

Challenges to the view that all propositions are
truth-functions of elementary propositions
(§§5.52-5.5423)

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We have interrupted our sequential exposition the *Tractatus* to jump ahead to the statement of the general form of the proposition. On the way to this conclusion, Wittgenstein considers three kinds of propositions which challenge his view that all propositions are truth-functions of elementary propositions.

1 Generality (5.52-5.5262)

One initially problematic class of propositions is the class of general propositions, including those that we would symbolize as

$$\forall x Fx$$

Such propositions are problematic because there is an intuitive problem in representing them as truth-functions of propositions about particular objects. The only plausible way to analyze such general propositions seems to be as long conjunctions:

$$Fa \ \& \ Fb \ \& \ \dots \ \& \ Fn$$

where a - n collectively name each object. But it seems as though the conjunction is not equivalent to the general claim. After all, it seems that there could have been some other object which doesn't actually exist and so is not named by any of the terms in the long

conjunction - the general statement, but not the conjunction, will then imply that this object is F.

Nonetheless, Wittgenstein claims that general claims are truth-functions of elementary propositions:

5.52 If ξ has as its values all the values of a function f_x for all values of x , then $N(\bar{\xi}) = \neg(\exists x).fx$.

Remember the definition of ' N '. The idea is that if we consider the set of all propositions of the form ' fx ', then the joint negation of that set - the conjunction of the negations of every member of the set - will be equivalent to the claim that there is no object which is 'f'. The importance of this point for general propositions follows from the fact that the negation of an existential claim like this one is a general proposition: the claim that everything is 'not-f.' So here Wittgenstein claims that we can analyze this general claim as, in effect, a long conjunction of the negations of attributions of 'f' to particular objects.

From this it is but a short step to the claim that we can analyze all general claims in similar terms. If we can analyze

$$\forall x\neg Fx$$

as the joint negation of a set S of propositions, then we can analyze

$$\forall xFx$$

as the joint negation of the set of propositions consisting of the negations of every member of S .

But, as we discussed earlier, there's a problem with the idea that a universal generalization is just a conjunction of particular claims. The problem is that for any collection of particular claims

$$Fn_1 \ \& \ Fn_2 \ \& \ Fn_3 \ \& \ \dots \ \& \ Fn_m$$

and the corresponding universal generalization

$$\forall x(Fx)$$

it always seems possible for the first to be true and the second false (if there is some object which is not one of $n_1 \dots n_m$ which is not F), or the second true and the first false (if one of $n_1 \dots n_m$ does not exist). How does Wittgenstein deal with this problem?

The answer to this problem comes from another part of Wittgenstein's system. The problem for the conjunctive analysis of general claims seemed to be tied to the possibility of the truth conditions of the general claim and its analysis diverging in cases where one

possible situation contains an object which does not exist in the other possible situation. But recall that for Wittgenstein, possibility and necessity are alike explained in terms of a stable, necessarily existing supply of objects. So, according to Wittgenstein's metaphysics, the problematic possibilities are not genuine possibilities.

What about general claims which are not about Wittgenstein's necessarily existing objects, but rather about 'ordinary' objects like tables and chairs? What should Wittgenstein say about claims like these?

2 Identity (5.53-5.5352)

A second class of claims which pose a problem for Wittgenstein's theory of propositions are identity statements like

$$a = b$$

and statements of non-identity like

$$a \neq b$$

To see why these claims are problematic, it's important to note that there is no place in Wittgenstein's system for claims about how many objects exist. Wittgenstein realized this:

4.1272 ...

So one cannot say, for example, 'There are objects', as one might say 'There are books.' And it is just as impossible to say, 'There are 100 objects' ...

And it is nonsensical to speak of the *total number of objects*.

Recall that Wittgenstein thought that all propositions were truth-functions of elementary propositions. Further, he thought that elementary propositions were all contingent, and independent of all other elementary propositions. This seems to indicate that every truth-function of elementary propositions – and hence every proposition – must fall into one of three categories: (i) contingent claim, (ii) tautology (logical truth), or (iii) contradiction.

The problem is that claims about how many objects there are do not fit comfortably into any of (i)-(iii). First, they seem to be neither tautologies nor contradictions, for two reasons: they seem to be of the same form as paradigmatically non-logical claims like 'There are 100 books', and, while tautologies like 'Either it is raining or it is not raining,' say nothing, existence claims like 'There are 100 objects' do seem to make a claim about the world.'

So if claims like 'There are 100 objects' are to fit into Wittgenstein's system, they must be contingent claims. But there are two reasons why this does not work, either. First, the same objects exist across all possible ways the world could be; so the claim that there are 100 objects, if true, is necessarily rather than contingently true. Second, all contingent

claims are, according to Wittgenstein, made true by objects combining in certain ways; but the claim that there are 100 objects does not seem to require anything about the *combination* of objects.

The problem with statements of identity and non-identity is that they provide a way to formulate claims about how many objects there are. Consider, for example, the following claims about how many objects there are and their translations into claims of identity and non-identity:

<p style="margin: 0;">There is at least one object.</p> <p style="margin: 0;">There are at least two objects.</p> <p style="margin: 0;">There is exactly one object.</p> <p style="margin: 0;">There are exactly two objects.</p>	<p style="margin: 0; text-align: center;">$\exists x(x = x)$</p> <p style="margin: 0; text-align: center;">$\exists x\exists y(x \neq y)$</p> <p style="margin: 0; text-align: center;">$\exists x\forall y(x = y)$</p> <p style="margin: 0; text-align: center;">$\exists x\exists y\forall z(x \neq y \ \& \ (z = x \vee z = y))$</p>
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Each of these claims would, for the above reasons, be problematic from the perspective of the *Tractatus*.

Wittgenstein's response is to claim that propositions about identity and non-identity are only pseudo-propositions:

- 5.533 The identity-sign, therefore, is not an essential constituent of conceptual notation.
- 5.534 And now we see that in a correct conceptual notation pseudo-propositions like ' $a = a$ ' ... cannot even be written down.
- 5.535 This also disposes of all the problems that were connected with such pseudo-propositions. ...

It is one thing to say this; but it does seem as though there are propositions which are *not* pseudo-propositions but which we would naturally formalize using the identity sign. Consider, for example,

'Someone loves no one besides herself.'

which would naturally be analyzed as

$$\exists x\forall y (x \text{ loves } y \rightarrow x = y)$$

If Wittgenstein's rejection of statements of identity is to be plausible, he must find some other way of analyzing such claims. His suggestion is contained in the following:

- 5.53 Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs.

Wittgenstein's suggestion is, in effect, that we reform our logical vocabulary, so that we use different variables only for different objects. So we could analyze the above claim about as

$$\exists x \forall y \neg (x \text{ loves } y)$$

But there is no way, it may seem, to give an analysis of ‘There are at least two objects’ which does not make use of the identity sign. So it may seem that Wittgenstein’s notation does exactly what we would want it to.

How about the claim that there is at least one object – isn’t this a necessary truth, and can’t we express it without use of the identity sign as follows:

$$\exists x (x \text{ is an object})$$

Wittgenstein’s response to this kind of claim is similar: ‘object’ does not express a real concept, but only a pseudo-concept.

One might still wonder whether every claim which we should be able to express will be analyzable using this logical notation. Consider, for example, the sentence ‘Someone loves someone.’ Intuitively, this sentence could be true either because someone loves himself or because someone loves someone else – the sentence is simply noncommittal on the relation between the person doing the loving and the person loved. The problem is that there seems no way to analyze this claim which remains similarly noncommittal. If, in Wittgenstein’s system, we analyze the sentence as

$$\exists x \exists y (x \text{ loves } y)$$

then we are committed to someone loving someone *else* - and this falsifies the meaning of the original sentence. If we analyze it as “ $\exists x (x \text{ loves } x)$ ” then we face the opposite problem. The best we seem to be able to do is the slightly unnatural

$$\exists x \exists y (x \text{ loves } y) \vee \exists z (z \text{ loves } z)$$

But there is a further worry: even granting that claims about how many objects there are are pseudo-propositions, can we generate other claims which will pose the same problem — of being necessary truths which are not plausibly regarded as logical truths?

Consider, e.g.:

Quantification over properties:

$$\exists F \exists x (Fx)$$

This says, intuitively, ‘Something is some way.’ This is not plausibly a logical truth, but it may not seem to be contingent either.

Wittgenstein would presumably not have allowed this sort of quantification over properties, and would have offered a different analysis of sentences which seem to involve this sort of quantification. This is easier said than done, however.

3 Psychology (5.54-5.5423)

The third and final challenge to Wittgenstein's view that all propositions are truth-functions of elementary propositions which we will consider are propositions about psychological facts, like

John believes that grass is green.

It looks as though the proposition "Grass is green" is a part of this proposition about John. If this is so, as it seems, then the proposition about John must be a truth-function of this proposition about grass; this is, after all, the only way for one proposition to be a part of another.

However, if a proposition p is a truth-function of another proposition q (along with, perhaps, other propositions) it should be possible to substitute another proposition q^* for q without changing the truth-value of p , so long as q and q^* have the same truth-value. (This is just what it means to say that one proposition is a *truth*-function of another.)

But in the case of propositions about an agent's psychology, we cannot perform this kind of substitution. For consider: even though 'Grass is green.' and 'There are an infinity of prime numbers.' both have the same truth-value (true), it does not follow from

John believes that grass is green.

that

John believes that there are an infinity of prime numbers.

It follows that, in general, propositions of the form ' A believes that S ' are not truth-functions of the propositions substituted for ' S .' But those pose a problem for Wittgenstein's view that the only way for one proposition to be a part of the analysis of another is via truth-functional combination.

Wittgenstein saw that this was a problem:

- 5.541 At first sight it looks as if it were also possible for one proposition to occur within another in a different way.
Particularly with certain forms of proposition in psychology, such as ' A believes that p is the case' and ' A has the thought p ', etc.
For if these are considered superficially, it looks as if the proposition p stood in some kind of relation to an object A .

Wittgenstein was inclined to solve this problem by rejecting this analysis – by rejecting the thought that ' p ' is a part of these propositions about psychology.

- 5.542 It is clear, however, that ' A believes that p ', ' A has the thought p ' and ' A says p ' are of the form " p says p ": and this does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.

It is not easy to see what Wittgenstein is saying here. Here's one interpretation: recalling his previous claim that propositions are facts, when he denies that claims about psychology express correlations of a fact with an object, he is denying that such claims correlate an agent with a proposition. This expresses the denial of the view that the propositions which (seem to) come after 'that' in such claims are parts of such claims.

His positive suggestion is harder to understand: that these claims are correlations of facts by means of the correlation of their objects. Perhaps the idea is that claims like 'A believes that *S*' can be thought of as analyzable into two parts: 'A says '*S*'' and "'*S*' says that *S*.' The problem here is that the second claim would be a claim about representation which is, according to Wittgenstein, unsayable. We'll see why shortly.