Kripkenstein

The "rule-following paradox" is a paradox about how it is possible for us to mean anything by the words of our language. More precisely, it is an argument which seems to show that it is impossible for us to mean anything by the words we use - which seems absurd.

To understand this paradox, a first step is to grasp clearly the distinction between an **expression** and **what that expression refers to** - the thing in the world that it is used to talk about.

This distinction is clear enough as applied to the names of ordinary objects. We can distinguish between the name, "Fido," and the flesh-and-blood animal it stands for. To make a clear distinction even clearer, we might point out that a single dog can have more than one name without being more than one dog.

Now consider a verb - like "is pretty." Does this also stand for something?

It seems that it does - it stands for a certain way that something can be, the property of being pretty. And this, no less than Fido, is a non-linguistic thing. Being pretty would be exactly the property that it is if English had never come into being, and "pretty" had never been a word - just like Fido could have existed, and been the dog he is, even if "Fido" had never been given to him as a name. And just like Fido could have had several names, we know that in different languages there are different ways of talking about the property of being pretty - "est jolie" is a different collection of words than "is pretty", but in French the former words stand for just the same thing that the latter words stand in English.

But this distinction might seem less clear in the case of mathematical language; it is important to see that we have the same distinction here as elsewhere, since the example we will be discussing is an example which involves the use of mathematical language.

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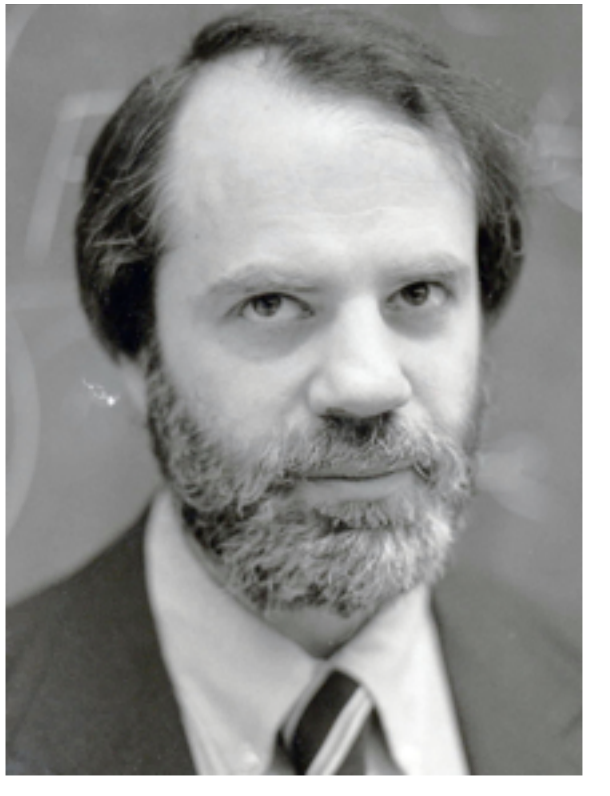
One way to bring this out is by thinking about different mathematical symbol systems. Consider, for example, the following three expressions:

23

XXIII

10111

These all stand for the same number: the first in the system of Arabic numerals, the second in the system of Roman numerals, and the last in base-2 notation. Hence the number - what these all stand for, and are used to talk about - must be something distinct from each of these expressions (just as in the case of Fido).



The paradox which we'll be discussing today is a paradox about how we can use symbols to talk about things. The paradox is due to Saul Kripke, a contemporary American philosopher.

Kripke develops the paradox in terms of our use of the "+" symbol. Just as we can distinguish between numerals in various systems of representation and the numbers they are used to talk about, so we can distinguish between this symbol and the thing it is used to talk about - namely, the addition function.

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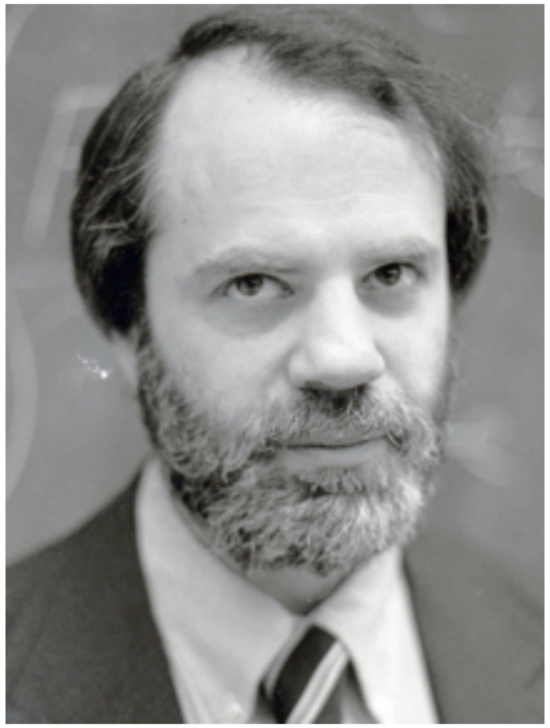

Kripke develops the paradox in terms of our use of the "+" symbol. Just as we can distinguish between numerals in various systems of representation and the numbers they are used to talk about, so we can distinguish between this symbol and the thing it is used to talk about - namely, the addition function.

For our purposes, we can think of a function as a rule which takes you from arguments to values. The addition function is a rule which, given a pair of numbers as argument, gives you their sum as value. A function can thus be thought of as a kind of machine, into which you give some input and receive some output. It is important that functions, so conceived, are machines which, for any specific input, always give you the same output.

There is no mystery about what the addition function is. And it also seems obvious that the addition function is the thing that we use the symbol "+" to talk about - it is what that symbol stands for. However, Kripke gives us an argument which seems to show that this is not the case; that it is, in fact, not true that "+" stands for the addition function.

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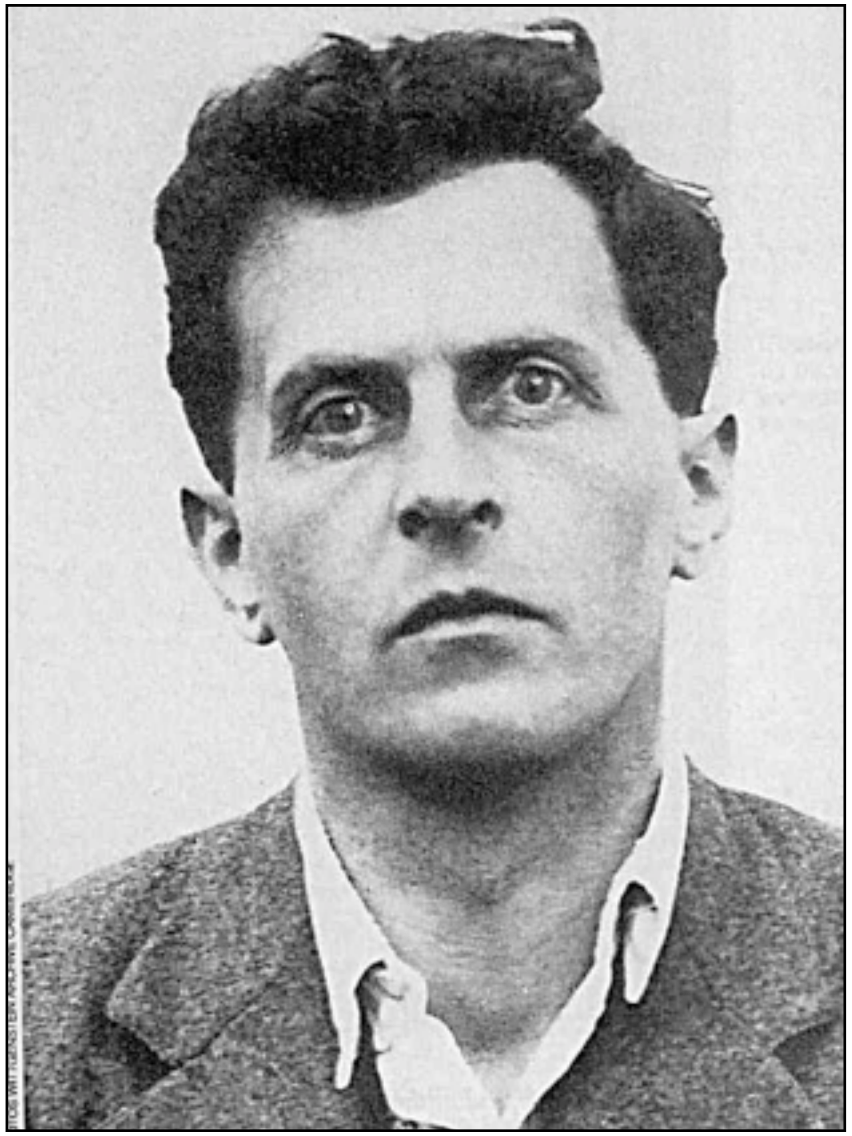

Rather, it derives from a general paradox about meaning developed by the 20th century Austrian philosopher Ludwig Wittgenstein.

Wittgenstein summed up his paradox in the following words:

> "this was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule." (*Philosophical Investigations* §201)

This is the paradox that Kripke tries to explicate using the example of addition. Because, as Kripke says, the paradox which results is neither original to him nor to be found in that form in Wittgenstein's work, it is sometimes called the Kripkensteinian paradox.

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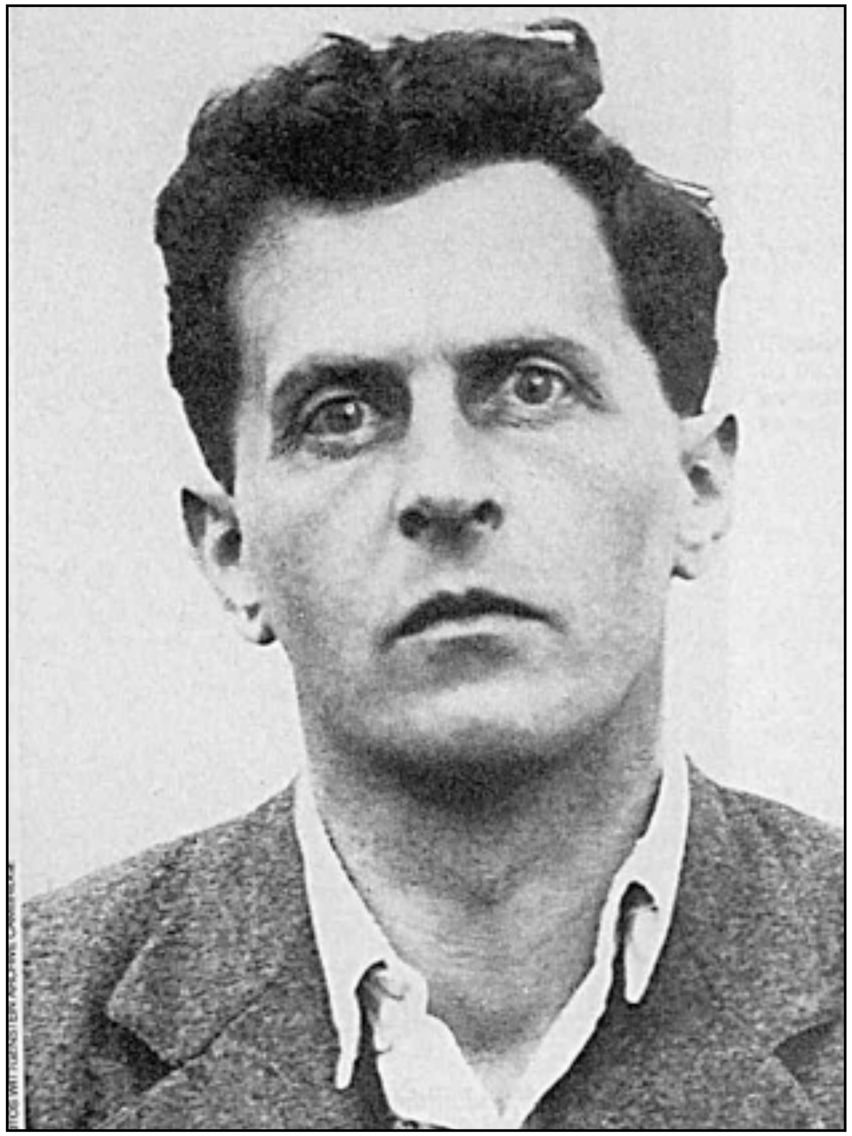

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Kripke develops his argument using the example of a mathematical function which is closely related to the addition function: the quaddition, or quus, function. As addition is symbolized by "+", we can follow Kripke by symbolizing the quus function as "⊕."

The quaddition function can be defined as follows:

Definition of quaddition

$x \oplus y = x + y$ if $x, y < 57$
$x \oplus y = 5$ otherwise

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It is important to see that quaddition is a perfectly well-defined function; it is no more and no less well-defined than addition itself.

One way to bring out the Kripkensteinian paradox is to ask yourself: yesterday, did you mean addition or quaddition by your use of the "+" sign?

The answer seems pretty straightforward: you mean addition, rather than quaddition. Why do we think this? Well, one reason has to do with our views about truth and falsehood.

More specifically, it seems clear that yesterday, had you written the sentence

$68 + 57 = 125$

this sentence would have been true, whereas if you had written the sentence

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that sentence would have been false.

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What does it take for a sentence to be true? If you think about it, you will see that for a sentence to be true, two things have to be the case: (1) the sentence must mean something; it must say that the world is a certain way; (2) the world must be that way. Consider, for example, the following two sentences:

$$10111 + 1 = 11000$$

$$10111 + 1 = 10112$$

Which is true, and which is false? It depends who is using the sentences. If we are using the sentences - people who by default speak a base-10 mathematical language - then the second is true, and the first false. But if a base-2 speaker is using these sentences, then the first is true, and the second nonsensical. Does this mean that the mathematical facts are relative to speakers? Obviously not; this is just a case in which two speakers are using a single sentence to express different mathematical claims.

In general, the truth or falsity of sentences is dependent on both the meaning of those sentences, and the relevant state of the world.

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In the case of the sentences above, we know the relevant state of the world; we know that the addition function gives 125 as value for 68, 57 as arguments, and that the quaddition function gives 5 as value for these arguments. Hence if we think that the first sentence out of your mouth was true yesterday and the second sentence false, we must think that yesterday you meant plus rather than quus by the "+" symbol.

This is, perhaps, unsurprising. What is surprising is the argument of Kripke's skeptic against this seemingly obvious truth.

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The skeptic's challenge

Now suppose I encounter a bizarre sceptic. This sceptic questions my certainty about my answer, in what I just called the 'metalinguistic' sense. Perhaps, he suggests, as I used the term 'plus' in the past, the answer I intended for '68+57' should have been '5'! Of course the sceptic's suggestion is obviously insane. My initial response to such a suggestion might be that the challenger should go back to school and learn to add. Let the challenger, however, continue. After all, he says, if I am now so confident that, as I used the symbol '+', my intention was that '68+57' should turn out to denote 125, this cannot be because I explicitly gave myself instructions that 125 is the result of performing the addition in this particular instance. By hypothesis, I did no such thing. But of course the idea is that, in this new instance, I should apply the very same function or rule that I applied so many times in the past. But who is to say what function this was? In the past I gave myself only a finite number of examples instantiating this function. All, we have supposed, involved numbers smaller than 57. So perhaps in the past I used 'plus' and '+' to denote a function which I will call 'quus' and symbolize by '⊕'. It is defined by:

$$
\begin{aligned}
x \oplus y &= x + y, \text{ if } x, y < 57 \\
&= 5 \quad \text{otherwise.}
\end{aligned}
$$

Who is to say that this is not the function I previously meant by '+'?

The sceptic claims (or feigns to claim) that I am now misinterpreting my own previous usage. By 'plus', he says, I *always meant* quus;[8] now, under the influence of some insane frenzy, or a bout of LSD, I have come to misinterpret my own previous usage.

Ridiculous and fantastic though it is, the sceptic's hypothesis is not logically impossible. To see this, assume the common sense hypothesis that by '+' I *did* mean addition. Then it would be *possible*, though surprising, that under the influence of a momentary 'high', I should misinterpret all my past uses of the plus sign as symbolizing the quus function, and proceed, in conflict with my previous linguistic intentions, to compute 68 plus 57 as 5. (I would have made a mistake, not in mathematics, but in the supposition that I had accorded with my previous linguistic intentions.) The sceptic is proposing that I have made a mistake precisely of this kind, but with a plus and quus reversed.

Now if the sceptic proposes his hypothesis sincerely, he is crazy; such a bizarre hypothesis as the proposal that I always meant quus is absolutely wild. Wild it indubitably is, no doubt it is false; but if it is false, there must be some fact about my past usage that can be cited to refute it. For although the hypothesis is wild, it does not seem to be *a priori* impossible.

What, exactly, is the skeptic's argument? The skeptic's main contention is that if, in the past, I meant plus rather than quus by the "+" sign, there must be some fact about me in the past which explains this - some fact which makes it the case that I meant plus rather than quus.

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But, the skeptic will argue, there is no such fact; hence in the past I did not mean plus rather than quus by the "+" sign.

One might therefore think of the argument as follows:

The skeptical argument	
1	If in the past I meant plus rather than quus by "+", there must be some fact about me which determines that I meant plus rather than quus.
2	There is no such fact.
C	In the past I did not mean plus rather than quus by "+."

Is this really a paradox? Is the conclusion of this argument really surprising?

To bring out the surprising nature of the conclusion, one might focus on practical uses of mathematical language. Imagine that you were in Notre Dame's bookstore yesterday, and bought two textbooks, one of which cost \$68 and the other of which cost \$57. Suppose that you had then said to yourself: "OK, $57 + 68 = 5$, so I owe a total of \$5." Isn't it very clear that that you would have spoken falsely? But if you did, then you must have mean addition rather than quaddition by the "+" sign since, if you had meant quaddition, you would have spoken truly. not falsely.

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The first premise of the skeptic's argument seems hard to disagree with. After all, it is surely possible that there be people who use "+" to mean quaddition - intuitively, it seems clear that we could have decided to do this. This seems just as obvious as that we could have decided to use a numeric system other than base-10.

So attention naturally focuses on the second premise: the claim that there is no fact about me in the past which determines that I meant plus rather than quus. Surely, one thinks, there must be such a fact!

The skeptic's argument consists in a series of arguments designed to show that whatever sort of fact one comes up with is in fact not suited to do the job.

What would it take for some fact about me to satisfy the skeptic, and show premise 2 to be false?

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What would it take for some fact about me to satisfy the skeptic, and show premise 2 to be false?

> An answer to the sceptic must satisfy two conditions. First, it must give an account of what fact it is (about my mental state) that constitutes my meaning plus, not quus. But further, there is a condition that any putative candidate for such a fact must satisfy. It must, in some sense, show how I am justified in giving the answer '125' to '68+57'. The 'directions' mentioned in the previous paragraph, that determine what I should do in each instance, must somehow be 'contained' in any candidate for the fact as to what I meant. Otherwise, the sceptic has not been answered when he holds that my present response is arbitrary.

The first condition is that the fact must rule out the skeptical hypothesis that I meant quus rather than plus. The second condition is a bit trickier. Consider this: it is not just the case that it is true that, had I said "$57 + 68 = 125$" yesterday, I would have spoken truly; it is also the case that I knew this to be true. But that means that I must have known yesterday that I meant plus rather than quus; after all, had I not known this, I would have had no reason to be confident that this sentence was true. Hence an answer to the skeptic's challenge must not only rule out the skeptical hypothesis; it must also be the sort of thing that can explain my being so sure that "125" was the right answer.

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Let's now consider some attempts to answer the skeptical challenge.

A natural first thought is that the skeptical hypothesis is ruled out by my past usage of the "+" sign. After all, in the past I always responded to questions of the form "What is x + y?" by giving the sum of x and y, rather than their quum. And, furthermore, I know this about myself; so isn't my past usage of this sign just the fact we were looking for?

No. After all, we are supposing that I have never computed an addition problem involving any numbers greater than 56; so my past usage of the "+" sign is equally consistent with my having meant quus and plus, since the quus and plus functions agree perfectly for all numbers < 56.

But one might reasonably protest at this point: look, everyone in this room has computed addition problems involving numbers much higher than this. So doesn't our past usage discriminate between quus and plus?

How might the skeptic respond to this objection?

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Candidates for the fact that I meant plus rather than quus

past usage of "+"

rules for use of "+"

dispositions to use "+"

an experience of meaning plus

A different attempt to answer the skeptical challenge focuses not on past usage of the "+" sign, but rather on the rules which I used in the past to govern my usage of this sign - the instructions, or algorithms, that I mastered. On this view, it is the rule that I mastered which rules out quus, not the list of past applications of that rule.

What rule did I master? Here is one suggestion:

> What was the rule? Well, say, to take it in its most primitive form: suppose we wish to add *x* and *y*. Take a huge bunch of marbles. First count out *x* marbles in one heap. Then count out *y* marbles in another. Put the two heaps together and count out the number of marbles in the union thus formed. The result is $x+y$.

Perhaps it was my adoption of this rule that rules out the skeptical hypothesis.

Despite its initial appeal, as Kripke says, this response to the skeptic is open to an immediate objection.

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> Despite the initial plausibility of this objection, the sceptic's response is all too obvious. True, if 'count', as I used the word in the past, referred to the act of counting (and my other past words are correctly interpreted in the standard way), then 'plus' must have stood for addition. But I applied 'count', like 'plus', to only finitely many past cases. Thus the sceptic can question my present interpretation of my past usage of 'count' as he did with 'plus'. In particular, he can claim that by 'count' I formerly meant *quount*, where to 'quount' a heap is to count it in the ordinary sense, unless the heap was formed as the union of two heaps, one of which has 57 or more items, in which case one must automatically give the answer '5'. It is clear that if in the past 'counting' meant quounting, and if I follow the rule for 'plus' that was quoted so triumphantly to the sceptic, I must admit that '68+57' must yield the answer '5'. Here I have supposed that previously 'count' was never applied to heaps formed as the union of sub-heaps either of which has 57 or more elements, but if this particular upper bound does not work, another will do. For the point is perfectly general: if 'plus' is explained in terms of 'counting', a non-standard interpretation of the latter will yield a non-standard interpretation of the former.[12]

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Of course, one might give more sophisticated rules for defining addition than the one given in terms of counting; one might give something more like an algorithm for addition one gets in math classes.

But this would not help, for two reasons.

First, it would be open to the "count"/"quount" problem. Any algorithm will be given in terms of other symbols, which raise just the same problems as "+". (For example, the symbol for the successor function.) Hence we are again delaying the skeptical problem rather than solving it.

Second, it is not the sort of answer that could work; after all, many people mean addition by the "+" sign without being in a position to produce anything like an algorithm for addition.

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So, as Kripke says, one cannot explain the fact that I followed the rule of using "+" to mean addition in terms of other rules. So let's turn to a different sort of response.

One might think that the "past usage" response to the skeptic can be modified in a way which solves the problems we found with that response. After all, even if I never considered any addition problem involving a number higher than 57, there are surely facts about how I would have responded, were I given such an addition problem. It is likely that you have never considered the addition problem "57+68" - but it was true of you yesterday that, had you been given this addition problem, you would have answered "125." So even if the skeptical hypothesis is not ruled out by my past usage of "+", perhaps it is ruled out by my past dispositions for the use of this symbol. This is what Kripke calls the "dispositional solution" to the skeptical problem.

According to the dispositional solution, the fact that I meant addition by "+" consists in the fact that I was disposed, for any numbers x and y, to respond to "x+y" by giving their sum, rather than their quum.

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According to the dispositional solution, the fact that I meant addition by "+" consists in the fact that I was disposed, for any numbers x and y, to respond to "x+y" by giving their sum, rather than their quum.

This simple dispositional solution is open to three objections, Kripke thinks.

Error. Typically we are not disposed to respond to every instance of '$x + y$' with the sum of x and y: everyone is disposed, in at least some cases, to make mistakes. But then it follows that if the function attached to '+' is determined by the equations involving '+' which we are disposed to accept, we do not mean addition by '+.'

Finitude. We are finite creatures capable only of understanding numbers of limited size; so "not only my actual performance, but also the totality of my dispositions, is finite" (26). But the addition function is defined over numbers of arbitrary size. So, once again, the dispositional theory delivers the wrong result: if our dispositions for using '+' really did determine the function attached to it, then the function expressed by '+' would apply to only numbers small enough for us to grasp. But addition is not like this.

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Normativity. The dispositionalist tries to explain the nature of meaning in terms of what we are disposed to do. But, Kripke says, this is the wrong sort of answer. He says: "Suppose I do mean addition by '+'. What is the relation of this supposition to the question how I will respond to the problem '68 + 57'? The dispositionalist gives a descriptive account of this relation: if '+' meant addition, then I will answer '125.' But this is not the proper account of the relation, which is normative, not descriptive. The point is not that, if I meant addition by '+', I *will* answer '125', but that, if I intend to accord with my past meaning of '+', I *should* answer '125.' . . . The relation of meaning and intention to future action is normative, not descriptive" (37).

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One might respond to these problems by giving a more sophisticated dispositional solution to the problem. One might appeal not to what my dispositions for using "+" actually are, but what they would be if my brain were radically expanded, or my cognitive abilities radically increased, or ...

One might think that this can help with the objections to the simple dispositional theory. But Kripke thinks not:

> How in the world can I tell what would happen if my brain were stuffed with extra brain matter, or if my life were prolonged by some magic elixir? Surely such speculation should be left to science fiction writers and futurologists. We have no idea what the results of such experiments would be. They might lead me to go insane, even to behave according to a quus-like rule.

This indeterminacy, Kripke thinks, is inconsistent with the fact that we are quite sure that "125" is the correct response to "57 + 68" (the second condition on responses to the skeptic above). Surely, to know that this is the right answer, one need not know how one would be disposed to respond after undergoing a brain-expanding experiment of some sort.

The skeptical argument
1 If in the past I meant plus rather than quus by "+", there must be some fact about me which determines that I meant plus rather than quus.
2 There is no such fact.
C In the past I did not mean plus rather than quus by "+."

Definition of quaddition

$$x \oplus y = x + y \text{ if } x, y < 57$$
$$x \oplus y = 5 \quad \text{otherwise}$$

Candidates for the fact that I meant plus rather than quus

past usage of "+"
rules for use of "+"
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It is extremely natural to think that we can answer the skeptic by looking to facts related to my past use, or dispositions to use, the "+" sign. But at this point, it might seem that we have exhausted all such avenues.

So let's look in another direction. Consider a mental event other than meaning addition by the "+" sign: having a headache. Surely we have headaches, and we know that we have headaches. How does this work? How can one tell that one is having a headache? A plausible answer is: it just feels a certain way, and one knows that one is having a headache by simply recognizing that feeling.

Maybe meaning is more like this than we have supposing. Maybe meaning addition by "+" is simply an irreducible mental event, with a certain "feel" to it - one which we can recognize (and use to justify our answers to addition problems) just as surely as we can recognize headaches.

There are two problems here. The first is that it is somewhat mysterious how an inner "feel" of this sort could tell us how to answer indefinitely many addition problems; how does this inner feeling tell us what the answer to "57+68" is?

But there is another, more obvious problem as well.

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Kripke (along with Wittgenstein) thinks that, if we simply examine our inner lives, we will find that there is no "qualitative feel" analogous to the feeling of a headache which could be the mental state of meaning addition by "+." It might "feel just the same" to mean quaddition and addition by "+."

> It takes relatively little introspective acuteness to realize the dubiousness of the attribution of a special qualitative character to the 'experience' of meaning addition by 'plus'. Attend to what happened when I first learned to add. First, there may or may not have been a specifiable time, probably in my childhood, at which I suddenly felt (*Eureka!*) that I had grasped the rule for addition. If there was not, it is very hard to see in what the suppositious special experience of my learning to add consisted. Even if there was a particular time at which I could have shouted "*Eureka!*" – surely the exceptional case – in what did the attendant experience consist? Probably consideration of a few particular cases and a thought – "Now I've got it!" – or the like. Could just *this* be the content of an experience of 'meaning addition'? How would it have been different if I had meant quus? Suppose I perform a particular addition now, say '5+7'. Is there any special quality to the experience? Would it have been different if I had been trained in, and performed, the corresponding quaddition? How different indeed would the *experience* have been if I had performed the corresponding multiplication ('5×7'), other than that I would have responded automatically with a different answer? (Try the experiment yourself.)

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One thing you might want to think about is whether there are any promising candidates for the fact that I meant addition by "+" that Kripke fails to consider.