Modals

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1. SYNTAX OF MODALS

Sometimes, in English, modals seem to combine with sentences to form senences, as in

Necessarily, 2+2=4. It is possible that the number of beer bottles on Notre Dame's campus is odd.

Other times, though, devices for expressing modality seem to occur as parts of verb phrases, as in

John must die. No dog can play chess.

These also seem to be claims about necessity and possibility in some sense.

I'll follow the text in treating these uses of modal expressions in pretty much the same way we treated tense, and thinking of them as undergoing movement, and functioning syntactically like sentence operators. Hence we can think of the logical form of the first sentence above as roughly [$_{\rm S}$ MUST [$_{\rm S}$ John dies]].

2. Semantics for modals

One confusing aspect of modal expressions in English is that they appear to have different interpretations. A good example is the word "must". Consider the following three sentences:

The smallest prime number greater than 10 billion must be odd.

The murder must have been committed by someone who was in Malloy Hall at midnight.

Students must respect, indeed revere, their professors.

Here 'must' seems to have a different meaning in each sentence. These are examples of, respectively, alethic, epistemic, and deontic uses of 'must.' As you might expect, each of these uses must get a different semantic treatment.

2.1. Alethic modals

Alethic uses of modals make claims about how the world could (or could not) be. Alethic modals are naturally interpreted in just the way that we interpreted ' \Box ' and ' \diamondsuit ' in the intensional predicate calculus. So a first step would be to divide alethic modals into devices for expressing possibility ('can', 'might', 'could') and devices for expressing necessity ('must') (keeping in mind that these same words can be used to express something other than alethic modality), and then to treat them with the same semantics suggested before. So we have

 $[\![MUST \ S]\!]^{M, \, w, \, i, \, g} = 1 \text{ iff for every } i^* \in I \text{ and every } w^* \in W, \, [\![S]\!]^{M, \, w^*, \, i, ^* g} = 1$

 $[\![CAN \ S]\!]^{M, \, w, \, i, \, g} = 1 \text{ iff for some } i^* \in I \text{ and some } w^* \in W, \, [\![S]\!]^{M, \, w^*, \, i, ^*g} = 1$

$$\label{eq:can't S]} \begin{split} \llbracket & \texttt{CAN'T S]}_{^{M,\,w,\,i,\,g}} = 1 \ \text{iff} \ \llbracket & \texttt{CAN S}_{^{M,\,w,\,i,\,g}} = 0 \\ & \text{iff for every } i^* \in I \ \text{and every } w^* \in W, \ \llbracket \phi \rrbracket^{^{M,\,w^*,\,i,^*\,g}} = 0 \end{split}$$

Does this capture every alethic use of, for example, 'can't'? How about

Cats can't talk.

Does this really imply that there is no possible worlds where cats talk?

One way to approach this sort of sentence is by thinking of them as a kind of counterfactual conditional — which is another way of expressing claims about alethic modality which we have not yet discussed. Consider the sentence

If cats could talk, they would say profound things.

How might we understand the truth conditions of this sentence in the sort of framework we have been developing?

Using counterfactuals, is there a way to understand the truth conditions of 'Cats can't talk'?

A different sort of example is

Every massive object in the universe must attract every other massive object in the universe.

Could we understand this as a claim about truth in some subset of the possible worlds — perhaps all the worlds that meet a certain condition?

2.2. Epistemic modals

The second use of 'must' above seems to express a claim which has something to do with our knowledge. The claim is not that the murderer is in Malloy Hall in every possible world, but rather, intuitively, something more like the claim that the murderer is in Malloy Hall in every world compatible with what we know about the situation.

This seems to fit nicely with one interesting feature of epistemic modals, which is their apparent context-sensitivity. I might say, at the beginning of the inquiry,

The murderer might have been in Decio at midnight.

and it seems that this might be true. Then, as more information comes in, I might say, truly,

The murderer must have been in Malloy at midnight.

and, at this stage, the sentence about Decio which was true will now be false. This, it seems, of a piece with other context-sensitive expressions. If I say 'Coleman Morse Hall is where I am now' this is true — but the same sentence will be false in a few hours.

There are, however, ways, in which epistemic modals seem to differ from other contextsensitive expressions. Suppose that you have extra information, and overhear my first utterance of 'The murderer might have been in Decio at midnight.' You will be inclined to judge my utterance false; you might say something like 'No, he couldn't have been. The chief just told me that ...' And this can't be explained by saying that out of your mouth the sentence would have been false. This is true, but doesn't explain the data. If you say 'I am hungry' then I don't correct you on the grounds that the same sentence would be false if I uttered it. It's a matter of current controversy how these data are best to be explained.

The above sort of analysis also implies that, in the epistemic use of 'might', for any sentence S which expresses a necessary truth,

It might be the case that not-S.

will be false. Is this problematic?

2.3. Deontic modals

In effect, the suggestion above was to understand epistemic modals by restricting the set of relevant worlds in a certain way. Deontic modals, like

Students must respect, indeed revere, their professors.

involve the notions of permission and obligation. Can you think how these might, similarly, be understood using restrictions on the set of possible worlds?

3. MODALS, QUANTIFIERS, AND SCOPE AMBIGUITIES

Consider the sentence

The greatest mathematician is necessarily a mathematician.

Our discussion of modals so far suggests that, in a sentence like this with a quantifier phrase and a modal which will undergo movement, we should get an ambiguity in this sentence depending on whether the modal, or the quantifier, has wide scope. Do we? What are the two interpretations?

Brief discussion of Quine's objection to the intelligibility of the interpretation on which th e description has wide scope.