

Analyticity and reference determiners

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1. THE LANGUAGE MYTH

Russell introduces her definition of analyticity via a criticism of the *language myth*, a false view of language characterized by its failure to distinguish between three “meaning properties” of an expression which correspond to the following platitudes:

- (1) To understand an expression is to know what it means.
- (2) The meaning of an expression in a sentence contributes to what the sentence as a whole says.
- (3) Which object(s) an expression applies to is determined by what it means.

Corresponding to the these platitudes are three different types of meaning:

- **character:** the thing speakers must know (perhaps tacitly) to count as understanding an expression
- **content:** what the word contributes to what a sentence containing it says (the proposition it expresses)
- **reference Determiner:** a condition which an object must meet in order to be the referent of, or fall in the extension of, an expression

Arguments that these come apart: Kripke on names shows that reference determiner \neq content and character; Kaplan on indexicals shows that content \neq character.

Once we have distinguished these properties, we can ask which of these is relevant for defining analyticity.

Why it can't be content: names and other devices of direct reference.

Why it can't be character: (i) arguably, to understand a name, you need only know what it refers to, which again would make "Hesperus is Phosphorus" analytic; and (ii) we want some sentences, like "Stick S is one meter long" to come out analytic.

Russell's choice is to define analyticity in terms of the reference determiners of the expressions in the sentence. To get a feel for how she's thinking about reference determiners, consider what she says about "Hesperus":

I will assume that names are directly referential. One gives a name a meaning, and hence introduces it to a language, by giving it a referent. This can be done using a description (e.g. *Let 'Hesperus' refer to the evening star.*) The referent of the name is then whatever single object falls under this description in the context of introduction, so long as there is one, and if there is not then the expression is meaningless. The name will refer to that object regardless of the agent, time or place specified in the context of utterance (in this names are unlike indexicals) and regardless of the context of evaluation (names are rigid designators). I will assume that the name *Hesperus* was introduced when someone pointed to a bright speck near the horizon one evening and said: *Let's call that bright speck 'Hesperus'.* *Hesperus* thus refers to whatever (if anything) the baptiser demonstrated whilst saying this in the context of introduction. The story for *Phosphorus* is similar, except that the baptiser was pointing at a bright speck in the sky one *morning.*

This shows how names are “sensitive to context of introduction.” But not all expressions are like this. Examples: logical constants, color words.

2. THE DEFINITION OF ANALYTICITY

Here is her first attempt at a definition of analyticity in terms of reference determination:

Definition 6 (Truth in Virtue of Meaning (modal definition)) *A sentence S is true in virtue of meaning just in case for all pairs of context of introduction and context of utterance, the proposition expressed by S with respect to those contexts is true in the context of evaluation.*

One worry you might have here: if we take any sentences which contain only expressions which are not sensitive to context of introduction, and are not sensitive to context of utterance, and are necessary, those will automatically be analytic. Why this seems bad: mathematical claims; color incompatibilities.

Russell is aware of this worry and, later (in part of the book that I did not assign), gives a different, improved definition of analyticity.

Though her account is general, let's focus on a monadic predication of the form

n is F

Russell's view is that the sentence will be analytic iff the reference determiner for $\ulcorner n \urcorner$ is *contained in* the reference determiner for $\ulcorner F \urcorner$. Similarly, for a simple identity sentence $\ulcorner n$ is $m \urcorner$ the sentence will be analytic iff the reference determiner for $\ulcorner n \urcorner$ is contained in the reference determiner for $\ulcorner m \urcorner$. This raises the question: what is it for a reference determiner to be contained in another?

Russell gives the following necessary condition on containment:

Whatever else is true of the containment relation on reference determiners, it ought to satisfy the following principle:

Containment Principle

If the reference determiner for an expression E contains the reference determiner for an expression F, then for all x , if x satisfies E with respect to an ordered pair $\langle c_i, c_u \rangle$, where c_i is a context of introduction and c_u a context of utterance, then x satisfies F with respect to $\langle c_i, c_u \rangle$.

More loosely: where A and B are reference determiners, if A contains B, then B is satisfied by any object that satisfies A. For example, since the reference determiner for *bachelor* contains that of *is a man*, anything which satisfies *bachelor* also satisfies *is a man*. Similarly, since

Russell also says that identity of reference determiner is a special case of containment — which indicates that identity of reference determiner is a *sufficient* condition for containment.

But this does not, so far, give us necessary and sufficient conditions for containment. Let's think about how we might do this.

3. DEFINING CONTAINMENT

To do this it will be useful to introduce the following way of thinking about reference determiners:

Reference determiners for expressions are functions from pairs of a context of introduction and a context of utterance to a property; an object will then be in the reference of — i.e., satisfy — the expression iff it instantiates the relevant property which is the value of the reference determiner for the relevant introduction/utterance pair.

Given this, there are a couple of different ways in which we might define containment. The simplest, and weakest, is just to let Russell's necessary condition on containment — expressed in the containment principle — also be a sufficient condition for containment. This might be expressed as follows:

EXTENSIONAL CONTAINMENT

If R1 is the reference determiner for e1, and R2 is the reference determiner for e2, then:

The reference determiner for e1 contains the reference determiner for e2 iff for every introduction/context pair $\langle i, u \rangle$ and every object o, if o instantiates R1($\langle i, u \rangle$) (at the world of $\langle i, u \rangle$) then o instantiates R2($\langle i, u \rangle$) (at the world of $\langle i, u \rangle$).

There are a few reasons why, I think, Russell does not want to define containment as extensional containment. One is that it will make certain putative 'substantive necessities'— like "God exists" (or "God does not exist", depending on your views) — analytic. Another is that, given that the reference determiners for numerals and other mathematical expressions are constant functions, this trivializes the claim that truths of arithmetic are analytic. Same with color incompatibilities.

So how could we formulate a stronger condition? One promising idea begins with the thought that properties, like propositions, are structured. Then, given that we are thinking of reference determiners as functions from introduction/utterance pairs to properties, we might define containment as follows:

CONSTITUENT CONTAINMENT

If R1 is the reference determiner for e1, and R2 is the reference determiner for e2, then:

The reference determiner for e1 contains the reference determiner for e2 iff for every introduction/context pair $\langle i, u \rangle$,

$\forall F \forall G ((F=R1(\langle i, u \rangle) \ \& \ G=R2(\langle i, u \rangle)) \rightarrow$ (i) G is a constituent of F & (ii) for every object o, if o instantiates F, then o instantiates G (at the world of $\langle i, u \rangle$)

Each property will count as a constituent of itself, in order to secure the result that identity is sufficient for containment.

Why include clause (ii)? The reason why we need clause (ii) is that not all ways of being a constituent will be sufficient for containment. For example, F is a constituent of the complex disjunctive property (F or G) — but we don't want, for example, the reference

determiner for ‘odd or even’ to contain the reference determiner for ‘odd’, on pain of making ‘Every number which is odd or even is odd’ come out analytic.

Constituent containment is a stronger condition than extensional containment. We’ve already seen Russell’s reasons for thinking that extensional containment is a bit too weak — it really does seem odd for “God exists” or “God does not exist” to come out analytic.

4. SOME REMAINING QUESTIONS

4.1. *Reference determiners: content or character?*

Here is an intuitive problem with the idea of a reference determiner, as developed so far. Suppose that one evening someone introduced the name ‘Hesperus’ by saying (or thinking to himself)

Hesperus is *that* (pointing at the brightest object visible in the evening sky).

Given that this is the way that ‘Hesperus’ was introduced, how should we think about its reference determiner?

It is very natural to think that a name’s reference determiner should have something to do with the sentence uttered (or the thought thought) to introduce the name, and in particular (in the present case) that it should be closely related to the words

is *that* (pointing at the brightest object visible in the evening sky).

A natural first thought is that the reference determiner should be recoverable from the *content* of this predicate in the context of utterance which, assuming a direct reference view of demonstratives, will be the property corresponding to the open sentence

$x = o$.

relative to an assignment of Venus to the free variable ‘ o ’. Ignoring the possibility of sensitivity to the context of utterance for now, the idea would then be that we should look at the context of introduction, and ask: which thing has this property? That will then be the reference of ‘Hesperus’ relative to that context of introduction.

But it is pretty clear that this is not what we want. After all, we want names to be sensitive to the context of introduction, and if this is the reference determiner for “Hesperus”, it won’t be — it will single out Venus relative to every context of introduction in which Venus exists, and nothing otherwise. One way to see the problems which would result from this would be to imagine that “Phosphorus” was similarly introduced using a demonstrative, except when looking at the morning sky — this would give “Phosphorus” the same reference determiner as “Hesperus”, and hence, given that identity is sufficient for containment, would make “Hesperus is Phosphorus” analytic. Similar problems would

result if we imagined the names as introduced by slightly more complex demonstrative phrases, like “that planet” or “that bright planet.”

Russell never explicitly considers this problem; and never explicitly gives us a recipe for obtaining the reference determiner for a name from the way in which it was introduced. Here’s a suggestion, which fits most of the examples of reference determiners that she gives in the book. Even if the reference determiner should be recoverable from something about the words used to introduce the name, and in particular the predicate

is *that* (pointing at the brightest object visible in the evening sky)

maybe what matters is not the content of the predicate, but rather it’s *character* — in Kaplan’s sense, of a function from contexts to contents. Here we can (following Kaplan’s “Fregean theory of demonstrations”) think of the character as the description associated with the demonstration which accompanies the utterance of the demonstrative, which would make the relevant content not the property corresponding to

$x = o$.

relative to an assignment of Venus to the free variable ‘o’, but rather the relation corresponding to the open sentence

x is the object in the evening sky the speaker of ci is demonstrating at the time of ci

4.2. *How specific should reference determiners be?*

This still leaves us with the question of which descriptive content should get into the reference determiner. And here, it seems to me, we face an intuitive dilemma.

Let’s imagine that “Hesperus” was introduced as above, so that its reference determiner is that function from contexts of introduction to properties which, given a context of introduction I as argument, has as its value the property which is expressed by the open sentence

x is the object in the evening sky the speaker of ci is demonstrating at the time of ci

relative to an assignment of I as value to ‘ ci ’.

Now suppose that, the next night, a rival astronomer goes out and sees a bright object in the evening sky, which he dubs “Twinkle.” It is hard to see how “Twinkle”’s reference determiner could differ from that just given to “Hesperus”; after all, both were introduced as names for a demonstrated bright object in the evening sky.

But this is problematic. Suppose that the next day our two astronomers get together and consider the sentence

Hesperus is Twinkle.

This sentence seems synthetic — or, at least, if we are sure that ‘Hesperus is Phosphorus’ is synthetic, we should be sure that this sentence is synthetic. But if they are associated with the same reference determiner, then by either definition of containment given above, this sentence comes out analytic.

This is a problem. Let’s consider a few solutions.

4.2.1. Make reference determiners more specific

It would, of course, be possible to avoid the unwanted result by changing our view of the reference determiners associated with the two names for Venus. Suppose, for example, that the introduction of “Hesperus” occurred on April 21, 1845. Then instead of identifying the reference determiner for ‘Hesperus’ with that function from contexts of introduction to properties which, given a context of introduction I as argument, has as its value the property which is expressed by the open sentence

x is the object in the evening sky the speaker of ci is demonstrating at the time of ci

relative to an assignment of I as value to ‘ci’, we could identify it with the function which has as its value

x is the object in the evening sky the speaker of ci is demonstrating on April 21, 1845.

(This would be to partially reverse the move from the content of the demonstrative to the sense of the associated description suggested in the preceding section.) The reference determiner for “Twinkle”, by contrast, would be that function which, given a context I as argument, has value

has as its value

x is the object in the evening sky the speaker of ci is demonstrating on April 22, 1845.

This makes the reference determiners different, and — even using the weaker extensional definition of containment — makes “Hesperus is Twinkle” come out synthetic.

But this leads to other problems, which seem to me just as bad. Consider the predicate

was demonstrated on April 22, 1845

It seems that the reference determiner for this predicate will be contained — on either definition sketched above — by the reference determiner just suggested for “Hesperus.” But this would make

Hesperus was demonstrated on April 22, 1845.

analytic — which seems like a mistake.

(This is an analogue of Kripke’s epistemic argument against reference fixing descriptivism — except that here the claim is not the theory in question overgenerates a priori sentences, but that it overgenerates analytic sentences.)

4.2.2. Revising the definition of analyticity

Fortunately, there is another option which is very much in the spirit of Russell’s account. We can simply give up the idea that identity of reference determiner is sufficient for containment.

To see how this might work, return to our original suggestion of a reference determiner for “Hesperus” and “Twinkle”, according to which each is the function from contexts of introduction to properties which, given a context of introduction I as argument, has as its value the property which is expressed by the open sentence

x is the object in the evening sky the speaker of ci is demonstrating at the time of ci

relative to an assignment of I as value to ‘ci’. The reason why, despite sharing this reference determiner, “Hesperus is Twinkle” seems to be analytic is, I think, that there is no guarantee that the two names were introduced in the same context. Hence, despite sharing a reference determiner, there is no guarantee that they have the same reference — given that their reference determiners determine different references for different contexts of introduction.

This suggests a revision of our definition of containment. Consider first extensional containment. Rather than the definition:

EXTENSIONAL CONTAINMENT

If R1 is the reference determiner for e1, and R2 is the reference determiner for e2, then:

The reference determiner for e1 contains the reference determiner for e2 iff for every introduction/context pair $\langle i, u \rangle$ and every object o, if o instantiates R1($\langle i, u \rangle$) (at the world of $\langle i, u \rangle$) then o instantiates R2($\langle i, u \rangle$) (at the world of $\langle i, u \rangle$).

I suggest that we should go for something like this:

EXTENSIONAL CONTAINMENT*

If R1 is the reference determiner for e1, and R2 is the reference determiner for e2, then:

The reference determiner for e1 contains the reference determiner for e2 iff for every pair of introduction/context pairs $\langle i, u \rangle$, $\langle i^*, u \rangle$, and every object o, if o instantiates R1($\langle i, u \rangle$) (at the relevant world) then o instantiates R2($\langle i^*, u \rangle$) (at the relevant world).

Analogous changes could be made to the definition of constituent containment. The idea is that rather than require that the reference determiners for our pair of terms give the same result for every introduction/utterance pair, we need to require that they give the same result even when they are assigned different introduction/utterance pairs — so long as the ‘utterance’ member of the pair is held fixed. (We have to hold the ‘utterance’ pair fixed if we want ‘I am here’ et. al. to come out analytic, as Russell does.)

Extensional containment* and extensional containment will coincide for terms which are not sensitive to their context of introduction; but for terms which are, like names, they will come apart. And, importantly for our present concerns, this will give us the result that ‘Hesperus is Twinkle’ comes out synthetic.

This gives up Russell’s claim that identity of reference determiner is sufficient for containment; but it seems to give us the results we should, intuitively, want.

4.3. *Cassius Clay & Mohammed Ali*

However, the move from containment to containment*, nice as it is for helping with the case of “Twinkle”, threatens another of the claims about analyticity Russell wants to preserve: the claim that “Cassius Clay is Mohammed Ali” is analytic, if the name “Mohammed Ali” was introduced in a certain way. Here’s what she says:

We’ll stipulate, in order to have a clear example, that the name *Cassius Clay* was introduced when Cassius Clay’s parents baptised him (*Let’s call him* (pointing) ‘*Cassius Clay*.’) The referent of *Mohammed Ali* was introduced in a slightly different way, when Elijah Muhammad, the leader of the Nation of Islam, said *Let’s use ‘Mohammed Ali’ to name Cassius Clay. Mohammed Ali* thus refers to whatever object, if any, *Cassius Clay* refers to.

There are two different interpretations of the reference fixer for “Mohammed Ali” that we might take away from this example. If we focus on Elijah Muhammad’s words, then the natural choice for a reference fixer is the function from a context of introduction ci to the property corresponding to the open sentence

x is Cassius Clay in ci .

But this can’t be quite right, since Russell is elsewhere happy to treat names as devices of direct reference, which would make this property equivalent to

x is o in ci .

relative to an assignment of Ali to ‘ o ’. But then the reference fixer for “Mohammed Ali” would not even extensionally contain — let alone constituent contain or extensionally contain* — the reference fixer for “Cassius Clay.” (This would also be an odd view of the reference determiner, since it would make the name insensitive to context of introduction.)

Instead, I think, Russell has in mind the reference determiner which, for context of introduction ci , has as value

x is named by “Cassius Clay” in ci .

However, if we are thinking in terms of containment* rather than containment, it is not clear that even this will help “Cassius Clay is Mohammed Ali” come out analytic. For there could, obviously, have been (and presumably are) multiple people named “Cassius Clay.” Let’s imagine a world w in which Cassius Clay is named “Cassius Clay” — and someone else — let’s call him “Bob” — is also named “Cassius Clay.” Now imagine that the above metalinguistic condition is indeed the reference determiner for “Mohammed Ali”, and that, as should be consistent with this, this name is introduced in w to stand for Bob.

Now suppose that I am acquainted in w with Bob, whom I know only under the name “Muhammed Ali”, and that in w I know Cassius Clay — the Cassius Clay who was actually a great boxer. I might come to suspect that they are the same person, and utter the sentence “Cassius Clay is Mohammed Ali.” This would be false out of my mouth — *even though I was using the names with the reference determiners they actually have*. But if this can happen, then the sentence is not guaranteed to be true by the reference determiners of its expressions (plus the way they are combined) and hence should *not*, by Russell’s lights, count as analytic.

I’m not sure quite what to say about this case. On the one hand, if Russell had to give up on her claim about the analyticity of “Cassius Clay is Mohammed Ali”, this would not be so bad for her theory — it’s not like it is an uncontroversial case of analyticity in the first place.

But the worry is that there are other cases which are relevantly like this one which really do seem to be analytic. Suppose that “gray” was introduced like this:

Let “gray” stand for whatever color “grey” stands for.

Presumably we should want “Gray is grey” to be analytic; but this is just like the Cassius Clay example. (We can imagine a world where “grey” is ambiguous between, e.g., a name for a color and a name for a shape.)

It looks like we need to add something to the account. One idea would be to in effect complicate the reference determiner for “Mohammed Ali” to require that it refer to the referent of “Cassius Clay”. where the reference determiner for the latter is held fixed — but this won’t work, since we can imagine both uses of “Cassius Clay” as associated with the same reference determiner.

We might say instead that, in the example of w and Bob, when I falsely say “Cassius Clay is Mohammed Ali,” there is a sense in which I am not uttering the *same sentence* as the actual sentence, “Cassius Clay is Mohammed Ali,” which Russell thinks is analytic. What this must mean is that I am not uttering a sentence of the same type; and then what we need is a specification of what the relevant type is. The problem is that none of the meaning properties — content, character, or reference determiner — will give us the result we want. So it is not obvious how to specify the relevant type.

We could get around this by requiring that all the terms in the sentence be evaluated with respect to the same context of introduction/utterance — but this would be to give up containment* and return to containment.

4.4. *Paderewski*

A further problem results from an example from Kripke’s “A Puzzle About Belief” of Paderewski, the stateman/pianist. In these contexts, it seems that “Paderewski is Paderewski” should be synthetic, for just the same reasons as “Hesperus is Phosphorus” is.

A way to accommodate this case: let reference determiners include causal chains leading up to the relevant tokens.

5. EPISTEMOLOGICAL CONSEQUENCES OF TRUTH IN VIRTUE OF REFERENCE DETERMINERS