1 Frege's criterion

The best way into Fregean semantics is via Frege's criterion of difference for senses, which can be stated as follows:

If it is possible to understand two sentences $S$ and $S^*$ while (after reflection) taking different attitudes toward their truth-values, then $S$ and $S^*$ differ in sense.

Remember that ‘intension’ should be thought of not as a generic term for ‘meaning’, but rather as a name for the entities that proponents of possible worlds semantics identify with meanings. An analogous point here holds with respect to ‘sense’: it is not a generic term for meaning, but rather a term for those values of expressions (if such there are) which obey Frege’s criterion.

It is a bit tricky to turn this into a criterion for sameness of sense; the obvious way of doing so would yield the result that, for example, any two sentences which are such that anyone who understands them knows that they are true have the same sense, which is clearly incorrect.

It is a bit easier to do this with subsentential expressions. Consider, for example, the following:

$e$ and $e^*$ differ in sense $\iff$ there are two sentences $S$ and $S^*$, which differ only in the substitution of $e$ and $e^*$, and differ in sense.

We’ll take this formula to define sameness and difference of Fregean sense.
2 Fregean propositions

So what does the Fregean say about propositions? Fregeans agree that propositions are structured, but think that the constituents of propositions are not objects and properties, but modes of presentation of objects and properties. (This is more neo-Fregean than Fregean; Frege would have said something more like ‘modes of presentation of objects and concepts’.) These modes of presentation are called ‘senses’, as are the complexes built out of them. Frege also called the latter ‘thoughts.’

This has the advantage that it makes propositions very fine-grained; we’re never in the position that two sentences intuitively differ in meaning, and yet have the same Fregean sense. For example, unlike the proponent of possible worlds semantics, the Fregean is under no theoretical pressure to say that the following two sentences have the same meaning:

- South Bend is in Indiana.
- South Bend is in Indiana and arithmetic is incomplete.

This, in turn, helps with the semantics of propositional attitude ascriptions. If we adopt the naive relational theory that sentences of the form "A V’s that S" express a relation between the referent of ‘A’ and the meaning of ‘S’ in the context of the ascription, we are under no pressure to say that the following two sentences must have the same truth-value:

- Bob believes that South Bend is in Indiana.
- Bob believes that South Bend is in Indiana and arithmetic is incomplete.

and, in general, we are under no pressure to say that expressions which have the same reference — even if they have the same reference with respect to all possible worlds — must have the same meaning.

3 Arguments for Frege’s criterion

We can separate out at least two arguments for Frege’s criterion of difference, one based on propositional attitude ascriptions, and one based on a view about understanding.

Argument from attitude ascriptions
1. A understands each sentence and, upon reflection, thinks that \( S \) is true and \( S^* \) is false.
2. \( \neg A \) believes that \( S \) is true. (1)
3. \( \neg A \) believes that \( S^* \) is false. (1)
4. If two sentences differ in truth-value, they differ in meaning.
5. \( \neg A \) believes that \( S \) and \( \neg A \) believes that \( S^* \) differ in meaning.(2,3,4)
6. If two sentences which differ only in the substitution of a pair of expressions differ in meaning, then that pair of expressions differs in meaning.
   
C. \( S \) and \( S^* \) differ in meaning. (5,6)

By conditional proof, Frege's criterion for difference of sense follows.

Argument from the transparency of meaning

1. To understand an expression is to know its meaning.
2. If two expressions have the same meaning, then anyone who understands both will (on reflection) believe this. (1)
3. If someone (on reflection) takes two sentences to have different truth-values, he does not believe that they have the same meaning.
   
C. If two expressions have the same meaning, then no one can (on reflection) take them to have different truth-values. (2,3)

4 The nature of Fregean senses

So far we’ve talked about how to tell when expressions do or do not have the same sense — but what sorts of things are senses? Here’s Frege’s explanation:

“The reference of a proper name is the object itself which we designate by its means; the idea, which we have in that case, is wholly subjective; in between lies the sense, which is indeed no longer subjective like the idea, but is yet not the object itself. The following analogy will perhaps clarify these relationships. Somebody observes the Moon through a telescope. I compare the Moon itself to the reference; it is the object of the observation, mediated by the real image projected by the object glass in the interior of the telescope, and by the retinal image of the observer. The former I compare to the sense, the latter is like the idea or experience. The optical image in the telescope is indeed one-sided and dependent upon the standpoint of observation; but it is still objective, inasmuch as it can be used by several observers. At any rate it could be arranged for several to use it simultaneously. But each one would have his own retinal image.” (‘On sense and reference,’ 30)

Fregean senses are abstract objects, like properties. However, they can’t be properties. To see this, note that we could have two linguistic expressions which stand for the same
property, and yet fail Frege’s test for sameness of meaning. The relationship between Fregean senses and things like objects and properties is many-one. The most plausible interpretation seems to be that on the Fregean view, meanings are a sui generis category of abstracta.