

The logic of the *Tractatus* and Church's theorem

PHIL 43904
Jeff Speaks

November 20, 2007

Wittgenstein makes a number of bold claims about logical truth in the *Tractatus*. Consider, for example, the following two remarks:

- 5.32 All truth functions are results of successive applications to elementary propositions of a finite number of truth-operations.
- 6.126 One can calculate whether a proposition belongs to logic, by calculating the logical properties of the *symbol*. . . .

The remarks seem to indicate that Wittgenstein thought that, since every proposition is the result of a finite number of truth operations, it was effectively determinable, for any proposition, whether that proposition was a truth of logic or not.

However, that there is a problem here is shown by Church's theorem. This theorem says that any logical system involving quantification and two-place predicates will be undecidable: that is, it will not be the case that, in a finite number of steps, one will be able to determine whether a given formula is a truth of logic.

Given the view that all propositions are truth-functions of elementary propositions, Wittgenstein's faces the following dilemma:

- The logic of the *Tractatus* lacks the expressive power to formulate some claims involving quantification and two-place predicates; hence, since all propositions are truth-functions of elementary propositions, these are not meaningful propositions.
- It is not always possible to calculate whether a proposition belongs to logic.

Which of these options should Wittgenstein choose, given his other commitments in the *Tractatus*?

(For more discussion, See Fogelin, *Wittgenstein*, and Soames, *Philosophical Analysis in the Twentieth Century*, v. 1)