Pascal’s wager

So far we have discussed a number of arguments for or against the existence of God. In the reading for today, Pascal asks not "Does God exist?" but "Should we believe in God?" What is distinctive about Pascal's approach to the latter question is that he thinks that we can answer it without first answering the former question.

Here is what he has to say about the question, "Does God exist?":

> "Let us then examine this point, and let us say: 'Either God is or he is not.' But to which view shall we be inclined? Reason cannot decide this question. Infinite chaos separates us. At the far end of this infinite distance a coin is being spun which will come down heads or tails. How will you wager? Reason cannot make you choose either, reason cannot prove either wrong."

Pascal is here expressing a kind of skepticism about the ability of human reason to deliver an answer to this question.

But, he notes, this does not remove the necessity of our choosing to believe, or not believe, in God:

> "Yes, but you must wager. There is no choice, you are already committed. Which will you choose then? Let us see: since a choice must be made, let us see which offers you the least interest. You have two things to lose: the true and the good; and two things to stake: your reason and your will, your knowledge and your happiness. . . . Since you must necessarily choose, your reason is no more affronted by choosing one rather than the other. . . . But your happiness? Let us weigh up the gain and the loss involved in calling heads that God exists. Let us assess the two cases: if you win you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist."

This is quite different than the sorts of arguments we have discussed so far for belief in God. Each of those arguments made a case for belief in God on the basis of a case for the *truth* of that belief; Pascal focuses on the *happiness* that forming the belief might bring about.

But why does happiness give us a reason to believe in God, rather than not believe? Pascal spells out his reasoning, using an analogy with gambling:

> "...since there is an equal chance of gain and loss, if you stood to win only two lives for one you could still wager, but supposing you stood to win three?
>
> ...it would be unwise of you, since you are obliged to play, not to risk your life in order to win three lives at a game in which there is an equal chance of winning and losing. ...But here there is an infinity of happy life to be won, one chance of winning against a finite number of chances of losing, and what you are staking is finite. That leaves no choice; wherever there is infinity, and where there are not infinite chances of losing against that of winning, there is no room for hesitation, you must give everything. And thus, since you are obliged to play, you must be renouncing reason if you hoard your life rather than risk it for an infinite gain, just as likely to occur as a loss amounting to nothing."

Pascal was one of the first thinkers to systematically investigate the question of how it is rational to act under certain kinds of uncertainty, a topic now known as "decision theory." We can use some concepts from decision theory to get a bit more precise about how Pascal's argument here is supposed to work.

Suppose I offer you the chance of choosing heads or tails on a fair coin flip, with the following payoffs: if you choose heads, and the coin comes up heads, you win $10; if you choose heads, and the coin comes up tails, you win $5. If you choose tails, then if the coin comes up heads, you get $2, and if it comes up tails, you get $5.

We can represent the possibilities open to you with the following table:

Courses of action	**Possibility 1: Coin comes up heads**	**Possibility 2: Coin comes up tails**
Choose 'heads'	Win $10	Win $5
Chose 'tails'	Win $2	Win $5

Obviously, you should choose heads. One way to put the reason for this is as follows: there is one possibility on which you are better off having chosen heads, and no possibility on which you are worse off choosing heads. This is to say that choosing heads *dominates* choosing tails.

It seems very plausible that if you are choosing between A and B, and choosing A dominates choosing B, it is rational to choose A.

One interpretation of Pascal's argument is that belief in God dominates not believing. Pascal says, after all,

> "if you win, you win everything, if you lose you lose nothing."

This indicates that he is thinking of the choice about whether to believe as follows:

Courses of action	**Possibility 1: God exists**	**Possibility 2: God does not exist**
Believe in God	Infinite reward	Lose nothing, gain nothing
Do not believe in God	Infinite loss	Lose nothing, gain nothing

If this is the right way to think about the choice between belief and non-belief, then believing seems to dominate not believing.

Is this the right way to think about the choice?

If we think about a scenario in which one believes in God but God does not exist as involving some loss -- either because one would not do things which one might like to do, or because having a false belief is in itself a loss -- then believing does not dominate not believing.

But there is another way to reconstruct Pascal's argument, using another key concept from decision theory, *expected utility*.

Recall the example of the coin toss above, in which, if you bet heads and the coin comes up heads, you win $10, and if it comes up tails, you win $5. Suppose I offer you the following deal: I will give you those payoffs on a fair coin flip in exchange for you paying me $7 for the right to play. Should you take the bet?

Here is one way to argue that you should take the bet. There is a 1/2 probability that the coin will come up heads, and a 1/2 probability that it will come up tails. In the first case I win $10, and in the second case I win $5. So, in the long run, I'll win $10 about half the time, and $5 about half the time. So, in the long run, I should expect the amount that I win per coin flip to be the average of these two amounts -- $7.50. So the *expected utility* of my betting heads is $7.50. So it is rational for me to pay any amount less than the expected utility to play (supposing for simplicity, of course, that my only interest is in maximizing my money, and I have no other way of doing so).

To calculate the expected utility of an action, we assign each outcome of the action a certain probability, and a certain value (in the above case, the relevant value is just the money won). In the case of each possible outcome, we then multiply its probability by its value; the expected utility of the action will then be the sum of these results.

In the above case, we had (1/2 * 10) + (1/2 * 5) = 7.5.

The notion of expected utility seems to lead to a simple rule for deciding what to do:

> If deciding between a number of possible actions, it is rational to choose the action with the highest expected utility.

How could Pascal's argument for belief in God be reconstructed using the notion of expected utility?

Pascal says two things which help us here. First, he says that "there is an equal chance of gain and loss" -- an equal chance that God exists, and that God does not exist. This means that we should assign each a probability of 1/2.

Second, he says that in this case the amounts to be won and lost are *infinite*. We can represent this by saying that the utility of belief in God if God exists is ∞, and that the utility of non-belief if God exists is -∞. So on this view, the choices would look like this:

Courses of action	**Possibility 1: God exists**	**Possibility 2: God does not exist**
Believe in God	Positive infinity (∞)	Finite loss (-n, where n is some finite number)
Do not believe in God	Negative infinity (-∞)	Finite gain ($+m$, where m is some finite number)

If this is right, then the expected utility calculations would be as follows:

$$\begin{aligned}
\text{Expected utility of belief in God} &= \text{(1/2)*(Utility of belief in God, given that God exists) + (1/2)*(Utility of belief in God, given that God does not exist)} \\
&= (1/2)^*(\infty) + (1/2)^*(\text{-}n) \\
&= \infty
\end{aligned}$$

$$\begin{aligned}
\text{Expected utility of non-belief in God} &= \text{(1/2)*(Utility of non-belief in God, given that God exists) + (1/2)*(Utility of non-belief in God, given that God does not exist)} \\
&= (1/2)^*(\text{-}\infty) + (1/2)^*(m) \\
&= \text{-}\infty
\end{aligned}$$

This looks like a pretty strong "expected utility" argument for the rationality of belief in God.

Now consider the following objections:

1. The probability that God exists is not 1/2, but some much smaller number -- say, 1/100.

2. Even if God exists, it is very unlikely that God would punish non-believers for eternity. So the probability that non-belief would lead to infinite suffering, even if we assume that God exists, is quite small.

A different sort of worry arises not from the particulars of Pascal's assumptions about the probabilities of various claims about God, but from the use of the rule that we ought to maximize expected utilities in cases where the expected utilities of actions are infinite.

Consider the following bet: I am going to flip a fair coin until it comes up heads. If the first time it comes up heads is on the 1st toss, I will give you $2. If the first time it comes up heads is on the second toss, I will give you $4. If the first time it comes up heads is on the 3rd toss, I will give you $8. And in general, if the first time the coin comes up heads is on the *n*th toss, I will give you $\$2^n$.

Suppose I offer you the chance to play for $2. Should you take it?

Suppose I raise the price to $3. Should you take it?

Suppose now I raise the price to $10,000. Should you be willing to pay that amount to play the game?

Many people have the very strong intuition that it is not rational to pay $10,000 to play this game. But expected utility calculations say otherwise. The expected utility of playing the game is, remember, the sum of odds * value for each possible outcome. That is:

(1/2 * $2) + (1/4 * $4) + (1/8 * $8) + ...

or, simplifying,

1 + 1 + 1 + ...

since there are infinitely many possible outcomes, the expected utility of playing this game is infinite. So the rule of expected utility tells you that you ought to be willing to pay any finite amount of money to play. But this seems wrong. This is known as the *St. Petersburg paradox*.

This sort of case relies on an example in which there are infinitely many outcomes, whereas Pascal's argument relies on a case in which a single outcome has infinite value, so the arguments are not clearly analogous. But perhaps this does show that we should be cautious when relying on expected utility calculations in the infinite case.

A different sort of reply focuses on the impossibility of believing things at will. If I offer you $5 to raise your arm, you can do it. But suppose I offered you $5 to believe that you are not now sitting down. Can you do that (without standing up)?

Pascal considers this reply:

> "... is there really no way of seeing what the cards are? ... I am being forced to wager and I am not free; I am begin held fast and I am so made that I cannot believe. What do you want me to do then?"

and has this to say in response:

> "That is true, but at least get it into your head that, if you are unable to believe, it is because of your passions, since reason impels you to believe and yet you cannot do so. Concentrate then not on convincing yourself by multiplying proofs of God's existence, but by diminishing your passions. ..."

Denis Diderot, a French philosopher who lived a century after Pascal, objected against this argument that “An Imam could reason just as well this way.”

By that, Diderot meant that someone with quite different beliefs about God than Pascal -- such as a Muslim -- could use Pascal’s argument to support belief in the existence of a God with the characteristics assigned to God by his religion. But it is impossible to believe both that God exists and is the way the Catholic Church believes God to be, and that God exists and is the way the Islamic faith says God is.

More dramatically, a polytheist could use Pascal’s argument to support belief in a collection of deities.

So what beliefs, exactly, should Pascal’s argument lead us to have?

How could Pascal respond to this objection?