The theory of elementary propositions

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1 Elementary propositions

The heart of the *Tractatus* is Wittgenstein's theory of propositions; and the heart of his theory of propositions is his theory of elementary propositions. Beginning in §4.2, Wittgenstein turns to this topic.

1.1 Elementary propositions and states of affairs (4.2-4.28)

Given Wittgenstein's repeated reliance on the existence of a correspondence between language and the world, it should be no surprise that his philosophy of language – i.e., his theory of propositions and their constituents – mirrors his metaphysics – i.e., his theory of facts and their constituents. Just as the facts are all determined in some sense by a class of basic facts – states of affairs – so propositions are all, in a sense to be explained, determined by a class of basic propositions – the elementary propositions.

Elementary propositions stand for states of affairs:

> 4.21 The simplest kind of proposition, an elementary proposition, asserts the existence of a state of affairs.

So just as all states of affairs are independent of all other states of affairs, all elementary propositions are independent of all other elementary propositions:

> 4.211 It is a sign of a proposition's being elementary that there can be no elementary proposition contradicting it.

This is already enough to show that elementary propositions are not the sorts of propositions with which we are familiar, and are certainly not the same kinds of things as Russell's atomic propositions. Consider, for example, two apparently simple claims:

That is red.

That is exactly 6 feet tall.

Claims like these about the colors and sizes of objects cannot be elementary, since they are contradicted by other propositions about the colors and sizes of the same objects:

That is green.

That is exactly 5 feet tall.

Another clue that elementary propositions are different than the propositions with which we are familiar is Wittgenstein's claim that

> 4.22 An elementary proposition consists of names. It is a nexus, a concatenation, of names.

This obviously parallels Wittgenstein's thesis about the nature of states of affairs:

> 2.03 In a state of affairs objects fit into one another like the links of a chain.

The idea seems to be that names stand for objects, and so that, since a state of affairs is just an arrangement of objects and elementary propositions stand for states of affairs, an elementary proposition will be an arrangement of names.

1.2 Truth possibilities of elementary propositions (4.3-4.45)

We know that elementary propositions are correlated with states of affairs; it follows that

> 4.3 Truth possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs.

There is also a connection between Wittgenstein's view about the relationship between elementary propositions and other propositions, and his view about the relationship between states of affairs and other facts. If elementary propositions are matched up one-to-one with states of affairs, then a list of all the true elementary propositions will state all there is to know about the states of affairs. And if the states of affairs determine all the facts, then it seems that all the true elementary propositions will say all that there is to be said about the facts. But we know further that every proposition has as its sense some (possible) fact – and whether a proposition is true depends on whether the fact is actual. So it must be that the list of all the true elementary propositions determine the truth values of all the propositions.

Wittgenstein expresses this conclusion when he says,

> 4.41 Truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions.

2 Tautologies and contradictions (4.46-4.4661)

But this leaves open the question of what we should say about those propositions which are true, or false, no matter what is the case. Wittgenstein calls the former 'tautologies' and the latter 'contradictions' (§4.46).

We know that these cannot be elementary propositions. We know that every elementary proposition corresponds to a state of affairs, and that for any two possible states of affairs, it is consistent that both obtain, that either obtain, or that neither obtain. So contradictions can't be elementary propositions, since they can't be true at all, and so a fortiori can't be true when an arbitrary elementary proposition is true. Tautologies can't be elementary propositions since they can't fail to be true, and so a fortiori can't fail to be true when an arbitrary elementary proposition is true.

There is a sense in which, according to Wittgenstein, these kinds of claims do not say anything; as Wittgenstein rightly says, "...I know nothing about the weather when I know that it is either raining or not raining" (§4.461). But neither are they nonsensical:

> 4.4611 Tautologies and contradiction are not, however, nonsensical. They are part of the symbolism, much as '0' is part of the symbolism of arithmetic.

It is not transparent what the analogy here is supposed to be. But the basic point is that tautologies and contradictions are a kind of artifact of the ways in which we can combine propositions — they are the kind of limiting case in which the propositions combined cancel each other out.

One way to make sense of what Wittgenstein has in mind here is via his view that the meaning of a proposition is the condition required for it to be true. Then tautologies and contradictions only seem to have meaning in a kind of vacuous way; one is true in every condition, and one is true under no conditions. This is presumably what Wittgenstein has in mind in §4.465 when he says that "the logical product of a tautology and a proposition says the same thing as the proposition" — the logical product (i.e., the conjunction) of a proposition and a tautology is true under just the same conditions as the proposition alone.

3 The general form of the proposition

3.1 Elementary propositions and other propositions (4.5-5.02)

We already know from the connections between states of affairs and facts, and between elementary propositions and states of affairs, that the truth conditions of all propositions must be in some way based on the truth conditions of elementary propositions. Beginning in §4.5, Wittgenstein uses this point to sketch his general theory of the nature of propositions:

> 4.51 Suppose that I am given *all* elementary propositions: then I can simply ask what propositions I can construct out of them. And there I have *all* propositions, and *that* fixes their limits.

What we have to understand is what Wittgenstein means when he talks about 'constructing' one proposition out of some others. It turns out that what he means is very simple:

> 5 A proposition is a truth-function of elementary propositions.

One proposition p is a truth-function of a set of propositions if and only if, once you determine the truth value of every proposition in the set, you also fix the truth-value of p.

The connection between truth-functions and truth-tables; the idea that one proposition is a truth-function of some others if and only if you can construct the right sort of truth table.

3.2 Elementary propositions and the foundations of probability (5.1-5.156)

After introducing the view that all propositions are truth-functions of elementary propositions, Wittgenstein moves to an application of that view: propositions about the probabilities of events. One of Wittgenstein's explanatory ambitions is to explain the nature of probability: to say what it is that we are saying when we say that an event has a certain probability.

He begins with the claim

> 5.153 In itself a proposition is neither probable nor improbable. Either an event occurs or it does not: there is no middle way.

It is difficult to see what the second half of this claim does to justify the first half. Here's one idea: states of affairs either obtain or do not obtain. So when we say that a certain event has a certain probability, we are not making a claim that a certain state of affairs obtains: the state of affairs of such and such having x chance of happening. Rather it must be the case that "probability is a generalization" (§5.156).

How the apparatus of elementary propositions may be used to support this claim.

3.3 Internal relations between propositions are the result of truth-functions (5.2-5.32)

We know that elementary propositions are all independent of each other – so we know that there are no necessary relations between the truth of elementary propositions. But many propositions do have internal relations to each other — and this is a phenomenon that Wittgenstein thinks that we can explain in terms of the ways that elementary propositions are combined.

In fact, this is an explanatory task to which the view that all propositions are truth-functions of elementary propositions commits him. For consider any propositions p and q such that, necessarily, if p is true then q is true. Now consider the proposition r which expresses the claim that if p is true, q is true. This is a necessary truth, and hence is a tautology, and hence is not an elementary proposition. So we should be able to explain the fact that it is a tautology by explaining it as a truth-function of elementary propositions.

3.4 Logical constants (5.4-5.476)

We will mostly pass over these remarks, which discuss issues in the philosophy of logic. The core idea is that logical expressions do not stand for anything; there are no logical objects, or logical relations. Why Wittgenstein's view of propositions (as opposed to, for example, Russell's) makes this sort of view possible.

Wittgenstein expresses this key idea in a (typically) metaphorical way when he says:

> 5.4611 Signs for logical operations are punctuation-marks.

Just as no one would be tempted to say that commas must stand for constituents of a proposition, so we should not be tempted to say that 'and' an 'or' stand for special sorts of relations. Rather, like commas, they are just ways of grouping sets of words which really do stand for things.

3.5 Joint negation (5.5-5.5151)

We know by now that Wittgenstein thinks that all propositions are truth-functions of elementary propositions. In these sections he points out that from this we can get the result that all propositions are the result of repeated applications of a certain operation, which Wittgenstein symbolizes with the letter 'N', and which is sometimes called 'joint negation.' Wittgenstein explains joint negation as follows:

> 5.51 If $\bar{\xi}$ has only one value, then $N(\xi) = \neg p$ (not p); if it has two values, then $N(\xi) = \neg p . \neg q$ (neither p nor q).

Here 'ξ' is a variable which has propositions as its value; the expression '$N(\bar{\xi})$' signifies the complex formula obtained by negating each proposition which can be substituted for 'ξ' and conjoining them. If there is one such proposition, the result is its negation; if there are two such propositions, the result is the negation of the first conjoined with the negation of the second; and so on.

To get an idea of how one can define other truth-functions in terms of joint negation, consider the following translations;

Table 1: Wittgenstein's operator 'N'

$\neg p$	$N(p)$
p & q	$N[N(p), N(q)]$
$p \vee q$	$N(N[p, q])$

3.6 $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ *(6-6.031)*

Wittgenstein then uses joint negation to state the result toward which his theory of propositions has been working: his statement of the general form of the proposition.

> 6 The general form of a truth-function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$.
> This is the general form of a proposition.
>
> 6.001 What this says is that every proposition is a result of successive applications to elementary propositions of the operation $N(\xi)$.

This is just a succinct way of stating the combination of the two theses that all propositions are truth-functions of elementary propositions, and that all truth-functions can be defined in terms of joint negation. What the formula in §6 says is that every proposition may be generated by the following procedure: begin with the class of elementary propositions; apply joint negation to some subset of those propositions to arrive at a new proposition; apply join negation again to some subset of the set of propositions consisting of the elementary propositions plus the one just obtained; apply joint negation again ... and so on and so on.