Pascal’s wager

Blaise Pascal was a 17th century French philosopher, theologian, and mathematician; he made foundational contributions to, among other areas, the early development of the theory of probability.

So far we have discussed a number of arguments for or against the existence of God. In the reading for today, Pascal asks not **"Does God exist?"** but **"Should we believe in God?"** What is distinctive about Pascal's approach to the latter question is that he thinks that we can answer it without first answering the former question.

Here is what he has to say about the question of whether God exists:

> "Let us then examine this point, and let us say: 'Either God is or he is not.' But to which view shall we be inclined? Reason cannot decide this question. Infinite chaos separates us. At the far end of this infinite distance a coin is being spun which will come down heads or tails. How will you wager? Reason cannot make you choose either, reason cannot prove either wrong."

Pascal is here expressing a kind of skepticism about the ability of human reason to tell us whether there is a God.

So far we have discussed a number of arguments for or against the existence of God. In the reading for today, Pascal asks not **"Does God exist?"** but **"Should we believe in God?"** What is distinctive about Pascal's approach to the latter question is that he thinks that we can answer it without first answering the former question.

Here is what he has to say about the question of whether God exists:

> "Let us then examine this point, and let us say: 'Either God is or he is not.' But to which view shall we be inclined? Reason cannot decide this question. Infinite chaos separates us. At the far end of this infinite distance a coin is being spun which will come down heads or tails. How will you wager? Reason cannot make you choose either, reason cannot prove either wrong."

Pascal is here expressing a kind of skepticism about the ability of human reason to tell us whether there is a God.

But, as Pascal notes, this fact does not remove the necessity of our having to decide to believe, or not believe, in God:

> "Yes, but you must wager. There is no choice, you are already committed. Which will you choose then? Let us see: since a choice must be made, let us see which offers you the least interest. You have two things to lose: the true and the good; and two things to stake: your reason and your will, your knowledge and your happiness. . . . Since you must necessarily choose, your reason is no more affronted by choosing one rather than the other. . . . But your happiness? Let us weigh up the gain and the loss involved in calling heads that God exists. Let us assess the two cases: if you win you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist."

But, as Pascal notes, this fact does not remove the necessity of our having to decide to believe, or not believe, in God:

> "Yes, but you must wager. There is no choice, you are already committed. Which will you choose then? Let us see: since a choice must be made, let us see which offers you the least interest. You have two things to lose: the true and the good; and two things to stake: your reason and your will, your knowledge and your happiness. . . . Since you must necessarily choose, your reason is no more affronted by choosing one rather than the other. . . . But your happiness? Let us weigh up the gain and the loss involved in calling heads that God exists. Let us assess the two cases: if you win you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist."

This is quite different than the sorts of arguments we have discussed so far for belief in God. Each of those arguments made a case for belief in God on the basis of a case for the **truth** of that belief; Pascal focuses on the **happiness** that forming the belief might bring about. This is presumably the point of Pascal's emphasis on the question of **"gain and loss."**

But what is Pascal's argument that belief in God will lead to greater happiness? It seems to be contained in the last two sentences of this passage. Pascal is saying that if you believe in God and God exists ("you win"), you win eternal life ("you win everything"), whereas if God does not exist, it doesn't matter whether you believe in God ("you lose nothing").

Pascal was one of the first thinkers to systematically investigate the question of how it is rational to act under certain kinds of uncertainty, a topic now known as "decision theory." We can use some concepts from decision theory to get a bit more precise about how Pascal's argument here is supposed to work.

> "Yes, but you must wager. There is no choice, you are already committed. Which will you choose then? Let us see: since a choice must be made, let us see which offers you the least interest. You have two things to lose: the true and the good; and two things to stake: your reason and your will, your knowledge and your happiness. . . . Since you must necessarily choose, your reason is no more affronted by choosing one rather than the other. . . . But your happiness? Let us weigh up the gain and the loss involved in calling heads that God exists. Let us assess the two cases: if you win you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist."

But what is Pascal's argument that belief in God will lead to greater happiness? It seems to be contained in the last two sentences of this passage. Pascal is saying that if you believe in God and God exists ("you win"), you win eternal life ("you win everything"), whereas if God does not exist, it doesn't matter whether you believe in God ("you lose nothing").

We are facing a decision in which we have only **two options**: belief or nonbelief. And there is **one unknown factor** which will determine the outcome of our choice: whether or not God exists. **So, pairing each possible choice with each possible outcome, there are four possibilities.** Our question should be: when faced with a decision like this, what should guide our choice?

We can get clearer on this question by considering a simple bet:

> I offer you the chance of choosing heads or tails on a fair coin flip, with the following payoffs: if you choose heads, and the coin comes up heads, you win \$5; if you choose heads, and the coin comes up tails, you lose \$1. If you choose tails, then if the coin comes up heads, you get \$2, and if it comes up tails, you lose \$1.

As in Pascal's case, we have a decision with two options - heads or tails - and one relevant unknown - the way the coin will flip will turn out.

We are facing a decision in which we have only **two options**: belief or nonbelief. And there is **one unknown factor** which will determine the outcome of our choice: whether or not God exists. **So, pairing each possible choice with each possible outcome, there are four possibilities.** Our question should be: when faced with a decision like this, what should guide our choice?

We can get clearer on this question by considering a simple bet:

> I offer you the chance of choosing heads or tails on a fair coin flip, with the following payoffs: if you choose heads, and the coin comes up heads, you win \$5; if you choose heads, and the coin comes up tails, you lose \$1. If you choose tails, then if the coin comes up heads, you get \$2, and if it comes up tails, you lose \$1.

As in Pascal's case, we have a decision with two options - heads or tails - and one relevant unknown - the way the coin will flip will turn out.

We can represent this choice as follows:

Courses of action	Possibility 1: The coin comes up heads	Possibility 2: The coin comes up tails
Choose heads	win \$5	lose \$1
Choose tails	win \$2	lose \$1

Obviously, given this choice, you should choose heads. One way to put the reason for this is as follows: **there is one possibility on which you are better off having chosen heads, and no possibility on which you are worse off choosing heads.** This is to say that choosing heads **dominates** choosing tails.

This suggests the following rule for decision-making:

We are facing a decision in which we have only **two options**: belief or nonbelief. And there is **one unknown factor** which will determine the outcome of our choice: whether or not God exists. **So, pairing each possible choice with each possible outcome, there are four possibilities.** Our question should be: when faced with a decision like this, what should guide our choice?

This suggests the following rule for decision-making:

The rule of dominance

If you are choosing between A and B, and A dominates B, you should choose A

One way to read Pascal's argument that we should believe in God is as saying that believing in God **dominates** nonbelief. Recall the quote discussed above:

> "Yes, but you must wager. There is no choice, you are already committed. Which will you choose then? Let us see: since a choice must be made, let us see which offers you the least interest. You have two things to lose: the true and the good; and two things to stake: your reason and your will, your knowledge and your happiness. . . . Since you must necessarily choose, your reason is no more affronted by choosing one rather than the other. . . . But your happiness? **Let us weigh up the gain and the loss involved in calling heads that God exists. Let us assess the two cases: if you win you win everything, if you lose you lose nothing**. Do not hesitate then; wager that he does exist."

Pascal's claim that **if you lose, you lose nothing** is some indication that he thought that belief dominated non-belief; the thought would be that in the case where God exists (i.e., where you win), you are better off believing, and that in the case where God does not exist (i.e., where you lose) you are no worse off.

If Pascal is right about this, then we might represent our decision about whether or not to believe in God as follows:

We are facing a decision in which we have only **two options**: belief or nonbelief. And there is **one unknown factor** which will determine the outcome of our choice: whether or not God exists. **So, pairing each possible choice with each possible outcome, there are four possibilities.** Our question should be: when faced with a decision like this, what should guide our choice?

The rule of dominance

If you are choosing between A and B, and A dominates B, you should choose A

Pascal's claim that **if you lose, you lose nothing** is some indication that he thought that belief dominated non-belief; the thought would be that in the case where God exists (i.e., where you win), you are better off believing, and that in the case where God does not exist (i.e., where you lose) you are no worse off.

If Pascal is right about this, then we might represent our decision about whether or not to believe in God as follows:

Courses of action	Possibility 1: God exists	Possibility 2: God does not exist
Belief	you "win everything"	you lose nothing, win nothing
Non-belief	you win nothing	you lose nothing, win nothing

If this is the correct representation of our choice whether or not to believe, then belief dominates non-belief. Since the rule of dominance seems very plausible, this would be a very powerful argument that we ought to believe in God.

The rule of dominance

If you are choosing between A and B, and A dominates B, you should choose A

Courses of action	Possibility 1: God exists	Possibility 2: God does not exist
Belief	you “win everything”	you lose nothing, win nothing
Non-belief	you win nothing	you lose nothing, win nothing

If this is the correct representation of our choice whether or not to believe, then belief dominates non-belief. Since the rule of dominance seems very plausible, this would be a very powerful argument that we ought to believe in God.

However, it is not obvious that belief **does** dominate nonbelief, since it is not obvious that one really loses nothing if one believes in God in the scenario in which God does not exist. Wouldn’t one be undertaking religious obligations which one might have avoided? And isn’t having a false belief something bad in itself?

If we think about a scenario in which one believes in God but God does not exist as involving some loss - either because one would not do things which one might like to do, or because having a false belief is in itself a loss - then believing does not dominate not believing.

However, there is some indication that this is not the best interpretation of Pascal’s argument anyway.

However, there is some indication that this is not the best interpretation of Pascal's argument anyway.

Consider the following passage:

> "...since there is an equal chance of gain and loss, if you stood to win only two lives for one you could still wager, but supposing you stood to win three?
>
> ...it would be unwise of you, since you are obliged to play, not to risk your life in order to win three lives at a game in which there is an equal chance of winning and losing. ...But here there is an infinity of happy life to be won, one chance of winning against a finite number of chances of losing, and what you are staking is finite. That leaves no choice; wherever there is infinity, and where there are not infinite chances of losing against that of winning, there is no room for hesitation, you must give everything. And thus, since you are obliged to play, you must be renouncing reason if you hoard your life rather than risk it for an infinite gain, just as likely to occur as a loss amounting to nothing."

Here Pascal is thinking of bets **where you might win or lose something** by playing, but where what you win is greater than what you lose. But in bets of this sort, dominance reasoning will **not** tell us whether or not to take the bet, since it will not be the case that taking the bet will never leave you worse off than not taking it.

Let's consider how we might reason about decisions of this sort, where it is not the case that one option dominates the others, and so where the rule of dominance does not tell us what to do.

It's again useful to think about a simple bet.

Let's consider how we might reason about decisions of this sort, where it is not the case that one option dominates the others, and so where the rule of dominance does not tell us what to do.

It's again useful to think about a simple bet.

> I'm about to flip a coin, and offer you the following bet: if the coin comes up heads, then I will give you $5; if it comes up tails, you will owe me $3. You know that it is a fair coin. Should you take the bet?

We might represent this decision using the following table:

Courses of action	Possibility 1: Coin comes up heads	Possibility 2: Coin comes up tails
Take the bet	win $5	lose $3
Don't take the bet	$0	$0

Here neither course of action dominates the other; but it still seems that you should clearly take the bet. Why?

There is a ½ probability that the coin will come up heads, and a ½ probability that it will come up tails. In the first case I win $5, and in the second case I lose $3. So, in the long run, I'll win $5 about half the time, and lose $3 about half the time. So, in the long run, I should expect the amount that I win per coin flip to be the average of these two amounts a win of $1.

We can express this by saying that the **expected utility** of taking the bet is $1. It seems that one should take this bet because the expected utility of doing so is greater than the expected utility of not taking the bet.

Courses of action	Possibility 1: Coin comes up heads	Possibility 2: Coin comes up tails
Take the bet	win $5	lose $3
Don't take the bet	$0	$0

There is a ½ probability that the coin will come up heads, and a ½ probability that it will come up tails. In the first case I win $5, and in the second case I lose $3. So, in the long run, I'll win $5 about half the time, and lose $3 about half the time. So, in the long run, I should expect the amount that I win per coin flip to be the average of these two amounts a win of $1.

We can express this by saying that the **expected utility** of taking the bet is $1. It seems that one should take this bet because the expected utility of doing so is greater than the expected utility of not taking the bet.

To calculate the expected utility of an action, we assign each outcome of the action a certain **probability**, thought of as a number between 0 and 1, and a certain **value** (in the above case, the relevant value is just the money won). In the case of each possible outcome, **we then multiply its probability by its value; the expected utility of the action will then be the sum of these results.**

In the case of the above bet, the calculation looks like this:

Courses of action	Possibility 1: Coin comes up heads *Probability=0.5*	Possibility 2: Coin comes up tails *Probability=0.5*	Expected utility
Take the bet	win $5	lose $3	½ * $5 + ½ * (-$3) = $1
Don't take the bet	$0	$0	½ * $0 + ½ * $0 = $0

To calculate the expected utility of an action, we assign each outcome of the action a certain **probability**, thought of as a number between 0 and 1, and a certain **value** (in the above case, the relevant value is just the money won). In the case of each possible outcome, **we then multiply its probability by its value; the expected utility of the action will then be the sum of these results**.

Courses of action	Possibility 1: Coin comes up heads *Probability=0.5*	Possibility 2: Coin comes up tails *Probability=0.5*	Expected utility
Take the bet	win $5	lose $3	½ * $5 + ½ * (-$3) = $1
Don't take the bet	$0	$0	½ * $0 + ½ * $0 = $0

Reflection on this sort of example seems to make the following principle about rational action seem quite plausible:

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

This, as the example of the bet illustrates, tells us what we should do in certain situations about which the rule of dominance is silent. So, even if we think that belief does not dominate non-belief, perhaps we can use the notion of expected utility to reconstruct Pascal's argument.

Let's return to the passage discussed above.

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

This, as the example of the bet illustrates, tells us what we should do in certain situations about which the rule of dominance is silent. So, even if we think that belief does not dominate non-belief, perhaps we can use the notion of expected utility to reconstruct Pascal's argument.

Let's return to the passage discussed above.

> "...since there is an equal chance of gain and loss, if you stood to win only two lives for one you could still wager, but supposing you stood to win three?
>
> ...it would be unwise of you, since you are obliged to play, not to risk your life in order to win three lives at a game in which there is an equal chance of winning and losing. ...But here there is an infinity of happy life to be won, one chance of winning against a finite number of chances of losing, and what you are staking is finite. That leaves no choice; wherever there is infinity, and where there are not infinite chances of losing against that of winning, there is no room for hesitation, you must give everything. And thus, since you are obliged to play, you must be renouncing reason if you hoard your life rather than risk it for an infinite gain, just as likely to occur as a loss amounting to nothing."

Our question is: how might Pascal argue that believing in God has higher expected utility than nonbelief?

Pascal says two things which help us here. **First**, he emphasizes that "there is an equal chance of gain and loss" -- an equal chance that God exists, and that God does not exist. This means that we should assign each a probability of 1/2.

Second, he says that in this case the amount to be won is **infinite**. We can represent this by saying that the utility of belief in God if God exists is ∞.

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

Pascal says two things which help us here. **First**, he emphasizes that "there is an equal chance of gain and loss" -- an equal chance that God exists, and that God does not exist. This means that we should assign each a probability of 1/2.

Second, he says that in this case the amount to be won is **infinite**. We can represent this by saying that the utility of belief in God if God exists is ∞.

Let's concede the point made above in connection with dominance reasoning: if we believe in God, and God does not exist, this involves some loss of utility. This loss will be finite -- let's symbolize it by word "LOSS".

We can then think of the decision whether or not to believe as follows:

Courses of action	Possibility 1: God exists *(Prob. = 0.5)*	Possibility 2: God does not exist *(Prob. = 0.5)*
Believe in God	∞	LOSS
Don't believe	0	0

Supposing that this is the correct assignment of values, what is the expected utility of each action?

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

Supposing that this is the correct assignment of values, what is the expected utility of each action?

Courses of action	Possibility 1: God exists *(Prob. = 0.5)*	Possibility 2: God does not exist *(Prob. = 0.5)*
Believe in God	∞	LOSS
Don't believe	0	0

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

Supposing that this is the correct assignment of values, what is the expected utility of each action?

Courses of action	Possibility 1: God exists *(Prob. = 0.5)*	Possibility 2: God does not exist *(Prob. = 0.5)*	Expected utilities
Believe in God	∞	LOSS	.5* ∞ + .5-LOSS= ∞
Don't believe	0	0	.5*0 +.5*0 = 0

So it looks as though the expected utility of believing in God is infinite, whereas the expected utility of nonbelief is 0. If the rule of expected utility is correct, it follows that it is rational to believe in God - and it is not a very close call.

We will consider two sorts of objections to this expected utility argument for the rationality of belief in God. One sort is based on the values in the above table: one might dispute Pascal's claims about the values or probabilities of different outcomes. A second sort of objection is based on objections to the rule of expected utility itself.

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

Courses of action	Possibility 1: God exists *(Prob. = 0.5)*	Possibility 2: God does not exist *(Prob. = 0.5)*	Expected utilities
Believe in God	∞	LOSS	$.5 * \infty + .5\text{-LOSS} = \infty$
Don't believe	0	0	.5*0 +.5*0 = 0

Let's consider a few objections of the first sort. First, suppose that the probability that God exists is not 1/2, but some much smaller number -- say, 1/100. Would that affect the argument above?

It seems not. Let's suppose that the probability that God exists is some nonzero finite number *m*, which can be as low as one likes. Then it seems that we should think of the choice between belief and nonbelief as follows:

Courses of action	Possibility 1: God exists *(Prob. = m)*	Possibility 2: God does not exist *(Prob. = 1-m)*	Expected utilities
Believe in God	∞	LOSS	$m * \infty + (1-m)\text{-LOSS} = \infty$
Don't believe	0	0	m*0 + (1-m)*0 = 0

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

It seems not. Let's suppose that the probability that God exists is some nonzero finite number *m*, which can be as low as one likes. Then it seems that we should think of the choice between belief and nonbelief as follows:

Courses of action	Possibility 1: God exists *(Prob. = m)*	Possibility 2: God does not exist *(Prob. = 1-m)*	Expected utilities
Believe in God	∞	LOSS	$m * \infty + (1-m)$-LOSS$= \infty$
Don't believe	0	0	$m*0 + (1-m)*0 = 0$

This is a real strength of Pascal's argument: **it does not depend on any assumptions about the probability that God exists other than the assumption that it is nonzero**. In other words, he is only assuming that we don't know for sure that God does not exist, which seems to many people - including many atheists - to be a reasonable assumption.

But another objection might seem more challenging. It seems that Pascal is assuming that, if God exists, there is a 100% chance that believers will get infinite reward. But why assume that? Why not think that if God exists there is some chance that believers will get infinite reward, and some chance that they won't? How would that affect the above chart?

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

But another objection might seem more challenging. It seems that Pascal is assuming that, if God exists, there is a 100% chance that believers will get infinite reward. But why assume that? Why not think that if God exists there is some chance that believers will get infinite reward, and some chance that they won't? How would that affect the above chart?

To accommodate this possibility, we would have to add another column to our chart, to represent the two possibilities imagined. Let's call these possibilities "Rewarding God" and "No reward God", and let's suppose that each has a nonzero probability of being true -- respectively, *m* and *n*. The resulting chart looks like this:

Courses of action	Possibility 1: Rewarding God exists (Prob. = m)	Possibility 2: No reward God exists (Prob. = n)	Possibility 3: God does not exist (Prob. = $1-(m+n)$)	Expected utilities
Believe in God	∞	0	LOSS	$m^* \infty + n^*0 + (1-(m+n))$ *LOSS= ∞
Don't believe	0	0	0	$m^*0 + n^*0 + (1-(m+n))^*0 = 0$

As this chart makes clear, adding this complication has **no effect** on the result. Pascal needn't assume that God will certainly reward all believers; he need only assume that there is a nonzero chance that God will reward all believers.

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

As this chart makes clear, adding this complication has **no effect** on the result. Pascal needn't assume that God will certainly reward all believers; he need only assume that there is a nonzero chance that God will reward all believers.

However, one might think that there is yet another relevant possibility that we are overlooking. After all, isn't there some chance that God might give eternal reward to believers and nonbelievers alike? This is surely possible. Let's call this the possibility of "Generous God." Setting aside the possibility of No reward God, which we have seen to be irrelevant, taking account of the possibility of Generous God has a striking effect on the expected utilities of belief and nonbelief:

Courses of action	Possibility 1: Rewarding God exists *(Prob. = m)*	Possibility 2: Generous God exists *(Prob. = n)*	Possibility 3: God does not exist *(Prob. = 1-(m+n))*	Expected utilities
Believe in God	∞	∞	LOSS	$m^* \infty + n^* \infty + (1-(m+n))$ *LOSS= ∞
Don't believe	0	∞	0	$m^*0 + n^* \infty + (1-(m+n))^*0 = \infty$

Now, it appears, **belief and nonbelief have the same infinite expected utility**, which undercuts Pascal's argument for the rationality of belief in God.

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

Courses of action	Possibility 1: Rewarding God exists *(Prob. = m)*	Possibility 2: Generous God exists *(Prob. = n)*	Possibility 3: God does not exist *(Prob. = 1-(m+n))*	Expected utilities
Believe in God	∞	∞	LOSS	$m^* \infty + n^* \infty + (1-(m+n))$ *LOSS= ∞
Don't believe	0	∞	0	m*0 + $n^* \infty$ + (1-(m+n))*0 = ∞

Now, it appears, **belief and nonbelief have the same infinite expected utility**, which undercuts Pascal's argument for the rationality of belief in God.

However, Pascal seems to have a reasonable reply to this objection. It seems that the objection turns on the fact that any probability times an infinite utility will yield an infinite expected value. And that means that any two actions which have some chance of bring about an infinite reward will have the same expected utility.

But this is extremely counterintuitive. Suppose we think of a pair of lotteries, EASY and HARD. Each lottery has an infinite payoff, but EASY has a 1/3 chance of winning, whereas HARD has a 1/1,000,000 chance of winning. What is the expected utility of EASY vs. HARD? Which would you be more rational to buy a ticket for?

How might we modify our rule of expected utility to explain this case? Would this help Pascal respond to the case of Generous God?

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

How might we modify our rule of expected utility to explain this case? Would this help Pascal respond to the case of Generous God?

A natural suggestion is to say something like this: if two actions each have infinite expected utility, then (supposing that neither action has a very high chance of leading to a very bad outcome) it is rational to go with the action that has the higher probability of leading to the infinite reward. This sort of supplement to the rule of expected utility explains why it is smarter to buy a ticket in EASY than in HARD; and it also helps Pascal solve the problem of Generous God, since the believer receives an infinite reward if **either** Generous God or Rewarding God exists, whereas the nonbeliever only gets a reward in the first of these cases.

This does not help, however, with yet another objection. It is conceivable that God would find abhorrent the idea of believing in God on the basis of Pascal's wager, and decide to give eternal reward to just those people who do not choose to believe. Call this the possibility of Anti-Wager God.

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

This does not help, however, with yet another objection. It is conceivable that God would find abhorrent the idea of believing in God on the basis of Pascal's wager, and decide to give eternal reward to just those people who do not choose to believe. Call this the possibility of Anti-Wager God.

Taking into account this possibility makes the decision look like this:

Courses of action	Possibility 1: Rewarding God exists *(Prob. = m)*	Possibility 2: Anti-Wager God exists *(Prob. = n)*	Possibility 3: God does not exist *(Prob. = 1-(m+n))*	Expected utilities
Believe in God	∞	0	LOSS	$m^* \infty + n^* \infty + (1-(m+n))$ *LOSS= ∞
Don't believe	0	∞	0	$m^*0 + n^* \infty + (1-(m+n))^*0 = \infty$

It is no longer obvious that belief has a higher chance of reward than nonbelief: we need an argument that Rewarding God is more likely to exist than Anti-Wager God. This shows that Pascal's argument can't be completely free of commitments to the probabilities of certain theological claims.

How would Pascal handle the case of religions with differing views about God? Would the complications that these would introduce into the decision be analogous to those introduced by Anti-Wager God?

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

Even if we can solve the cases of Generous God and Anti-Wager God in this way, these possibilities raise a troubling sort of objection to Pascal's argument. His argument turns essentially on the possibility of decisions with infinite expected utility. But it is plausible that, in at least some cases which involve infinite expected utility, the rule of expected utility gives us incorrect results. Consider the following bet:

The St. Petersburg

I am going to flip a fair coin until it comes up heads. If the first time it comes up heads is on the 1st toss, I will give you \$2. If the first time it comes up heads is on the second toss, I will give you \$4. If the first time it comes up heads is on the 3rd toss, I will give you \$8. And in general, if the first time the coin comes up heads is on the *n*th toss, I will give you \$$2^n$.

Would you pay \$2 to take this bet? How about \$4?

Suppose now I raise the price to \$10,000. Should you be willing to pay that amount to play the game?

What is the expected utility of playing the game?

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

The St. Petersburg

I am going to flip a fair coin until it comes up heads. If the first time it comes up heads is on the 1st toss, I will give you \$2. If the first time it comes up heads is on the second toss, I will give you \$4. If the first time it comes up heads is on the 3rd toss, I will give you \$8. And in general, if the first time the coin comes up heads is on the *n*th toss, I will give you $\$2^n$.

What is the expected utility of playing the game?

We can think about this using the following table:

Outcome	First heads is on toss #1	First heads is on toss #2	First heads is on toss #3	First heads is on toss #4	First heads is on toss #5	.....
Probability	\$2	\$4	\$8	\$16	\$32	.....
Payoff	1/2	1/4	1/8	1/16	1/32	.....

The expected utility of playing = the sum of probability * payoff for each of the infinitely many possible outcomes. So, the expected utility of playing equals the sum of the infinite series

1+1+1+1+1+ 1+1+1+1+1+ 1+1+1+1+1+ 1+1+1+1+1+......

The rule of expected utility

It is always rational to pursue the course of action with the highest expected utility.

The St. Petersburg

I am going to flip a fair coin until it comes up heads. If the first time it comes up heads is on the 1st toss, I will give you $2. If the first time it comes up heads is on the second toss, I will give you $4. If the first time it comes up heads is on the 3rd toss, I will give you $8. And in general, if the first time the coin comes up heads is on the *n*th toss, I will give you $$2^n$.

Outcome	First heads is on toss #1	First heads is on toss #2	First heads is on toss #3	First heads is on toss #4	First heads is on toss #5	.....
Probability	$2	$4	$8	$16	$32	.....
Payoff	1/2	1/4	1/8	1/16	1/32	.....

The expected utility of playing = the sum of probability * payoff for each of the infinitely many possible outcomes. So, the expected utility of playing equals the sum of the infinite series

1+1+1+1+1+ 1+1+1+1+1+ 1+1+1+1+1+ 1+1+1+1+1+......

But it follows from this result, plus the rule of expected utility, that **you would be rational to pay any finite amount of money to have the chance to play this game once**. But this seems clearly mistaken. What is going on here?

Does this show that the rule of expected utility can lead us astray? If so, in what sorts of cases does this happen? Does this result depend essentially on their being infinitely many possible outcomes?

Let's now turn to a pair of quite different objections, which focus on the fact that Pascal is asking us to make decisions about what to **believe** on the basis of expected utility calculations. The objections are that this is (1) impossible and (2) the wrong way to form beliefs.

If I offer you $5 to raise your arm, you can do it. But, to illustrate objection (1), suppose I offered you $5 to believe that you are not now sitting down. Can you do that (without standing up)?

Pascal considered this objection, and gave the following response:

> "...is there really no way of seeing what the cards are? ...I am being forced to wager and I am not free; I am begin held fast and I am so made that I cannot believe. What do you want me to do then?"
>
> "That is true, but at least get it into your head that, if you are unable to believe, it is because of your passions, since reason impels you to believe and yet you cannot do so. Concentrate then not on convincing yourself by multiplying proofs of God's existence, but by diminishing your passions. ..."

What does he have in mind here?

Let’s now turn to a pair of quite different objections, which focus on the fact that Pascal is asking us to make decisions about what to **believe** on the basis of expected utility calculations. The objections are that this is (1) impossible and (2) the wrong way to form beliefs.

To see how objection (2) might be developed, consider the following claim about how we ought to form beliefs:

> **The rule of responsible belief formation**
>
> It is irrational to form a belief in a claim unless you have more reason to think that it true than that it is false.

It is plausible that, in at least many cases, we hold each other to this sort of rule. If your friend often forms beliefs despite having no reason for thinking that the belief is more likely to be true than false, you would likely take him to be irrational -- as well as untrustworthy.

But now recall the rule used in Pascal’s wager:

> **The rule of expected utility**
>
> It is always rational to pursue the course of action with the highest expected utility.

This also seemed plausible; and it led, without the assumption that we have any more reason to think that God exists than not, to the conclusion that it is rational to believe in God. This shows that, if we think of the rule of expected utility as applying to beliefs, these two rules sometimes come into conflict. This means that at least one must be, as it stands, false.