Zeno’s paradoxes

Our topic today will be a group of the oldest, and most historically important, paradoxes ever set forth: the paradoxes of motion credited to Zeno of Elea.

These paradoxes can be thought of as one of the earliest examples of a type of argument which has been quite common in the history of philosophy: an argument which, if successful, shows that some part of our ordinary picture of the world leads to contradiction. Zeno's idea was that a very basic part of our world-view - the view that things move - leads to contradiction.

You might wonder: how could anyone doubt that things move?

The idea of a thing moving is, to a first approximation, the idea of a certain physical thing - something which takes up space - occupying different bits of space at different times. One might think that nothing moves if one thinks that the physical world - the world of things which are extended in space - is illusory. This view is often called **idealism**.

Zeno (at least on the view handed down to us) had four central arguments against the reality of motion. These four paradoxes are:

The Racetrack

The Achilles

The Stadium

The Arrow

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To understand the reason for the grouping, we have to introduce the idea of a **continuous series**. For our purposes (though this is a simplification), a continuous series is one in which between every two members of the series, there is another member of the series.

Can you think of examples of continuous series (or continua) in this sense?

Our question is: **are space and time continuous?** If they are, then between any two points in space there is a third. Or, to put the point another way, for any length, there is such a thing as half of that length. Applied to time, the idea would be that for any amount of time, there is such a thing as half that time.

If space and time are **not** continuous, then we say that they are **discrete**. If space is discrete, then there are lengths which are not divisible; or, to put the point another way, there are points which have no point between them. If time is discrete, then there are indivisible instants; or, to put the point another way, there are pairs of times which are such that there is no time in between them.

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Space and time are discrete

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It is useful to begin with the most well-known of Zeno's paradoxes: the Achilles.

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We assume two things about Achilles and the Tortoise.

First, we assume that Achilles always takes some amount of time to cover a given distance.

Second, we assume that the Tortoise, even though slow, is quite persistent; in particular, the Tortoise is in constant motion, so that the Tortoise covers some distance in every interval of time, no matter how small that interval of time.

Remember that we are assuming that space and time are infinitely divisible; so the amount of distance covered by the Tortoise in very small amounts of time can be arbitrarily small.

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Achilles eventually makes it to the point where the Tortoise started the race; but of course it takes him some finite amount of time to do so. Let's call the this amount of time **t1**.

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Of course, Achilles has not given up; he sets off to make it to the point where the Tortoise is at the end of **t1**. But this journey takes him a finite amount of time; let's call this interval of time **t2**.

Given the Tortoise's persistence - and in particular the fact that he moves some distance in every time interval - we know that he has also gone some distance in **t2**.

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You get the idea. We could repeat this process indefinitely many times, and at the end of each time interval we considered, the Tortoise would still be ahead of Achilles. Hence it seems that Achilles can **never** catch the Tortoise. But we know that that is absurd.

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Whereas the Achilles attempts to show that nothing can ever catch anything else from behind (so long as the former is moving at a finite speed and the latter never stops moving), the Racetrack attempts to show directly that it is impossible for anything to move any distance at all.

The idea behind the argument can be laid out informally as follows:

> Imagine that you are trying to move from point A to point B. Suppose C is the midpoint of the distance from A to B. It seems that you have to first get from A to C, before you can get from A to B. Now suppose that D is the midpoint between A and C; just as above, it seems that you have to first get from A to D before you can get from A to C. Since space is infinitely divisible, this process can be continued indefinitely. So it seems that you need to complete an infinite series of journeys before you can travel any distance - even a very short one!

We can lay this out more carefully as an argument for the conclusion that it is impossible to move any finite distance in a finite time as follows:

1. Any distance is divisible into infinitely many smaller distances.
2. To move from a point x to a point y, one has to move through all the distances into which the distance from x to y is divisible.
3. To move from one point to another in a finite time, one has to traverse infinitely many distances in a finite time. (1,2)
4. It is impossible to traverse infinitely many distances in a finite time.

C. It is impossible to move from one point to another in a finite time. (3,4)

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The premises all seem plausible, the logic appears impeccable, but the conclusion is clearly false - what's going on here?

It is hard to reject premises 1 or 2, given our assumption that space and time are continuous. So attention focuses on premise 4: the assumption that it is impossible to traverse infinitely many distances in a finite time.

Why does premise 4 seem plausible? An initial thought is that premise 4 seems plausible because anyone who travels infinitely many finite distances will have to travel an infinite distance; and no one (at least, no one traveling at a finite speed) can do this in a finite time.

But this argument is not convincing. Why not?

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Let's suppose we grant that one can travel infinitely many distances (each of which has some finite length) without traveling an infinite distance. Given this, is there any reason to think that one **can't** travel infinitely many distances in a finite time?

One might try to show that there is something incoherent in the idea that infinitely many events of a certain sort could take place in a finite time. This is the target of the example of "Thomson's lamp."

Thomson's lamp

A lamp is turned on and off an infinite number of times between 3:00 and 4:00 one afternoon. The infinite series of events then can be represented as follows:

on, off, on, off, on, off ....

and so on, without end. Because there is no end to the series, every "on" is followed by an "off", and every "off" is followed by an "on."

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Does it follow from our description of the lamp that at the end of the series, the lamp is neither on nor off?

Does it make sense to ask whether the lamp is on or off at **the first moment** after the end of the series?

Does it make sense to ask whether the lamp is on or off at 4:01, given the stipulation that, after the series, it is never turned on or off?

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The fate of the Racetrack seems to depend on whether some argument for premise 4 of the sort exemplified by Thomson's lamp can succeed.

How could the falsity of premise 4 be used to solve the Achilles?

If premise 4 is true, then it looks like the Racetrack is a pretty strong argument against the possibility of motion **given the supposition that space and time are continuous**. So let's turn to the other possibility: the possibility that space and time are discrete.

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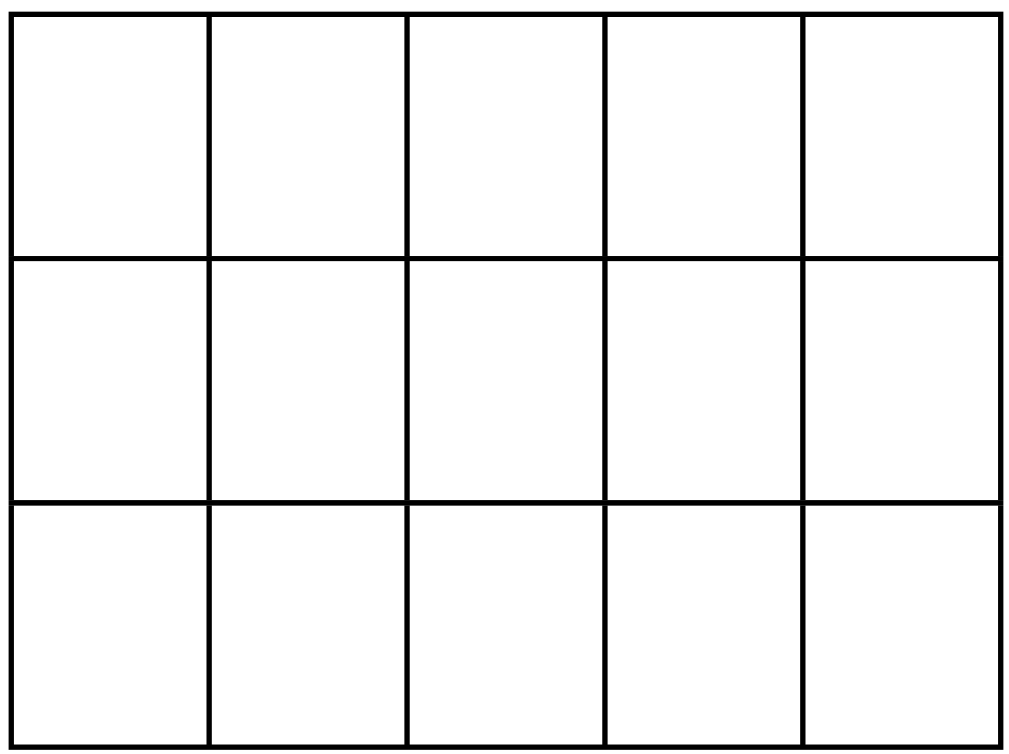
Let’s begin with the Stadium - an argument which Sainsbury does not discuss.

We are now assuming that space and time are discrete, which means that there can be points in space which are genuinely adjacent, in the sense that there are no points in between them. Suppose that the following is a grid of such adjacent points.

Now suppose that we occupy these points with certain particles, as follows:



Now suppose that we occupy these points with certain particles, as follows:

	A1	A2	A3	
	B1	B2	B3	
	C1	C2	C3	

Let's call this Time 1.

Now suppose that the A-particles are all about to move one space to the left, and the C-particles are about to move one space to the right.

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A1	A2	A3		
	B1	B2	B3	
		C1	C2	C3

Let's call the time after this movement is complete Time 2.

We are supposing that space and time are discrete, so we can assume that Time 1 and Time 2 are adjacent times, in the sense that there is no time between the two.

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Time 1

	A1	A2	A3	
	B1	B2	B3	
	C1	C2	C3	

Time 2

A1	A2	A3		
	B1	B2	B3	
		C1	C2	C3

Look at, for example, C2 and A3. At Time 1, C2 is to the left of A3. At Time 2, C2 is to the right of A3. **But the two never passed each other**. After all, they did not pass each other at Time 1, and did not pass each other at Time 2, **and there was no time in between**.

But this seems impossible. It seems impossible for objects to switch left-right orientations without at some point being “even” with each other. But if this really is impossible, then it seems to follow that motion is impossible if space and time are discrete.

Is this a convincing argument?

Zeno's final paradox is called "The Arrow."

Consider an arrow shot from a bow, and imagine that space and time are discrete.

Consider an indivisible moment in time. Does the arrow move **during** that instant? It seems that it cannot since, if it did, the instant would be divisible.

Can it move **between** instants? No, because there are no times between instants.

But if it cannot move during instants, and cannot move between them, it cannot move. So motion is impossible.

This argument can be laid out as follows:

1. At any one instant, an arrow does not move.
2. Nothing happens between one instant and the next.
3. The arrow does not move between instants. (2)

C. The arrow does not move. (1,3)

Is this argument valid? Could things move, even if they do not move either at individual instants or between them?