Zeno's paradoxes

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The Racetrack

1. Any distance is divisible into infinitely many smaller distances.
2. To move from a point x to a point y, one has to move through all the distances into which the distance from x to y is divisible.
3. To move from one point to another in a finite time, one has to traverse infinitely many distances in a finite time. (1,2)
4. It is impossible to traverse infinitely many distances in a finite time.

C. It is impossible to move from one point to another in a finite time. (3,4)

The Achilles

Suppose that the Tortoise and Achilles are racing to some point z, and that Achilles begins at point x, and the Tortoise begins at point y, where y is between x and z. Then we argue as follows that no matter what distances are involved, no matter how slow the Tortoise is, and no matter how fast Achilles is, Achilles can never catch the Tortoise, so long as the Tortoise never stops moving:

1. To traverse the distance between x and y, Achilles requires some interval of time.
2. During every interval of time, the Tortoise moves some distance.

C. By the time Achilles reaches y, the Tortoise is some distance beyond y. (1,2)

Since we made no particular assumptions about the distance between x and y or the length of the interval of time, the argument appears to generalize, and show that the Tortoise can never catch Achilles. That is, it appears to show that:

For every interval of time in the series, at the end of that interval the Tortoise is still ahead of Achilles.

Does this argument require any assumptions about the infinite divisibility of time or space?

The Arrow

1. At any one instant, an arrow does not move.
2. Nothing happens between one instant and the next.
3. The arrow does not move between instants. (2)

C. The arrow does not move. (1,3)

What assumption about time is required to make sense of talk about 'the next instant'?

The Stadium

Suppose that space and time are not continuous, so that we can speak of the point in space next to another (i.e., with none in between), and the moment in time after another (i.e., with no intervening moment).

Now consider the following arrangement of particles, each occupying a spot in a grid of adjacent points in space:

Moment 1

	a	a'	a''	
	b	b'	b''	
	c	c'	c''	

Now suppose that in the next moment, each of the a-particles moves one spot to the right, and each of the c-particles moves one spot to the left. Then we're left with the following configuration of particles in space:

Moment 2

		a	a'	a''
	b	b'	b''	
c	c'	c''		

Now consider, for example, particle a and particle c'. In Moment 1, a is to the left of c'; in Moment 2, a is to the right of c'. So it seems that at some point, a must have passed c'. But this is impossible, since it did not happen at Moment 1, and did not happen at Moment 2, and by hypothesis there is no moment between the two.

One way to view these paradoxes is as posing a dilemma for the believer in the reality of motion. Either space and time are continuous, or they are not. If they are, then the Racetrack and the Achilles show that motion is impossible. If they are not, then the Arrow and the Stadium show that motion is impossible.