## Omnipotence & omniscience

So far, in our discussion of theological paradoxes, we have been focusing on apparent contradictions between the existence of God, as traditionally conceived, and various apparent features of the world: the existence of evil, and of human free will. But attempts have also been made to show that there are contradictions inherent in the very idea of God. Today we focus on two of those, which are, respectively, attempts to show that there is a contradiction in the idea of an omnipotent being and in the idea of an omnipotent being.

We begin with the logical problem posed by omnipotence, which we've already come across in Mackie's discussion of the problem of evil. Here is how Mackie presents the problem:

This leads us to what I call the Paradox of Omnipotence: can an omnipotent being make things which he cannot subsequently control? Or, what is practically equivalent to this, can an omnipotent being make rules which then bind himself? (These are practically equivalent because any such rules could be regarded as setting certain things beyond his control, and *vice versa*.) The second of these formulations is relevant to the suggestions that we have already met, that an omnipotent God creates the rules of logic or causal laws, and is then bound by them.

It is clear that this is a paradox : the questions cannot be answered satisfactorily either in the affirmative or in the negative. If we answer "Yes", it follows that if God actually makes things which he cannot control, or makes rules which bind himself, he is not omnipotent once he has made them : there are *then* things which he cannot do. But if we answer "No", we are immediately asserting that there are things which he cannot do, that is to say that he is already not omnipotent.

It cannot be replied that the question which sets this paradox is not a proper question. It would make perfectly good sense to say that a human mechanic has made a machine which he cannot control : if there is any difficulty about the question it lies in the notion of omnipotence itself.

Some needless complications are introduced by the question of whether God can bind himself with rules; the basic question is just whether an omnipotent being can bring about a state of affairs in which that being is then unable to perform some action.

Some needless complications are introduced by the question of whether God can bind himself with rules; the basic question is just whether an omnipotent being can bring about a state of affairs in which that being is then unable to perform some action.

A traditional formulation of this problem is the paradox of the stone, which focuses on the question: Could God create a stone so large that God cannot lift it?

As Mackie says, it seems that if God is genuinely omnipotent, we cannot answer either "Yes" or "No" to this question.



We can also present this in premise/conclusion form, as a derivation of a contradiction from the assumption that God is essentially omnipotent.

1 God is essentially omnipotent.	assumed for reductio
2 Necessarily, God can bring about any state of affairs.	1
3 God can create a stone so large that God cannot lift it.	2
4 Possibly, there is a stone which is such that God cannot lift it.	3
5 Possibly, God can do anything and there is a stone which God cann lift.	ot 2, 4

Though Aquinas does not explicitly respond to this paradox, his remarks on omnipotence do provide the resources for a solution.

Could God create a stone so large that God cannot lift it?



We can also present this in premise/conclusion form, as a derivation of a contradiction from the assumption that God is essentially omnipotent.

1 God is essentially omnipotent.	assumed for reductio
2 Necessarily, God can bring about any state of affairs.	1
3 God can create a stone so large that God cannot lift it.	2
4 Possibly, there is a stone which is such that God cannot lift it.	3
5 Possibly, God can do anything and there is a stone which God cannot lift.	2, 4

Though Aquinas does not explicitly respond to this paradox, his remarks on omnipotence do provide the resources for a solution.

It remains therefore, that God is called omnipotent because he can do all things that are possible absolutely; which is the second way of saying a thing is possible. For a thing is said to be possible or impossible absolutely, according to the relation in which the very terms stand to one another, possible if the predicate is not incompatible with the subject, as that Socrates sits; and absolutely impossible when the predicate is altogether incompatible with the subject, as, for instance, that a man is a donkey. Aquinas is suggesting that we understand omnipotence to be defined in terms of possibility. To be omnipotent is not to be able to do anything; even an omnipotent being could not make a round square, or make a man a donkey.

But if this is the right view of omnipotence, then it seems as though our attempted reductio fails, because a premise in that argument is false: premise 2. Could God create a stone so large that God cannot lift it?



We can also present this in premise/conclusion form, as a derivation of a contradiction from the assumption that God is essentially omnipotent.

1 God is essentially omnipotent.	assumed for reductio
2 Necessarily, God can bring about any state of affairs.	1
3 God can create a stone so large that God cannot lift it.	2
4 Possibly, there is a stone which is such that God cannot lift it.	3
5 Possibly, God can do anything and there is a stone which God cannot lift.	2, 4

Though Aquinas does not explicitly respond to this paradox, his remarks on omnipotence do provide the resources for a solution.

Aquinas is suggesting that we understand omnipotence to be defined in terms of possibility. To be omnipotent is not to be able to do anything; even an omnipotent being could not make a round square, or make a man a donkey.

But if this is the right view of omnipotence, then it seems as though our attempted reductio fails, because a premise in that argument is false: premise 2. One might try to repair the argument by replacing premise 2 with the following:

2\*. Necessarily, God can bring about any **possible** state of affairs.

Would the resulting argument be sound?

Could God create a stone so large that God cannot lift it?



Then there is something that God cannot do, namely lift the stone.

Then there could be something that God cannot do, namely create such a stone.

1	God is essentially omnipotent.	assumed for reductio
2	Necessarily, God can bring about any state of affairs.	1
3	God can create a stone so large that God cannot lift it.	2
4	Possibly, there is a stone which is such that God cannot lift it.	3
5	Possibly, God can do anything and there is a stone which God cannot lift.	2, 4

However, one might also object to Aquinas' "restricted" view of omnipotence. Descartes is an example of someone who thought that, for God to be genuinely omnipotent, God had to be able to do **absolutely** anything, as the following excerpts from his correspondence suggest:

"The truths of mathematics ...were established by God and entirely depend on Him, as much as do all the rest of His creatures. Actually, it would be to speak of God as a Jupiter or Saturn and to subject Him to the Styx and to the Fates, to say that these truths are independent of Him ...You will be told that if God established these truths He would be able to change them, as a king does his laws; to which it is necessary to reply that this is correct. ...In general we can be quite certain that God can do whatever we are able to understand, but not that He cannot do what we are unable to understand. For it would be presumptuous to think that our imagination extends as far as His power. ...

As for the difficulty in conceiving how it was a matter of freedom and indifference to God to make it true that the three angles of a triangle should equal two right angles, or generally that contradictions should not be able to be together, one can easily remove it by considering that the power of God can have no limits. ... God cannot have been determined to make it true that contradictions cannot be together, and consequently He could have done the contrary

If this sort of "unrestricted" view of omnipotence is correct, then it seems that premise 2 of our reductio argument is true; which means that we are left without a response to the paradox of the stone.

How should a defender of Descartes' view of omnipotence reply to the paradox?

If God can make a round square, can God make a stone too large for him to lift, and also lift it?





It thus seems that, whichever view of omnipotence we adopt, the paradox of the stone poses no serious problems.

If God's power extends only to possible states of affairs, then the right answer to the above question is: No, God could not create such a stone; but, since it is impossible that there be such a stone, this is no objection to God's omnipotence.

If God can bring about any state of affairs, whether possible or impossible, then the right answer to the question is: Yes, God could create such a stone; but he could also lift it, so again we have no objection to God's omnipotence.

But the fact that the paradox of the stone dissolves under closer inspection does not show that the idea of omnipotence is unproblematic; after all, we still don't know exactly what it means to say that God is omnipotent.

Descartes' text suggests the following definition: **omnipotence is the ability to bring about anything**, whether possible or impossible.

But this seems to lead to absurd conclusions. For if his view is correct, then God could have made a round square; but, in general, if God could have brought about some state of affairs, then that state of affairs could have obtained; and if a state of affairs could have obtained, it is possible; from which it follows that it is possible that there be a round square. More generally, Descartes' view seems to lead to the conclusion that there is no distinction between the impossible and the possible.

One graphic way of bringing out the worry is by considering the impossible state of affairs that God never existed. Could it really be the case that God can bring it about that it was never the case that God existed?

So we might turn to Aquinas, whose text suggests the following definition: **omnipotence is the ability to bring about any possible state of affairs.** 

But the fact that the paradox of the stone dissolves under closer inspection does not show that the idea of omnipotence is unproblematic; after all, we still don't know exactly what it means to say that God is omnipotent.

Descartes' text suggests the following definition: omnipotence is the ability to bring about anything, whether possible or impossible.

But this seems to lead to absurd conclusions. For if his view is correct, then God could have made a round square; but, in general, if God could have brought about some state of affairs, then that state of affairs could have obtained; and if a state of affairs could have obtained, it is possible; from which it follows that it is possible that there be a round square. More generally, Descartes' view seems to lead to the conclusion that there is no distinction between the impossible and the possible.

One graphic way of bringing out the worry is by considering the impossible state of affairs that God never existed. Could it really be the case that God can bring it about that it was never the case that God existed?

So we might turn to Aquinas, whose text suggests the following definition: **omnipotence is the ability to bring about any possible state of affairs.** 

But we've already encountered some problems with this in connection with the free will defense. As Mackie pointed out, it is possible that every free agent always choose the morally best action; but it does not seem (at least if the free will defense is any good) to follow from this that God can make it that the case that they do so freely choose. So it seems that if Aquinas is right about what omnipotence involves, then even God is not omnipotent.

One might then move to an even more restricted view of omnipotence, along the following lines: a being X is omnipotent if and only if, for any state of affairs that it is possible for X to bring about, X can bring that state of affairs about.

But one might reasonably worry that this is too weak. Consider, for example, McEar, a mysterious being who is essentially such that he is able to do only one thing: scratch his ear. Then, for any action other than scratching his ear, it is impossible for McEar to perform that action. Hence the only state of affairs which is such that it is possible that McEar brings it about is the state of affairs of McEar's being scratched. And McEar can bring this state of affairs about. Hence it follows from the above definition that McEar is omnipotent - which seems clearly false.

This is only a good counterexample if it is possible for there to be a creature such as McEar. Is this possible? What exactly would this involve?

Omnipotence is only one of the attributes of God which has been thought to lead to paradox; another is omniscience.

Omniscience seems, at first glance, easy to define: for a being to be omniscient is for that being to know all the truths. This seems to imply that there is something - 'all the truths' - which such an omniscient being could know. But there is a plausible argument that there can be no such collection.

To show this, the first step is to understand the mathematical result called **Cantor's theorem**, after the mathematician Georg Cantor.

Think of a **set** as a certain collection of things, which may be either finite or infinite. A **subset** of a set S is some collection of the things in S - which may include all of those things, or just some of them. The **power set of S** is the set of all of its subsets.

So consider, for example, the small set

{1, 2}

The subsets of this set would be:

 $\{1\}, \{2\}, and \{1, 2\}$ 

So its power set would be the set with these three sets as members, namely

 $\{\,\{1\},\,\,\{2\}\,,\,\{1,\,2\}\,\}$ 

The **cardinality** of a set is the number of things in that set. Cantor's theorem states that the cardinality of the power set of S is always greater than the cardinality of S.

The proof of Cantor's theorem can be set forth in an intuitive way as follows.



The **cardinality** of a set is the number of things in that set. Cantor's theorem states that the cardinality of the power set of S is always greater than the cardinality of S.

The proof of Cantor's theorem can be set forth in an intuitive way as follows.

Consider an arbitrary set S, and its power set, which we can call P. Let's try to show first that S and P do not have the same cardinality. We show this by a *reductio* argument.

So let's assume for *reductio* that S and P have the same cardinality. Then there is a one-to-one pairing between the members of S and of P.

Since P is a set of sets, every member of S will be paired with a set. Now suppose we ask a question of each member of S: "Are you a member of the set with which you are paired?" If the answer is "Yes" we will say that that member of S is **self-paired**. If the answer is "No", we will say that it is **non-self-paired**.

Now consider the set of non-self-paired members of S - call this new set N. N will be a subset of S, and hence will be a member of P - P being the set of all of the subsets of S. So, N will be one of the things paired with members of S, since we're supposing that every member of P is paired with a member of S.

Now let's think about the member of S which happens to be paired with N; let's call the member paired with N, "Mr. X." Let's ask: is Mr. X self-paired, or non-self-paired? (I.e., is Mr. X a member of the set with which he is paired, or not?)

Suppose that Mr. X is self-paired. Then he must be a member of the set N with which he is paired. But this can't be, since N is the set of all the non-self-paired members of S.

So suppose instead that Mr. X is non-self-paired. Then he must be a member of N, since N is the set of all the non-self-paired members of S. But then Mr. X is a member of the set with which he is paired - which makes him self-paired, which means that he can't be non-self-paired.



So if Mr. X is self-paired, then he is non-selfpaired, and if he is non-self-paired, then he is self-paired - which is a contradiction. So there can be no Mr. X. But if there were a one-to-one pairing between S and P, there would have to be a Mr. X; hence there can be no such one-to-one pairing, and S and P must differ in cardinality.

How would you get from this result to the conclusion that the cardinality of P is not just different than that of S, but greater than it?

The **cardinality** of a set is the number of things in that set. Cantor's theorem states that the cardinality of the power set of S is always greater than the cardinality of S.

Let's return to the topic of omniscience. Suppose that what it means for God to be omniscient is for there to be a set of all the truths, and for God to know each one.

Let's call the set of all the truths T. Now consider the power set of T - the set of all the subsets of the set of all the truths. It seems that corresponding to each of these subsets - corresponding to each subset of all the truths - will be at least one truth. For example, if T<sup>\*</sup> is a subset of T, then it will either be true that

The proposition that 2+2=4 is a member of T\*

or that

The proposition that 2+2=4 is not a member of T\*.

But if there is a truth corresponding to each member of the power set of T, there must be as many truths as there are members of the power set of T. But we know from Cantor's theorem that there are more members of the power set of T than there are members of T - from which it follows that there are more truths than there are members of T. But this contradicts our initial supposition that T is the set of all the truths.

Since this argument is general, it seems to show that there can be no such thing as the set of all the truths.

This result is pretty much uncontroversial; what is controversial is the relationship between this result and the possibility of an omniscient being.

In the optional reading, Grim argues that this shows that the fact that there can be no set of all the truths shows that there can be no omniscient being - for if there is no such thing as the set of all the truths, how could some being know **all** the truths?



Since this argument is general, it seems to show that there can be no such thing as the set of all the truths.

This result is pretty much uncontroversial; what is controversial is the relationship between this result and the possibility of an omniscient being.

In the optional reading, Grim argues that this shows that the fact that there can be no set of all the truths shows that there can be no omniscient being - for if there is no such thing as the set of all the truths, how could some being know **all** the truths?

However, this last step of the attempt to show that omniscience is paradoxical can be questioned. One can state the traditional doctrine of omniscience without explicitly talking at a set of all the truths; rather than saying

(1) An omniscient being knows every member of the set of all the truths.

the defender of omniscience might instead say, simply,

(2) An omniscient being knows all the truths.

The question is then whether (2) is just a disguised version of (1) - or, to put the same question another way, whether (1) is a more explicit version of (2). If it is, then it seems that Cantor's theorem shows that know being could know all the truths, since the set of all the truths simply does not exist.

But one might reasonably question whether (2) is just a disguised version of (1), for two reasons.

First, it seems as though we can make universal claims about the true propositions which make sense - for example, "Every true proposition is non-false." If this makes sense, why not (2)?

Second, the argument against the intelligibility of omniscience seems itself paradoxical. How would the conclusion of that argument be stated?

However, this last step of the attempt to show that omniscience is paradoxical can be questioned. One can state the traditional doctrine of omniscience without explicitly talking at a set of all the truths; rather than saying

(1) An omniscient being knows every member of the set of all the truths.

the defender of omniscience might instead say, simply,

(2) An omniscient being knows all the truths.

The question is then whether (2) is just a disguised version of (1) - or, to put the same question another way, whether (1) is a more explicit version of (2). If it is, then it seems that Cantor's theorem shows that know being could know all the truths, since the set of all the truths simply does not exist.

But one might reasonably question whether (2) is just a disguised version of (1), for two reasons.

First, it seems as though we can make universal claims about the true propositions which make sense - for example, "Every true proposition is non-false." If this makes sense, why not (2)?

Second, the argument against the intelligibility of omniscience seems itself paradoxical. How would the conclusion of that argument be stated?

One might suggest: "There is no set of all the truths." But this is obviously problematic, since it uses exactly the phrase - "all the truths" - which the opponent of omniscience is claiming to be unintelligible. Hence it seems that the proponent of this sort of paradox of omniscience is in the awkward position of being unable to state the conclusion of his argument without, at the same time, contradicting it.

There are other ways to raise logical problems for the possibility of an omniscient being which involve two paradoxes which we'll be discussing later in the course - the Knower and the Liar - but we can defer discussion of those until then.