$

The term $-\frac{\alpha}{2\pi}$ is an electromagnetic vertex correction that should be evaluated using bound-state electron propagators.



Assume that the binding corrections modify

$$-\frac{\alpha}{2\pi} \to -\frac{\alpha}{2\pi} \pm \frac{\alpha}{2\pi},$$

and we find $Q_W^{\mathsf{SM}} = -73.09 \to -73.09 \pm 0.08$

[12]W. J. Marciano in *Precision Tests of the Standard Electroweak Model*, Ed. P. Langacker, (World Scientific, Singapore, 1995), p. 182

Conclusion

Using the average of the two most accurate theoretical values (plus Breit) of $E_{\mathrm{PNC}} \times 1.0041$:

$$E_{\mathrm{PNC}} = -9.057 \ \pm 0.037 \ iea_0 \times 10^{-12}(-Q_W/N),$$

where we use the 0.4% error estimate of Bennett and Wieman.[13] Combining this with the experimental value of E_{PNC}/β[14] leads to an experimental value for the weak charge

$$Q_W^{\mathrm{expt}}(^{133}\mathrm{Cs}) = -72.12 \pm (0.28)_{\mathrm{expt}} \pm (0.34)_{\mathrm{theor}}$$

This differs by 2.2 σ from the standard-model value[15],

$$Q_W^{\mathrm{SM}}(^{133}\mathrm{Cs}) = -73.09 \pm 0.03$$

[13] S. C. Bennett and C. E. Wieman, Phys. Rev. Lett. **82**, 2484 (1999); **82**, 4153 (1999); **83**, 889 (1999)

[14] C. S. Wood *et al.*, Science **275**, 1759 (1997)

[15] D. E. Groom et al., Euro. Phys. J. C **15**, 1 (2000).