An introduction to paradoxes

PHIL 20229
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January 16, 2008

1 What is a paradox?

Sainsbury gives the following description of a paradox:

“an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises.” (1)

This description is fine as far as it goes, but it also raises some questions. What are premises and conclusions? What does it mean for a premise or conclusion to be acceptable or unacceptable? What does it mean for reasoning to be acceptable or unacceptable?

1.1 Premises and conclusions

A good place to start here is by thinking about arguments as divisible into two parts, premises and conclusion. We’re all familiar in an informal way with arguments. Consider, for example, the following argument:

Notre Dame’s football team will win more games this year than they did last year. After all, they won only three games last year, and there’s at least four teams on the schedule for next year that they will definitely beat.
You can distinguish here what the speaker is arguing for — the conclusion of the argument — and what the speaker is using to support that conclusion — the premises. Here the conclusion seems to be:

Notre Dame’s football team will win more games this year than they did last year.

and there seem to be two premises:

Notre Dame’s football team won only three games last year.
Notre Dame’s football team will win at least four games next year.

Paradoxes are arguments, so in this course we will be concerned with lots of arguments. In order to talk about arguments, it is useful to have a way of writing them out which makes their premises and conclusion explicit, as follows:

1. Notre Dame’s football team won only three games last year.
2. Notre Dame’s football team will win at least four games next year.
C. Notre Dame’s football team will win more games this year than they did last year. (1,2)

Here the premises are numbered, the horizontal line marks the move from premises to conclusion, and the numbers after the conclusion indicate the premises from which the conclusion is supposed to follow.

1.2 Validity

What do we mean when we say that the conclusion follows from the premises? What we mean is that if the premises are true, then the conclusion must be true; or, equivalently, it is impossible for the premises to be true and the conclusion false. In this sense, the truth of the premises guarantees the truth of the conclusion.

If the conclusion follows from the premises, we say that the argument is valid.

It is important to see that saying that an argument is valid does not involve saying that the premises are true; it just involves saying that if the premises are true, the conclusion must be as well. For example, consider the following argument:

1. I am wearing bright purple pants.
C. I am not wearing black pants.

This argument is valid; if the premise is true, the conclusion must be true. The premise is not, as it turns out, true; but that is no objection to the argument’s validity.
1.3 A definition of paradoxes

Now remember the description of paradoxes with which we began:

“an apparently unacceptable conclusion derived by apparently acceptable reasoning from apparently acceptable premises.”

For our purposes, we’ll say that a premise or conclusion is apparently acceptable if and only if it is apparently true, and apparently unacceptable if and only if it is apparently false. A piece of reasoning is apparently acceptable if and only if it is apparently valid.

So a paradox is just an argument with the following three features:

1. The premises are all apparently true.
2. The conclusion is apparently false.
3. The argument is apparently valid.

2 Three ways to solve a paradox

Why is the word ‘apparently’ in the above list? Why not just say that a paradox is an argument with true premises and a false conclusion, which is also valid? Because it is impossible for there to be such an argument; it is impossible for there to be an argument which has true premises and a false conclusion and is such that it is impossible for its premises to be true and its conclusion false. A paradox is problematic because it seems to be an example of an argument of this sort, and so seems to be an actual example of something which we know to be impossible.

For this reason, we know that at least one of the three appearances catalogued above must be misleading. This yields three ways to solve a paradox: one can either (1) find a premise which, contrary to appearances, is false; (2) accept that the conclusion, which seems false, is really true; or (3) identify a flaw in the apparently valid reasoning which allows the premises to be true, and the conclusion false.

Some paradoxes really can be definitively solved. Next we’ll discuss a few.