

# Russell's view of propositions in the *Principles of Mathematics*

PHIL 43904  
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September 11, 2007

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## 1 Propositions, terms, things, concepts

Russell's aim in this chapter is the analysis of propositions. This tells us that he thinks that propositions are the sorts of things which can be analyzed — so they must be the sorts of things that, in some sense or other, have parts.

What are the parts of propositions? Russell says

“... every word occurring in a sentence must have *some* meaning ... The correctness of our philosophical analysis of a proposition may therefore be usefully checked by the exercise of assigning the meaning of each word in the sentence expressing the proposition.” (§46)

Like Moore, Russell thinks of propositions as what are expressed by sentences. The parts of propositions are then (to a first approximation) what are expressed by the parts of sentences — words.

Russell introduces a few bits of terminology for talking about the meanings of words. The first is ‘term’:

“Whatever may be an object of thought, or may occur in any true or false proposition . . . I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity. . . anything . . . that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false.” (§47)

So it seems as though Russell uses ‘term’ where Moore uses ‘concept’ — as a word for whatever the meanings of words, and the constituents of propositions, are. Russell also shares Moore’s assumption that everything which exists can be an object of thought — that there is no difference in kind between the constituents of thoughts and the constituents of reality.

We can also see the beginnings of the problem that we discussed briefly in connection with Moore — the puzzle of nonbeing. Russell says that “to deny that such and such a thing is a term must always be false.” But then consider a sentence like

Pegasus does not exist.

which seems true. ‘Pegasus’ seems to have a meaning, and we know that all meanings are terms, so it seems that this sentence denies that a certain term exists. But can something be a term without existing? This seems difficult to make sense of, even though Russell thinks that there are many nonexistent terms (§48). We’ll return to this problem, and Russell’s later views about it, when we discuss his theory of descriptions.

Russell divides the class of terms into two:

“Among terms, it is possible to distinguish two kinds, which I shall call respectively *things* and *concepts*. The former are the terms indicated by proper names, the latter those indicated by all other words.” (§48)

Russell then explains the distinction between proper names and other expressions in terms of an intuitive idea of what a proposition is about. In a simple sentence like

Socrates is human.

‘Socrates’ is the proper name, since Socrates is what the proposition is about.

## 2 Verbs and verbal nouns express the same concept

This raises the problem of how to classify the meanings of words like ‘humanity.’ It seems that ‘humanity’ stands for something which a simple proposition can be about, as in

Humanity is doomed.

So it seems that ‘humanity’ must stand for a thing, rather than a concept. But in a sentence like

Socrates is human.

it seems clear that ‘Socrates’ stands for a thing, and so that ‘human’ stands for a concept. So is the correct view that ‘human’ and ‘humanity’ have different meanings?

Russell thinks not, and gives the following argument:

“For suppose that *one* as adjective differed from 1 as term. In this statement, *one* as adjective has been made into a term; hence either it has become 1, which is self-contradictory; or there is some other difference between *one* and 1 in addition to the fact that the first denotes a concept not a term while the second denotes a concept which is a term. But in this latter hypothesis, there must be propositions concerning *one* as term, and we shall still have to maintain propositions concerning *one* as adjective as opposed to *one* as term; yet all such propositions must be false, since a proposition about *one* as adjective makes *one* the subject, and is therefore really about *one* the term. In short, if there were any adjectives which could not be made into substantives [i.e., proper names] without change of meaning, all propositions concerning such adjectives . . . would be false, and so would the proposition that all such propositions are false, since this turns the adjectives into substantives. But this state of things is self-contradictory.” (§49)

Here Russell is arguing against the view that some adjectives have a meaning which cannot be the meaning of any proper name. The argument seems to run as follows: suppose that there is such an adjective *A*, and let *C* be the concept *A* expresses. Now consider a sentence which expresses some proposition of which *C* is the subject, like ‘*C* is the concept expressed by *A*.’ This is true. But this contradicts our initial supposition, since in this sentence ‘*C*’ is a proper name and yet has the same meaning as the adjective *A*.

How should Russell’s opponent respond? What should she say about sentences like ‘*C* is the concept expressed by *A*’, and about words like ‘*C*’?

Suppose that Russell’s opponent concedes that sentences like ‘*C* is the concept expressed by *A*’ are false. It seems that Russell thinks that this response is self-contradictory. The above quote continues as follows:

“In short, if there were any adjectives which could not be made into substantives [i.e., proper names] without change of meaning, all propositions concerning such adjectives . . . would be false, and so would the proposition that all such propositions are false, since this turns the adjectives into substantives. But this state of things is self-contradictory.”

I am not sure exactly what contradiction Russell has in mind here. Suppose again that Russell’s opponent here says that

$C$  is the concept expressed by  $A$ .

is false. She should, it seems, say that the following is true:

All propositions which say that something is a concept are false.

Russell's charge seems to be that this view is self-contradictory, since the above sentence entails its own falsity. But it is hard to see why he thinks this, since the above sentence does not seem to express a proposition which says that something is a concept. We might see this sentence as having the following form:

$$\forall x [(x \text{ is a proposition} \ \& \ \exists y (x \text{ says that } y \text{ is a concept})) \rightarrow x \text{ is false}]$$

But this does not seem to express a proposition which attributes being a concept to something.

But even if Russell does not succeed in showing that his opponent is committed to a contradiction, his view does have considerable plausibility. If we take seriously the idea that adjectives stand for something, then it is hard to see how it could be impossible for us to name those things.

(This somewhat complicates the distinction between things and concepts. The two categories are supposed to be mutually exclusive, but jointly exhaustive of the more general category of terms. But then we can't define things as expressed by proper names, where proper names say what a sentence is about — as we have just seen, verbal nouns can contribute what a sentence is about, even though they stand for concepts rather than things. See the second paragraph in §48 for Russell's discussion of this point.)

### 3 The problem of the unity of the proposition

However, Russell's view — that verbs have the same meanings as the corresponding verbal nouns — leads to a notorious problem: the problem of the unity of the proposition:

“By transforming the verb, as it occurs in a proposition, into a verbal noun, the whole proposition can be turned into a single logical subject, no longer asserted, and no longer containing in itself truth or falsehood.” (§52)

Russell discusses the following pair of expressions:

Caesar died

The death of Caesar

He suggests that these are the same with respect to their meaning, since verbs have the same meaning as the corresponding verbal nouns; and yet the former expresses a proposition, since it can be true or false, whereas the latter does not.

In fact, the situation is even worse; not only can we switch from a sentence which expresses a proposition to a description without change in the meaning of any expression, we can also switch from a sentence to a combination of words which is not even grammatical:

Caesar dies

Caesar death

On the view being defended, these two combinations of words seem to have the same meaning. But if a proposition just is the meaning of a sentence, how can the first express a proposition, if the second does not?

We can solve the problem by saying that verbs have a kind of meaning which a proper name can never have; but that leads to the problems discussed in the preceding section, and it is clearly not a view that Russell will endorse.

A corresponding problem, as Russell notes in §54, arises when we consider relational propositions, such as the one expressed by

A differs from B.

This appears to have the same meaning as

A difference B.

even though the latter clearly does not express a proposition.

#### 4 Russell's solutions to the problem

In §52 and §54, Russell offers two apparently distinct solutions to the problem.

##### 4.1 *'an ultimate notion of assertion'*

In §51, Russell says

“There appears to be an ultimate notion of assertion, given by the verb, which is lost as soon as we substitute a verbal noun . . .”

Russell here seems to be suggesting that verbs contribute two things to sentences: their meaning, which is a concept, and the ‘ultimate notion of assertion’, whereas the corresponding verbal nouns only contribute the concept. His idea seems to be that the ultimate notion of assertion is required for a form of words to express a proposition.

Why the ‘ultimate notion of assertion’ couldn’t really have much to do with assertion; expressing propositions without asserting them. So what could it be?

#### 4.2 Relations that ‘actually relate’

In §54, Russell gives a different response to the problem:

“The twofold nature of the verb, as actual verb and as verbal noun, may be expressed, if all verbs are held to be relations, as the difference between a relation in itself and a relation actually relating. Consider, for example, the proposition ‘*A* differs from *B*’. The constituents of this proposition, if we analyze it, appear to be only *A*, difference, *B*. Yet these constituents, thus placed side by side, do not reconstitute the proposition. The difference which occurs in the proposition actually relates *A* and *B* . . .”

We run into an immediate problem here with false propositions. Suppose that ‘*A* differs from *B*’ is false. Then, we would naturally think, difference does *not* actually relate *A* and *B* — otherwise, the proposition would be true. Does this mean that the sentence does not, after all, express a proposition — and, by parallel argument, that no false sentences ever express propositions?

### 5 Russell’s bad habits

In this chapter of the *Principles*, Russell does two things which you should avoid, and which make the chapter more difficult to understand than it should be.

First, he introduces a word which he will use in the chapter, and then proceeds to use it in two different ways: ‘term.’ Russell moves back and forth between using ‘term’ as Moore used ‘concept’ — as a general term for the meaning of a word, whatever sort of word it might be — and using it as he himself uses ‘thing’ — as a term for the special sorts of things which are the meanings of proper names.

Second, he confuses talk about words and talk about what the words stand for. (This is always bad, but it is worse in a chapter about the relations between words and the things for which they stand, and yet worse from someone who criticized his opponents for failing to recognize this distinction.) Here are some examples of this:

“For suppose that *one* as adjective differed from 1 as term . . .”

An adjective is a kind of word, which might be contrasted with a proper name; a term (in the restrictive sense) is the meaning of a proper name, which might be contrasted with the meaning of an adjective, i.e. a concept. So Russell should have said either

“For suppose that *one* as concept differed from 1 as term (thing) . . .”

or

“For suppose that *one* as the meaning of an adjective differed from 1 as the meaning of a name . . .”

Another example can be found in §51, in which the following three passages occur:

- “a proposition . . . does not itself contain words: it contains the entities indicated by words”
- “when *a man* occurs in a proposition . . .”
- “the entity indicated by *John* does not have a meaning, . . .”

How is Russell using italics here?

A trickier case: try to make sense of Russell’s use of italics and quotation marks in the passage mentioned above:

“If we say ‘*kills* does not mean the same as *to kill*,’ we have already made *kills* a subject, and we cannot say that the concept expressed by the word *kills* cannot be made a subject.”

Suppose that italics here are being used as quotation marks, to form the name of a word. Then you should ask: does it follow from the fact that ‘kills’ is a subject that the concept expressed by the word ‘kills’ can be made a subject? What does it mean for ‘kills’ to be a subject — does it mean that ‘kills’ is a name, or that ‘kills’ is a thing which is part of the relevant proposition?