

# Parity Nonconservation in Cesium: Is the Standard Model in Trouble?

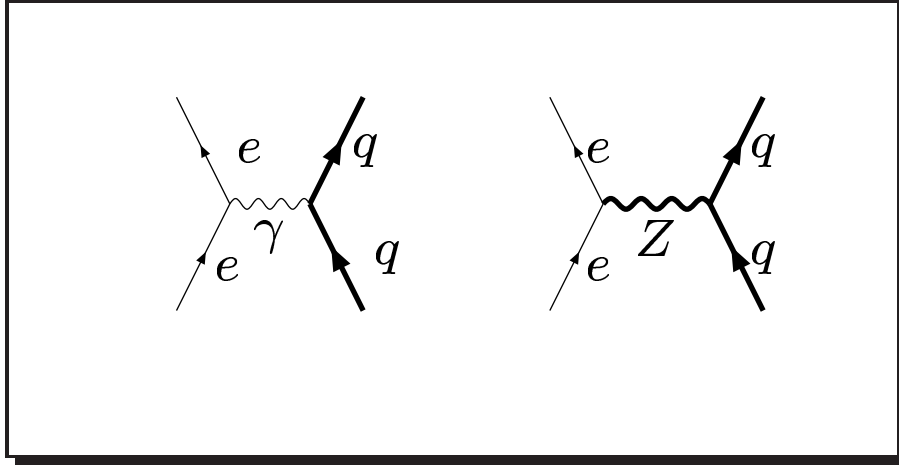
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## **Abstract**

This is a brief review of the current status of PNC in cesium including a discussion of the reported  $2.3\sigma$  disagreement between experiment and the Standard Model

## Overview



$$H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} Q_W \gamma_5 \rho(r)$$

where the *conserved* weak charge is

$$Q_W = -N + Z (1 - 4 \sin^2 \theta_W)$$

For states  $v$  and  $w$  that have the same parity  
 $\langle w | ez | v \rangle = Q_W \times$  “Structure factor”

Measure:  $E_{\text{PNC}} = \langle w | ez | v \rangle$

Calculate: “Structure factor”

Ratio gives  $Q_W$



## Z-e Coupling from Standard Model

$$\begin{aligned}\mathcal{H}^{(1)} &= \frac{G}{\sqrt{2}} (\bar{\psi}_e \gamma_\mu \gamma_5 \psi_e) \sum_i [c_{1p} (\bar{\psi}_{pi} \gamma^\mu \psi_{pi}) \\ &\quad + c_{1n} (\bar{\psi}_{ni} \gamma^\mu \psi_{ni})] \\ \mathcal{H}^{(2)} &= \frac{G}{\sqrt{2}} (\bar{\psi}_e \gamma_\mu \psi_e) \sum_i [c_{2p} (\bar{\psi}_{pi} \gamma^\mu \gamma_5 \psi_{pi}) \\ &\quad + c_{2n} (\bar{\psi}_{ni} \gamma^\mu \gamma_5 \psi_{ni})]\end{aligned}$$

where the standard-model coupling constants are

$$\begin{aligned}c_{1p} &= \frac{1}{2} (1 - 4 \sin^2 \theta_W) \approx 0.038, \\ c_{1n} &= -\frac{1}{2}, \\ c_{2p} &= \frac{1}{2} g_A (1 - 4 \sin^2 \theta_W) \approx 0.047, \\ c_{2n} &= -\frac{1}{2} g_A (1 - 4 \sin^2 \theta_W) \approx -0.047.\end{aligned}$$

In the above,  $g_A \approx 1.25$  is a scale factor for the partially conserved axial current  $A_N$ .

## Nonrelativistic Nucleons

$\mathcal{H}^{(1)}$ :

$$(\bar{\psi}_p \gamma^\mu \psi_p) \rightarrow \phi_p^\dagger \phi_p \delta_{\mu 0} \quad (\bar{\psi}_n \gamma^\mu \psi_n) \rightarrow \phi_n^\dagger \phi_n \delta_{\mu 0}$$

where  $\phi_p$  and  $\phi_n$  are nonrelativistic field operators. From this we extract an “effective” Hamiltonian to be used in the electron sector; namely,

$$H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 [2Z c_{1p} \rho_p(r) + 2N c_{1n} \rho_n(r)].$$

Here,  $\rho_p(r)$  and  $\rho_n(r)$  proton and neutron density functions normalized to 1. Assuming  $\rho_p(r) = \rho_n(r) = \rho(r)$ , we may rewrite the effective Hamiltonian as

$$H_{\text{eff}}^{(1)} = \frac{G}{2\sqrt{2}} \gamma_5 Q_W \rho(r)$$

where

$$Q_W = [2Z c_{1p} + 2N c_{1n}] = -N + Z (1 - 4 \sin^2 \theta_W)$$

## Axial Nuclear Current

$\mathcal{H}^{(2)}$ :

$$(\bar{\psi}_p \gamma^\mu \gamma_5 \psi_p) \rightarrow \phi_p^\dagger \sigma_i \phi_p \delta_{\mu i} \quad (\bar{\psi}_n \gamma^\mu \gamma_5 \psi_n) \rightarrow \phi_n^\dagger \sigma_i \phi_n \delta_{\mu i}$$

The corresponding effective Hamiltonian in the electron sector is obtained from

$$H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} \alpha \cdot [c_{2p} \langle \phi_p^\dagger \sigma \phi_p \rangle + c_{2n} \langle \phi_n^\dagger \sigma \phi_n \rangle]$$

Only unpaired valence nucleons (with polarization corrections) contribute, so the size of  $H^{(2)}$  is smaller than that from  $H^{(1)}$  by a factor of  $\approx 1/A$ .

For a single valence proton, this reduces to:

$$H_{\text{eff}}^{(2)} = -\frac{G}{\sqrt{2}} c_{2p} \frac{\kappa - 1/2}{I(I + 1)} \alpha \cdot \mathbf{I} \rho_p(r)$$

where  $\kappa = \mp(I + 1/2)$  for  $I = L \pm 1/2$ .

## Anapole Moment

PNC in nucleus  $\Rightarrow$  nuclear anapole:



The anapole is a toroidal electromagnetic current localized to the nucleus.

$$H_{\text{eff}}^{(a)} = \frac{G}{\sqrt{2}} K_a \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \rho_p(r)$$

Combining the two spin-dependent interactions:

$$H_{\text{eff}}^{(a)} + H_{\text{eff}}^{(2)} = \frac{G}{\sqrt{2}} K \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \rho_p(r)$$

with

$$K = K_a - \frac{\kappa - 1/2}{\kappa} c_{2p}$$

## Another Spin-Dependent Term

The action of  $\left[ H_{\text{hyperfine}} \times H_{\text{eff}}^{(1)} \right]$  gives yet another nuclear spin-dependent correction

$$H_{\text{eff}}^{(Q_W)} = \frac{G}{\sqrt{2}} K_{Q_W} \frac{\kappa}{I(I+1)} \alpha \cdot \mathbf{I} \rho_p(r)$$

with<sup>1</sup>

$$K_{Q_W}({}^{133}\text{Cs}) \approx 0.0307$$

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<sup>1</sup>C. Bouchiat and C. A. Piketty, Z. Phys. C 49, 91 (1991); Phys. Lett. B 269, 195 (1991).



## Atomic Structure

For the  $6s \rightarrow 7s$  transition in atomic cesium:

$$\langle 7s | ez | 6s \rangle = \sum_n \left\{ \frac{\langle 7s | ez | np_{1/2} \rangle \langle np_{1/2} | H^{(1)} | 6s \rangle}{E_{np} - E_{6s}} + \frac{\langle 7s | H^{(1)} | np_{1/2} \rangle \langle np_{1/2} | ez | 6s \rangle}{E_{np} - E_{7s}} \right\}$$

1.  $\sum_{n=6}^9$  with SD wave functions & energies (90%)
2.  $\sum_{n=10}^{\infty}$  “weak RPA” level (10%)
3. Breit interaction at “weak HF” level (0.2%)
4. Nucleon structure correction  $\rho_N(r)$  (<0.1%)
5.  $H^{(2')}$  contribution at “weak RPA” level

## Singles-Doubles Equations

$$\Psi_v = \Psi_{\text{DHF}} + \delta\Psi$$

$$\delta\Psi = \left\{ \begin{aligned} &\sum_{am} \rho_{ma} a_m^\dagger a_a + \frac{1}{2} \sum_{abmn} \rho_{mnab} a_m^\dagger a_n^\dagger a_b a_a \\ &+ \sum_{m \neq v} \rho_{mv} a_m^\dagger a_v + \sum_{bmn} \rho_{mnvb} a_m^\dagger a_n^\dagger a_b a_v \end{aligned} \right\} \Psi_{\text{DHF}}$$

$$E_C = E_C^{\text{DHF}} + \delta E_C$$

$$E_v = E_v^{\text{DHF}} + \delta E_v$$

(we also include limited triples)

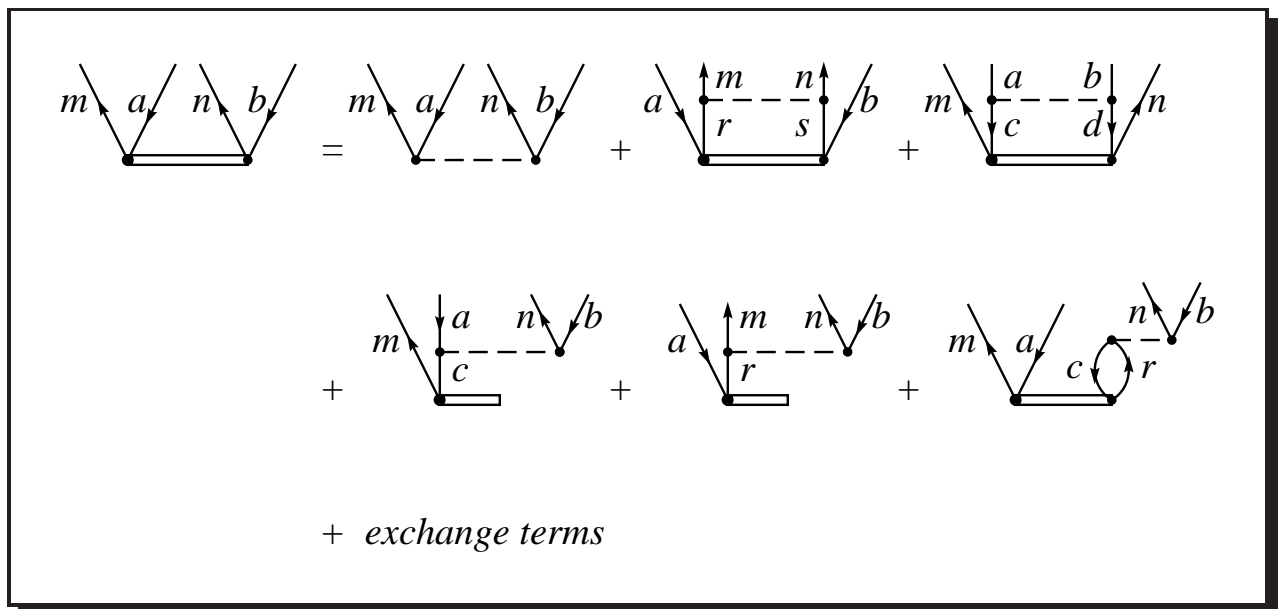
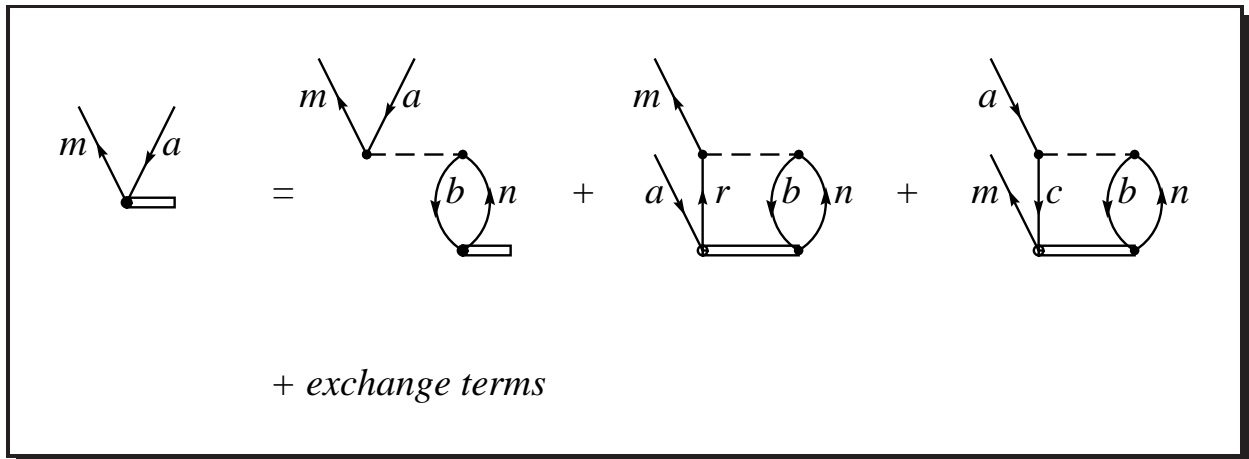
## Core Excitation Equations

$$\begin{aligned}
 (\epsilon_a - \epsilon_m)\rho_{ma} &= \sum_{bn} \tilde{v}_{mban}\rho_{nb} \\
 &+ \sum_{bnr} v_{mbnr}\tilde{\rho}_{nrab} - \sum_{bcn} v_{bcan}\tilde{\rho}_{mncb}
 \end{aligned}$$

$$\begin{aligned}
 (\epsilon_a + \epsilon_b - \epsilon_m - \epsilon_n)\rho_{mnab} &= v_{mnab} \\
 &+ \sum_{cd} v_{cdab}\rho_{mncd} + \sum_{rs} v_{mnr s}\rho_{rsab} \\
 &+ \left[ \sum_r v_{mnr b}\rho_{ra} - \sum_c v_{cnab}\rho_{mc} + \sum_{rc} \tilde{v}_{cnrb}\tilde{\rho}_{mrac} \right] \\
 &+ [ a \leftrightarrow b \quad m \leftrightarrow n ]
 \end{aligned}$$

$$\delta E_C = \frac{1}{2} \sum_{abmn} v_{abmn}\tilde{\rho}_{mnab}$$

$\approx 15,000,000$   $\rho_{mnab}$  coefficients for Cs ( $\ell = 6$ ).



Brueckner-Goldstone Diagrams for the core SD equations.

## Valence Equations

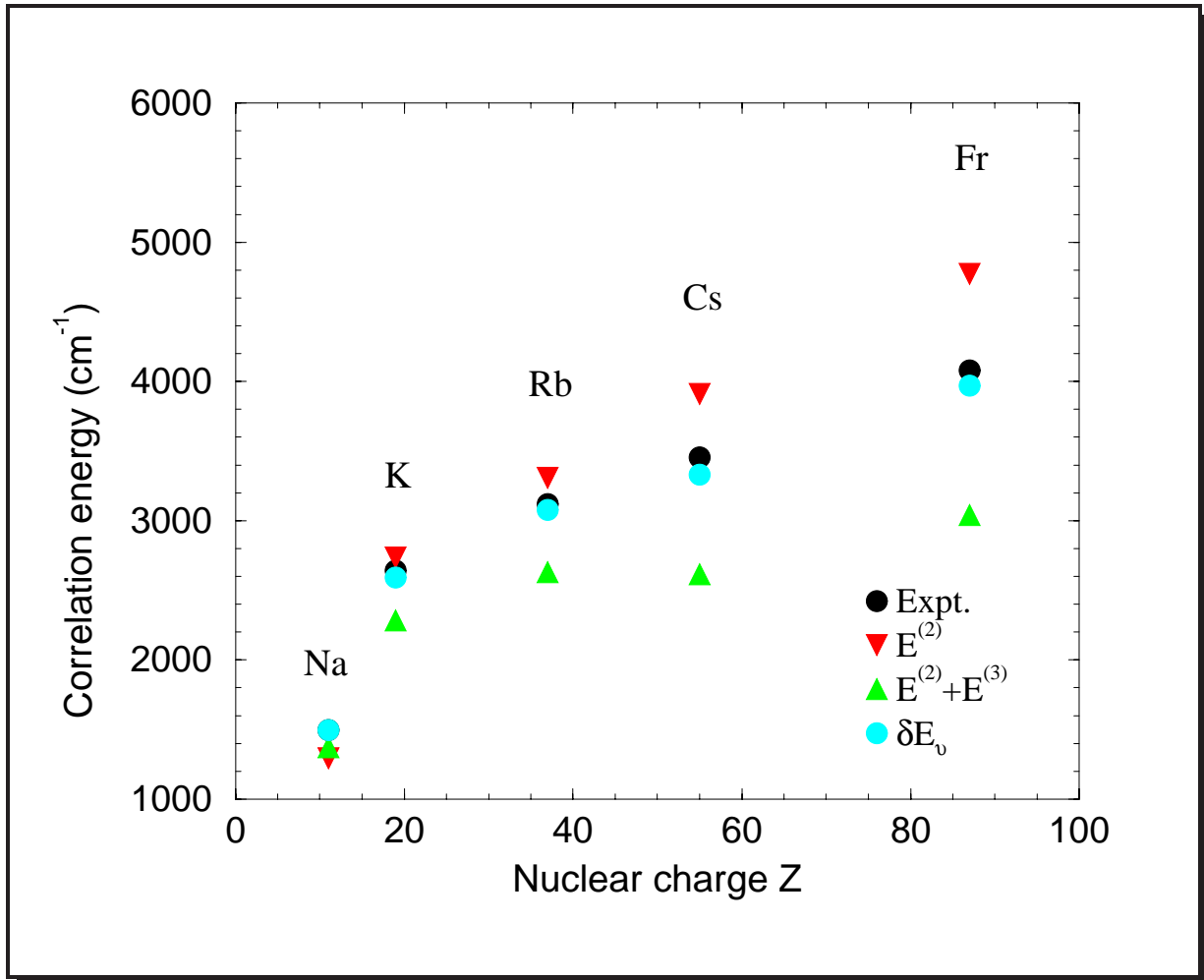
$$\begin{aligned}
 (\epsilon_v - \epsilon_m + \delta E_v) \rho_{mv} &= \sum_{bn} \tilde{v}_{mbvn} \rho_{nb} \\
 &+ \sum_{bnr} v_{mbnr} \tilde{\rho}_{nrub} - \sum_{bcn} v_{bcvn} \tilde{\rho}_{mncb}
 \end{aligned}$$

$$\begin{aligned}
 (\epsilon_v + \epsilon_b - \epsilon_m - \epsilon_n + \delta E_v) \rho_{mnvb} &= v_{mnvb} \\
 &+ \sum_{cd} v_{cdvb} \rho_{mncd} + \sum_{rs} v_{mnrsv} \rho_{rsvb} \\
 &+ \left[ \sum_r v_{mnrsv} \rho_{rv} - \sum_c v_{cnvb} \rho_{mc} + \sum_{rc} \tilde{v}_{cnrb} \tilde{\rho}_{mrv} \right] \\
 &+ [ v \leftrightarrow b \quad m \leftrightarrow n ]
 \end{aligned}$$

$$\begin{aligned}
 \delta E_v &= \sum_{ma} \tilde{v}_{vavm} \rho_{ma} + \sum_{mab} v_{abvm} \tilde{\rho}_{mvab} \\
 &+ \sum_{mna} v_{vbmna} \tilde{\rho}_{mnav}
 \end{aligned}$$

$\approx 1,000,000$   $\rho_{mnvb}$  coefficients for each state (Cs)

## SD Correlation Energy



Ground-state correlation energies for alkali-metal atoms

## Calculations of cesium $6s \rightarrow 7s$ PNC Amplitude

Group	$E_{\text{PNC}}$	Breit
Novosibirsk	$0.908 \pm 0.010$	-
Notre Dame	$0.905 \pm 0.010$	HF-level

$$\text{units: } i e a_0 \times 10^{-11} \frac{Q_W}{-N}$$

## Status of PNC Experiments

(a) Optical rotation:  $n_+ \neq n_- \quad \phi = E_{\text{PNC}}/M_1$

$6p_{1/2} - 6p_{3/2}$ transition		
Element	Group	$10^8 \times \phi$
Thallium	Oxford	-15.7(5)
Thallium	Seattle	-14.7(2)
Lead	Oxford	-9.8(1)
Lead	Seattle	-9.9(1)
Bismuth	Oxford	-10.1(20)

(b) Stark interference: Add  $E(t) = A \cos \omega t$  and detect the heterodyne signal  $R = E_{\text{PNC}}/\beta$

$6s_{1/2} - 7s_{1/2}$ (mV/cm)			
Element	Group	$R_{4-3}$	$R_{3-4}$
Cesium	Paris (1984)	-1.5(2)	-1.5(2)
Cesium	Boulder (1988)	-1.64(5)	-1.51(5)
Cesium	Boulder (1997)	-1.635(8)	-1.558(8)



## Bennett & Wieman<sup>2</sup>

- Measured  $\beta = 27.024(43)_{\text{expt}}(67)_{\text{theor}}a_0^3$
- Updated theory error estimates!

Expt	Tests	Diff $\times 10^3$		$\sigma_{\text{expt}}$
		Novo	ND	
Stark(6s-7s)	$\langle 7p    ez    ns \rangle$	-3.4	-0.7	1.0
$\tau_{6p_{1/2}}$	$\langle 6s    ez    6p \rangle$	-4.4	4.3	1.0
$\tau_{6p_{3/2}}$	$\langle 6s    ez    6p \rangle$	-2.6	7.9	2.3
$\alpha$	$\langle vs    ez    np \rangle$	-	-1.4	3.2
$\beta$	$\langle vs    ez    np \rangle$	-	-0.8	3.0
$A_{6s}$	$\Psi_{6s}(0)$	1.8	-3.1	-
$A_{7s}$	$\Psi_{7s}(0)$	-6.0	-3.4	0.2
$A_{6p_{1/2}}$	$\langle 1/r^3 \rangle_{6p}$	-6.1	2.6	0.2
$A_{7p_{1/2}}$	$\langle 1/r^3 \rangle_{7p}$	-7.1	-1.5	0.5

<sup>2</sup>S. C. Bennett & C. E. Wieman, Phys. Rev. Letts. **82**, 2484 (1999).

## Cesium: Theory vs. Experiment

$$\beta = 27.024 (43)_{\text{exp}} (67)_{\text{th}} a_0^3 \quad (1999)$$

(eliminating axial vector + anapole contribution)

$$\Im(E_{\text{PNC}}) = -0.8379 (37)_{\text{exp}} (21)_{\text{th}} \times 10^{-11} |e| a_0$$

(dividing by theoretical matrix element)

$$Q_W = -72.06 (29)_{\text{exp}} (34)_{\text{th}}$$

Marciano & Rosner (with radiative corrections)

$$Q_W^{SM} = -73.09 (03)_{\text{rad. corr.}}$$

$$\text{Expt.} - \text{Theory} = 2.3 \sigma$$

This difference has been cited as evidence for “new physics” beyond the Standard Model!

## Result for Anapole Moment

Difference  $R_{3-4} - R_{4-3}$  leads to:

$$\begin{aligned} K &= 0.441 \text{ (63)} \\ -(\kappa - 1/2)/\kappa K^{(2)} &= 0.055 \\ K^{(QW)} &= 0.031 \\ K^{(a)} &= 0.355 \text{ (63)} \end{aligned}$$

Theoretical estimates<sup>3</sup>

$$K^{(a)} = 0.25 - 0.75$$

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<sup>3</sup>W. C. Haxton and C. E. Wieman, arXiv:nucl-th/0104026

Table 10.4: (continued)

Quantity	Value	Standard Model	Pull
$m_t$ [GeV]	$174.3 \pm 5.1$	$172.9 \pm 4.6$	0.3
$M_W$ [GeV]	$80.448 \pm 0.062$	$80.378 \pm 0.020$	1.1
	$80.350 \pm 0.056$		-0.5
$M_Z$ [GeV]	$91.1872 \pm 0.0021$	$91.1870 \pm 0.0021$	0.1
$\Gamma_Z$ [GeV]	$2.4944 \pm 0.0024$	$2.4956 \pm 0.0016$	-0.5
$\Gamma(\text{had})$ [GeV]	$1.7439 \pm 0.0020$	$1.7422 \pm 0.0015$	—
$\Gamma(\text{inv})$ [MeV]	$498.8 \pm 1.5$	$501.65 \pm 0.15$	—
$\Gamma(\ell^+\ell^-)$ [MeV]	$83.96 \pm 0.09$	$84.00 \pm 0.03$	—
$\sigma_{\text{had}}$ [nb]	$41.544 \pm 0.037$	$41.480 \pm 0.014$	1.7
$R_e$	$20.803 \pm 0.049$	$20.740 \pm 0.018$	1.3
$R_\mu$	$20.786 \pm 0.033$	$20.741 \pm 0.018$	1.4
$R_\tau$	$20.764 \pm 0.045$	$20.786 \pm 0.018$	-0.5
$R_b$	$0.21642 \pm 0.00073$	$0.2158 \pm 0.0002$	0.9
$R_c$	$0.1674 \pm 0.0038$	$0.1723 \pm 0.0001$	-1.3
$A_{FB}^{(0,e)}$	$0.0145 \pm 0.0024$	$0.0163 \pm 0.0003$	-0.8
$A_{FB}^{(0,\mu)}$	$0.0167 \pm 0.0013$		0.3
$A_{FB}^{(0,\tau)}$	$0.0188 \pm 0.0017$		1.5
$A_{FB}^{(0,b)}$	$0.0988 \pm 0.0020$	$0.1034 \pm 0.0009$	-2.3
$A_{FB}^{(0,c)}$	$0.0692 \pm 0.0037$	$0.0739 \pm 0.0007$	-1.3
$A_{FB}^{(0,s)}$	$0.0976 \pm 0.0114$	$0.1035 \pm 0.0009$	-0.5
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	$0.2321 \pm 0.0010$	$0.2315 \pm 0.0002$	0.6

**Table 10.4:** (continued)

Quantity	Value	Standard Model	Pull
$A_e$	$0.15108 \pm 0.00218$	$0.1475 \pm 0.0013$	1.7
	$0.1558 \pm 0.0064$		1.3
	$0.1483 \pm 0.0051$		0.2
$A_\mu$	$0.137 \pm 0.016$		-0.7
$A_\tau$	$0.142 \pm 0.016$		-0.3
	$0.1425 \pm 0.0044$		-1.1
$A_b$	$0.911 \pm 0.025$	$0.9348 \pm 0.0001$	-1.0
$A_c$	$0.630 \pm 0.026$	$0.6679 \pm 0.0006$	-1.5
$A_s$	$0.85 \pm 0.09$	$0.9357 \pm 0.0001$	-1.0
$R^-$	$0.2277 \pm 0.0021 \pm 0.0007$	$0.2299 \pm 0.0002$	-1.0
$\kappa^\nu$	$0.5820 \pm 0.0027 \pm 0.0031$	$0.5831 \pm 0.0004$	-0.3
$R^\nu$	$0.3096 \pm 0.0033 \pm 0.0028$	$0.3091 \pm 0.0002$	0.1
	$0.3021 \pm 0.0031 \pm 0.0026$		-1.7
$g_V^{\nu e}$	$-0.035 \pm 0.017$	$-0.0397 \pm 0.0003$	—
	$-0.041 \pm 0.015$		-0.1
$g_A^{\nu e}$	$-0.503 \pm 0.017$	$-0.5064 \pm 0.0001$	—
	$-0.507 \pm 0.014$		0.0
$Q_W(\text{Cs})$	$-72.06 \pm 0.28 \pm 0.34$	$-73.09 \pm 0.03$	2.3
$Q_W(\text{Tl})$	$-114.8 \pm 1.2 \pm 3.4$	$-116.7 \pm 0.1$	0.5
$\frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow ce\nu)}$	$3.26^{+0.75}_{-0.68} \times 10^{-3}$	$3.15^{+0.21}_{-0.20} \times 10^{-3}$	0.1

## Possible Explanation of $2.3\sigma$

The previous result suggested the possible existence of a  $Z'$  particle to several authors:

1. R. Casalbuoni, S. De Curtis, D. Dominici, and R. Gatto, Phys. Lett. **B460**, 135 (1999).
2. J. L. Rosner, Phys. Rev. **D61**, 016006 (2000).
3. J. Erler and P. Langacker, Phys. Rev. Lett. **84**, 212 (2000).

## Breit Revisited

Weak HF level:

$$(h_0 + V^{\text{HF}} - \epsilon_v^{\text{HF}}) \tilde{\psi}_v^{\text{HF}} = -h_{\text{PNC}} \psi_v^{\text{HF}}$$

$$E_{\text{PNC}} = \langle \psi_{7s}^{\text{HF}} | ez | \tilde{\psi}_{6s}^{\text{HF}} \rangle + \langle \tilde{\psi}_{7s}^{\text{HF}} | ez | \psi_{6s}^{\text{HF}} \rangle$$

Type	$\langle 7s   ez   \tilde{6s} \rangle$	$\langle \tilde{7s}   ez   6s \rangle$	$E_{\text{PNC}}$
Coul	0.27492	-1.01439	-0.73947
+Breit	0.27411	-1.01134	-0.73722
$\Delta\%$	-0.29%	-0.30%	-0.30%

This correction was included in ND calculation

$$E_{\text{PNC}} = -0.907 + 0.002 = -0.905$$

but not in Novosibirsk calculation.

## Derevianko's observation

Brueckner level:

$$\left( h_0 + V^{\text{HF}} + \hat{\Sigma} - \epsilon_v^{\text{Br}} \right) \tilde{\psi}_v^{\text{Br}} = -h_{\text{PNC}} \psi_v^{\text{Br}} - \delta V_{\text{PNC}}^{\text{HF}} \psi_v^{\text{Br}}$$

$$E_{\text{PNC}} = \langle \psi_{7s}^{\text{Br}} | ez + \delta_{\text{RPA}}(ez) | \tilde{\psi}_{6s}^{\text{Br}} \rangle + \langle \tilde{\psi}_{7s}^{\text{Br}} | ez + \delta_{\text{RPA}}(ez) | \psi_{6s}^{\text{Br}} \rangle$$

Type	$\langle 7s   ez   \tilde{6s} \rangle$	$\langle \tilde{7s}   ez   6s \rangle$	$E_{\text{PNC}}$
Coul	0.43942	-1.33397	-0.89456
+ Breit	0.43680	-1.32609	-0.88929
$\Delta\%$	-0.60%	-0.59%	-0.59%

Using this result for the Breit correction, the final theoretical PNC amplitudes become

Group	Coul	Breit	$E_{\text{PNC}}$
Novosibirsk	0.908	-0.005	0.903
Notre Dame	0.907	-0.005	0.902



## Summary

With Breit corrections:

$$E_{\text{PNC}}^{\text{theor}} = -0.902(4) \quad \text{or} \quad (10)?$$

and the deviation of the  $Q_W$  from the standard model is reduced to  $1.5\sigma$  if 0.4% theoretical accuracy is still assumed. However, if a more realistic 1% theoretical uncertainty is assumed, the corresponding value of the weak-charge becomes

$$Q_W(^{133}\text{Cs}) = -72.42(0.28)_{\text{expt}}(0.74)_{\text{theor}}$$

and shows **NO** significant deviation from the standard model.<sup>4</sup>

**HELP! – accurate calculations needed – HELP!**

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<sup>4</sup>A. Derevianko, Phys. Rev. Lett. **85**, 1618 (2000); V. A. Dzuba *et al.*, Phys. Rev. A **63**, 044103 (2001); M. G. Kozlov *et al.*, arXiv:physics (0101053).