What is real?

Heaps, bald things, and tall things

Our topic today is another paradox which has been known since ancient times: the paradox of the heap, also called the sorites paradox ('sorites' is Greek for 'heap'). It calls into question the reality of every category of thing that admits of borderline cases — like bald people, or tall people, or mountains.

The paradox is sometimes attributed to Zeno, but most attribute it to the Greek philosopher Eubulides, who lived in the 4th century B.C. Like Zeno, all of his writings are lost. Our reading today is from the 2nd century Greek physician and philosopher, Galen. Galen, the author of the excellently titled book *That the Best Physician is also a Philosopher*, was probably the most influential physician in Western history.

Galen introduces the paradox by a series of questions which he imagines asking the reader:

> “tell me, do you think that a single grain of wheat is a heap? Thereupon you say: No. Then I say: What do you say about 2 grains? For it is my purpose to ask you questions in succession, and if you do not admit that 2 grains are a heap then I shall ask you about 3 grains. Then I shall proceed to interrogate you further with respect to 4 grains, then 5 and 6 and 7 and 8, and you will assuredly say that none of these makes a heap. ... For a heap ... has quantity and mass of some considerable size.”

So far, this is not especially problematic; we’ve just given an argument for the unsurprising conclusion that 8 grains of sand do not make a heap.

The problem which Galen points out is that he can keep asking questions like this indefinitely and that it will never seem plausible to say that the addition of a single grain of sand has made what was not a heap into a heap:

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> "I know of nothing worse and more absurd than that the being and not-being of a heap is determined by a grain of wheat. And to prevent this absurdity from adhering to you, you will not cease from denying, and will never admit at any time that the sum of this is a heap, even if the number of grains of wheat reaches infinity by the constant and gradual addition of more. And by reason of this denial the heap is proved to be non-existent…"

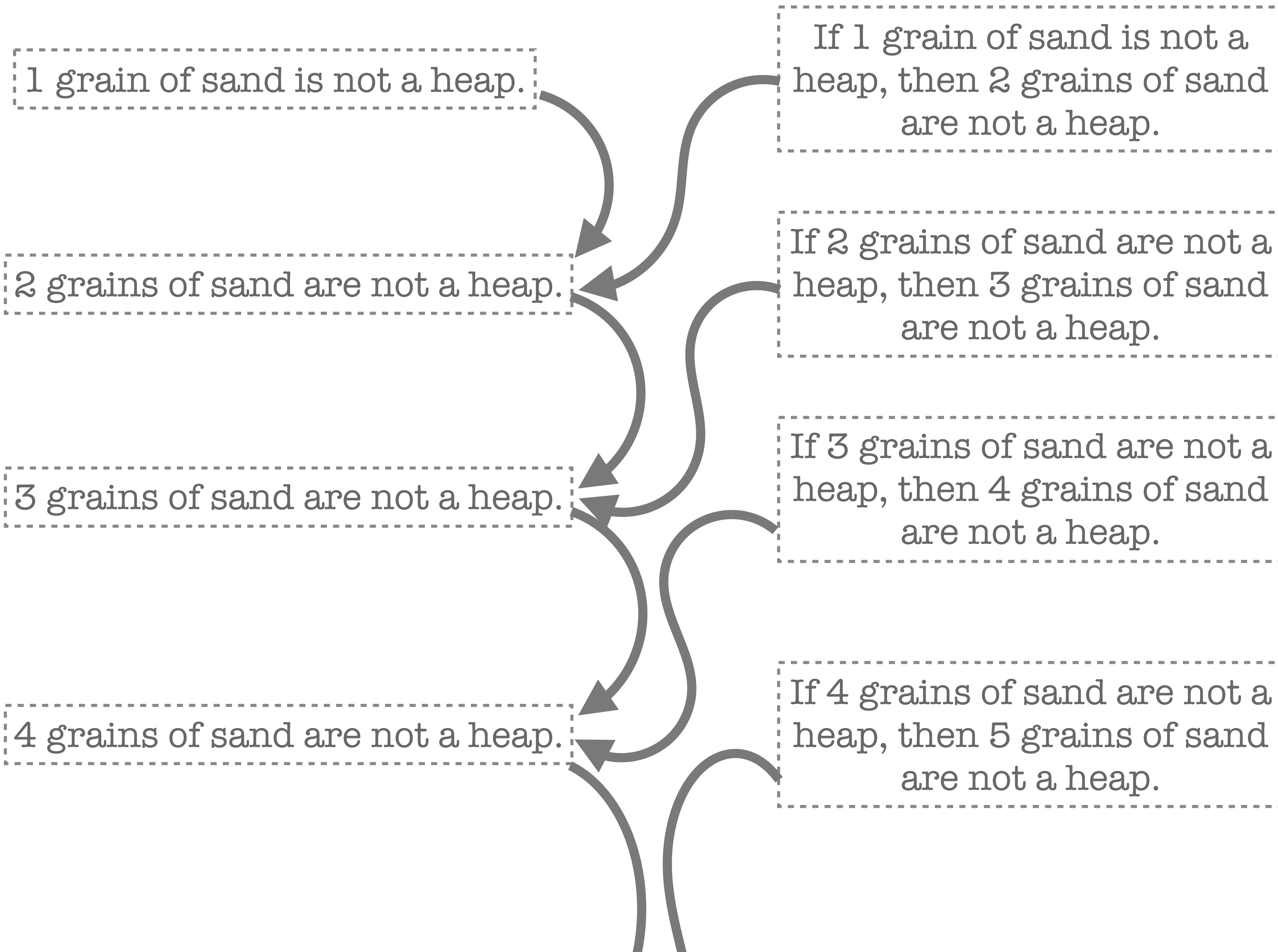
Maybe you don't care much about heaps. But similar lines of argument can be used to show (as Galen noticed) that there are no mountains — which is at least a bit surprising. A similar line of argument can be used to show that no one is tall, or rich, or famous, or …. just about anything.

Arguments of this sort can seem silly. As we'll see, though, they are not easily dismissed. They are also of interest because forms of this argument are often employed outside of the philosophy classroom, especially in public discourse about ethics and politics. One prominent example of this sort of argument is often employed in debates about the permissibility of abortion:

> "It is wrong to kill a fetus one second before it is born. But there is no morally significant difference between a fetus one second before it is born and a fetus two seconds before it is born. So it is wrong to kill a fetus two seconds before it is born. And, more generally, when you look at the development of a fetus from conception to birth, there is never a morally significant difference between the fetus at one time, and the fetus one second before that time. So the above line of reasoning can be repeated indefinitely to show that it is wrong to kill a fetus at conception."

This certainly looks like an application of the same form of reasoning used by Galen. It is thus of some practical importance to figure out what is going on with arguments of this sort. These are sometimes called "slippery slope" arguments (though not everything which is called a "slippery slope" argument is really of the same form as the sorites paradox).

Suppose that we wanted to present Galen’s reasoning as an explicit argument. How could we do it?



1. 1 grain of sand is not a heap.
2. If 1 grain of sand is not a heap, then 2 grains of sand are not a heap.
3. 2 grains of sand are not a heap. (1,2)
4. If 2 grains of sand are not a heap, then 3 grains of sand are not a heap.
5. 3 grains of sand are not a heap. (3,4)
6. If 3 grains of sand are not a heap, then 4 grains of sand are not a heap.
7. 4 grains of sand are not a heap. (5,6)
8. If 4 grains of sand are not a heap, then 5 grains of sand are not a heap.

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1997. 999 grains of sand are not a heap. (1995, 1996)

1998. If 999 grains of sand are not a heap, then 1000 grains of sand are not a heap.

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C. 1000 grains of sand are not a heap. (1997, 1998)

This is one way to represent the form of reasoning implicit in Galen's argument. It is a valid argument, and all of the premises seem plausible — and yet the conclusion is false. (If you don't think it is false, just add a few thousand more premises, and make the number mentioned in the conclusion bigger.)

Words like 'heap' which give rise to arguments like this are called 'vague.'

But it is somewhat unwieldy to use 1998 premise arguments. Fortunately, the argument can be shortened.

1. 1 grain of sand is not a heap.
2. For any number n, if n grains of sand are not a heap, then n+1 grains of sand are not a heap.

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C. 1000 grains of sand are not a heap. (1,2)

This is a form of reasoning often used in mathematical proofs, called ‘mathematical induction.’

This version of the sorites argument also appears to be valid, and also seems to give us a more general conclusion:

C*. No finite number of grains of sand make a heap.

Since this simple argument appears to be valid, we have exactly three choices about how to respond to it:

Reject the 1st premise

Reject the 2nd premise

Accept the conclusion

1. 1 grain of sand is not a heap.
2. For any number n, if n grains of sand are not a heap, then n+1 grains of sand are not a heap.

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C*. No finite number of grains of sand make a heap. (1,2)

Reject the 1st premise

Reject the 2nd premise

Accept the conclusion

It is hard to take the first of these options seriously. Remember that we can recreate this argument any number of ways:

1. A 4 foot tall person is not tall.
2. For any height h, if a person of height h is not tall, then a person of height h + 0.001" is not tall.

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C*. No person of finite height is tall. (1,2)

1. A person with 1¢ is not rich.
2. For any amount of money m, if a person with money m is not rich, then a person with money m + 1¢ is not rich.

———————————————————————-

C*. No person with a finite amount of money is rich. (1,2)

So I suggest that we throw out the first option.

1. 1 grain of sand is not a heap.
2. For any number n, if n grains of sand are not a heap, then n+1 grains of sand are not a heap.

————————————————————————-

C*. No finite number of grains of sand make a heap. (1,2)

Reject the 2nd premise

Accept the conclusion

It is also hard to get one's mind around accepting the conclusion. Accepting the conclusion would seem to be a bit like adopting the Nihilist response to the puzzle of the statue and the clay. One has to say that there are no mountains or heaps, and no tall things or bald things.

Indeed, it is in some respects worse, since it would rule out the existence of things which even the Nihilist about composite objects can accept. Consider the following sorites argument:

1. A temperature of 70° is not painful to ordinary humans.
2. For any n°, if a temperature of n° is not painful to ordinary humans, then a temperature of n+0.0001° is not painful to ordinary humans.

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C*. No finite temperature is painful to ordinary humans. (1,2)

1. 1 grain of sand is not a heap.
2. For any number n, if n grains of sand are not a heap, then n+1 grains of sand are not a heap.

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C*. No finite number of grains of sand make a heap. (1,2)

Reject the 2nd premise

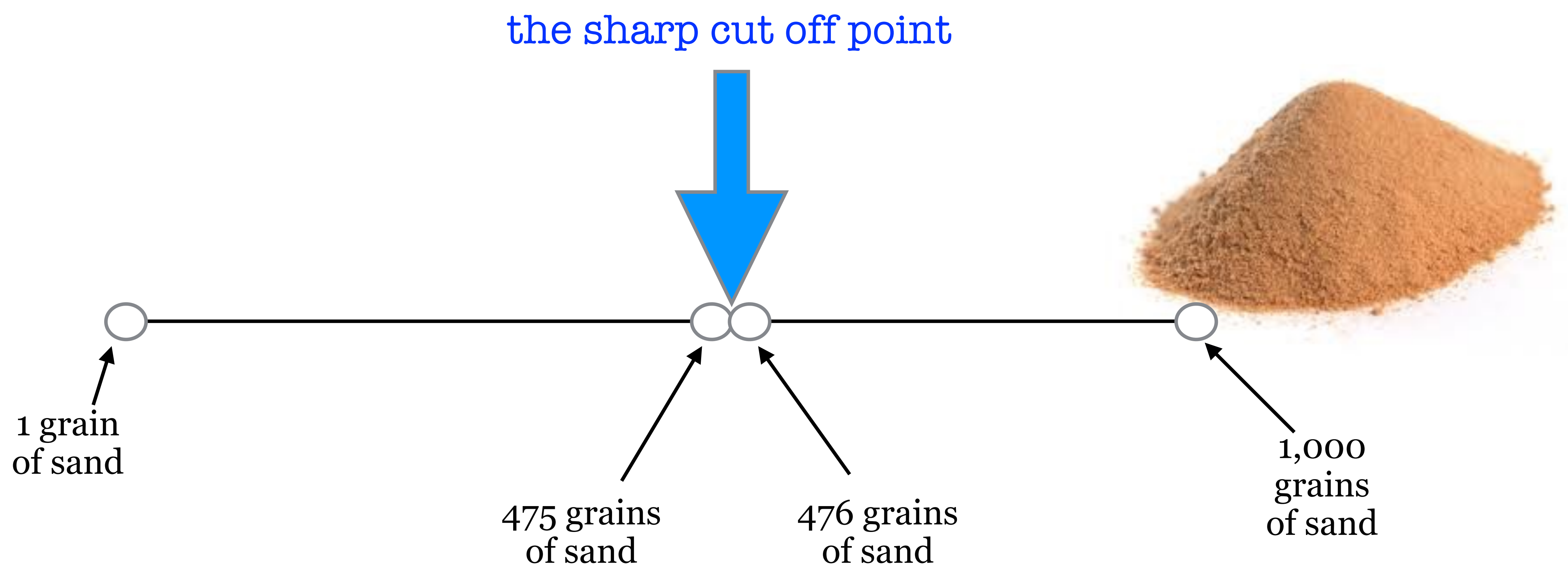
And so attention naturally focuses on the option of rejecting the second premise of the above argument, which is sometimes called the 'sorites premise.' By far the most popular response to the paradox of the heap is to reject the sorites premise.

Notice something important: if you agree with this, then it seems that you already agree that something is at least questionable about the argument about abortion we considered above. After all, if the sorites premise of our heap argument is not true, then why should we believe the corresponding premise of that argument?

Let's now consider two quite different ways in which one might try to reject the sorites premise of the heap argument.

For any number n, if n grains of sand are not a heap, then n+1 grains of sand are not a heap.

The first strategy is the simplest. We can call it the 'sharp cut off point' strategy.



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How exactly does this help with the form of argument Galen gives?

Recall the long, many-premise version of the argument. Just which premise will the sharp cut-off theorist reject?

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949. 475 grains of sand are not a heap.
(5,6)
950. If 475 grains of sand are not a heap, then 476 grains of sand are not a heap.
951. 476 grains of sand are not a heap.
(949, 950)

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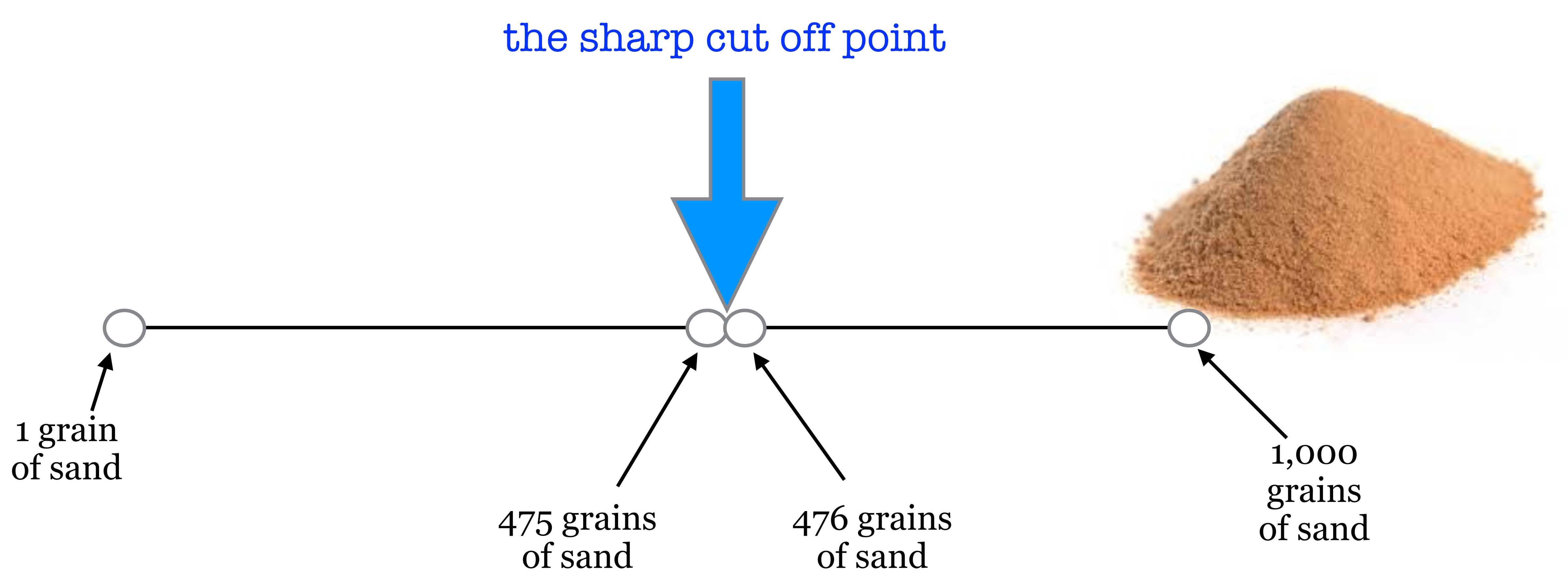
On this view, premise 949 is true. But ‘476 grains of sand are not a heap’ is false. So, on this view, premise 950 is the one which should be rejected.

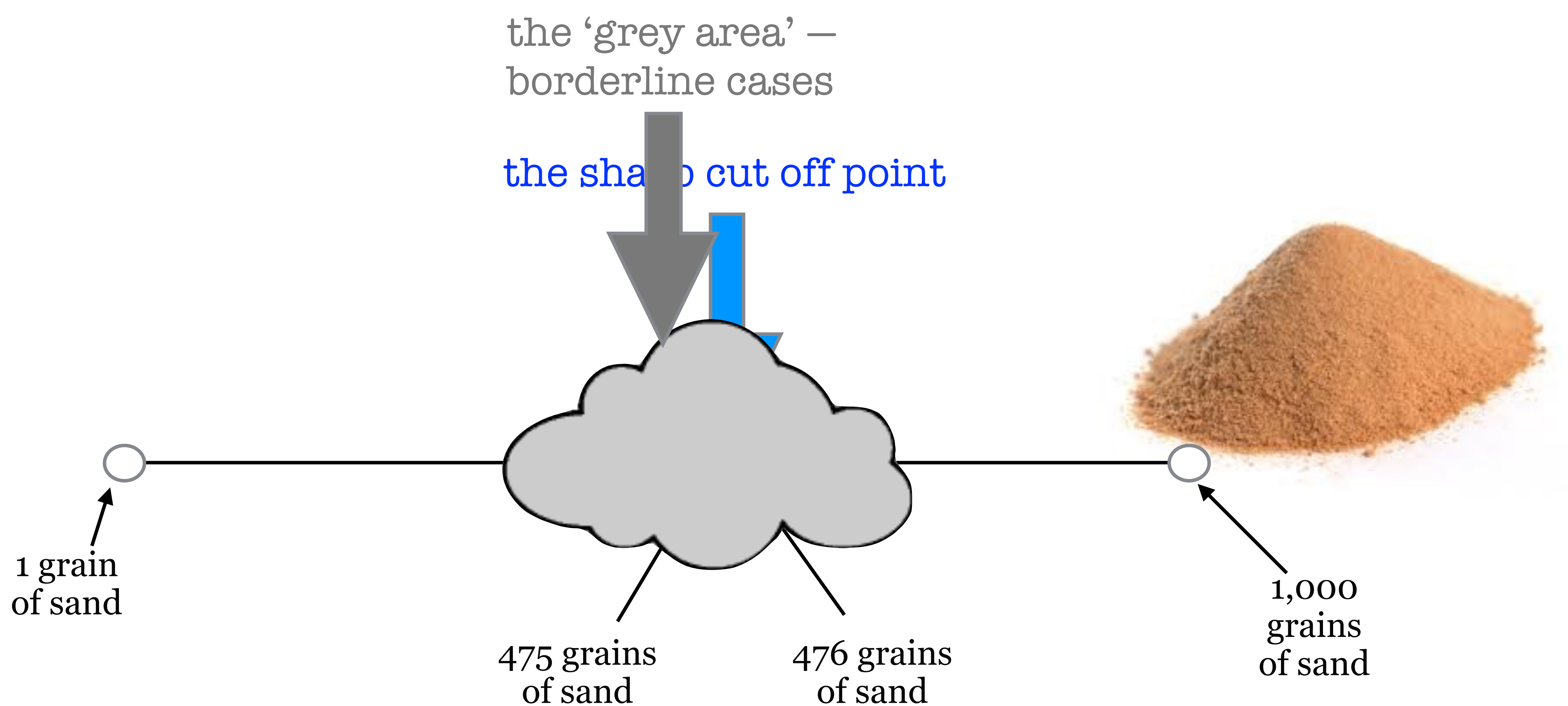


Many people find the sharp cut-off view hard to believe. Remember Galen's words: "I know of nothing worse and more absurd than that the being and not-being of a heap is determined by a grain of wheat."

One reason to doubt the view is that it seems that the sharp cut-off point is in principle unknowable. How could we go about figuring out just how much hair is required to avoid baldness, if we really wanted to?

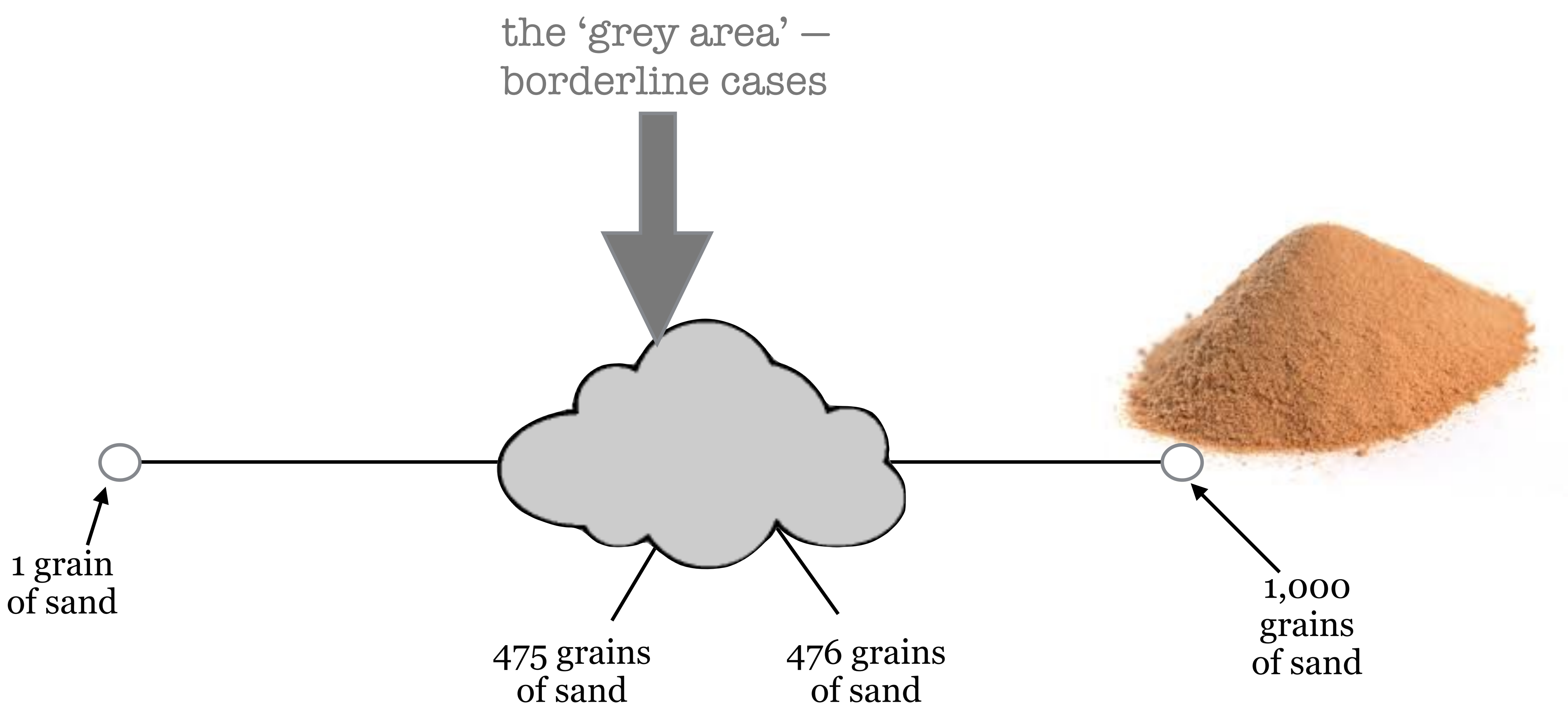
And it also just seems to invest individual grains of sand — or individual hairs in the case of baldness or individual units of height in the case of tallness — with undue importance. Suppose that I have 423 hairs on my head, and have a great fear of going bald. If the sharp cut-off point for baldness is between 422 and 423, should I be especially afraid of losing the next hair?

Many people feel inclined to respond to the sharp cut-off theory in something like the following way: there is no clear, simple, dividing line between the heaps and the non-heaps, or between the tall people and the not-tall people. Rather, there are some clear cases of tall people, and some clear cases of people who are not tall, and then just some borderline, indeterminate cases in between.



Collections of sand with 475 grains are borderline cases, in the grey area. What should we say about the sentence ‘475 grains of sand are a heap’?

We obviously do not want to say that it is true, since then it would be a heap, and hence not a borderline case.



Collections of sand with 475 grains are borderline cases, in the grey area. What should we say about the sentence ‘475 grains of sand are a heap’?

We obviously do not want to say that it is true, since then it would be a heap, and hence not a borderline case.

But we also do not want to say that it is false. For if it is false, then it looks like ‘475 grains of sand are not a heap’ would be true. And then again it would not be a borderline case — it would just be a case of a non-heap, and we’d be back to the sharp cut-off view.

So it looks like if we want a ‘grey area’ view rather than a ‘sharp cut-off’ view, we should say that sentences like ‘475 grains of sand are a heap’, which concern objects in the relevant grey area, are neither true nor false. You might say, instead, that they are ‘indeterminate’ or ‘undefined.’

For this reason this sort of view is sometimes called a ‘truth-value gap’ solution to the sorites paradox.

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949. 475 grains of sand are not a heap.
(5,6)
950. If 475 grains of sand are not a heap, then 476 grains of sand are not a heap.
951. 476 grains of sand are not a heap.
(949, 950)
....
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How, exactly, does this view respond to Galen's argument?

On this view, the following sentences are both neither true nor false:

475 grains of sand are not a heap.
476 grains of sand are not a heap.

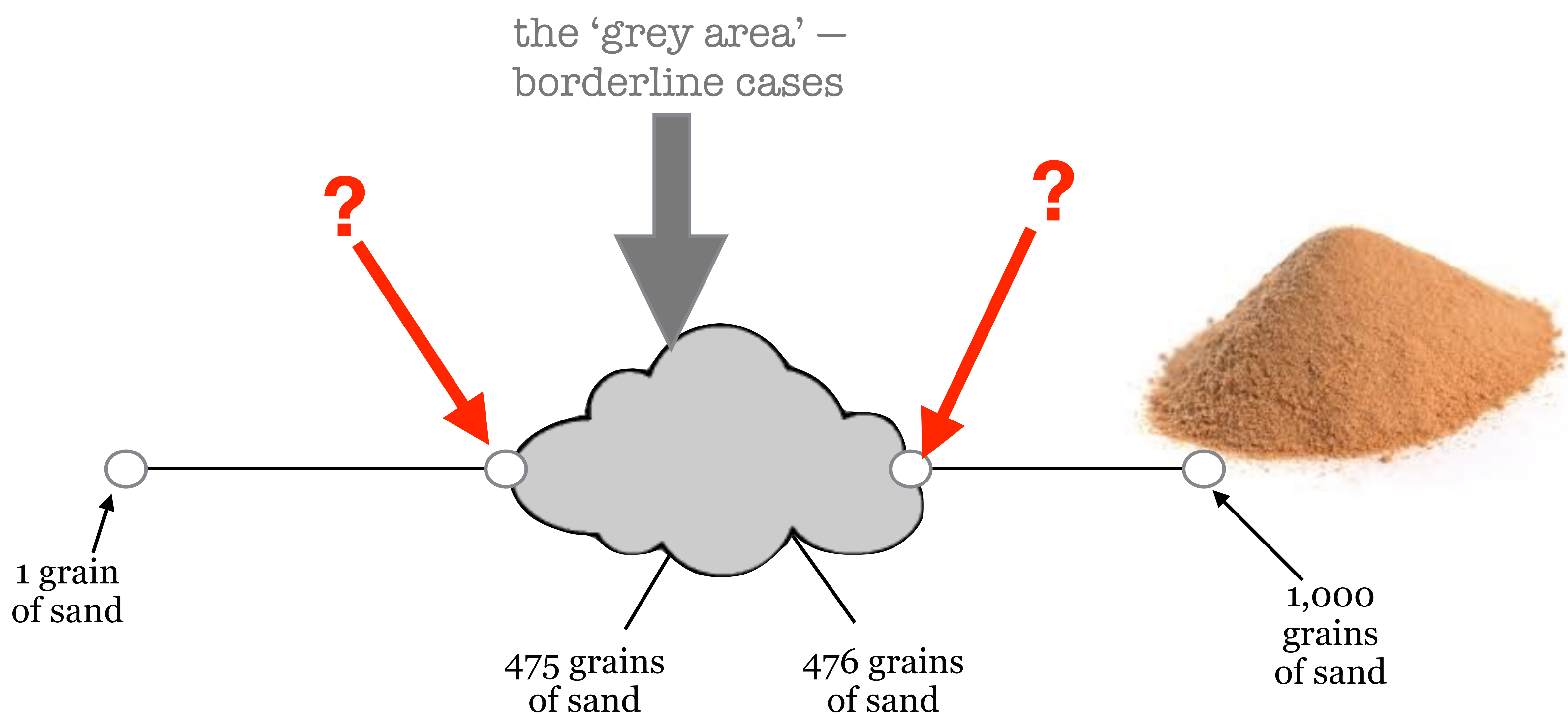
But now look at premise 950. It is an if-then sentence, and both the 'if' part and the 'then' part are neither true nor false. It is thus natural to say that the premise as a whole is neither true nor false. And of course, since the grey area is large, premise 950 won't be the only one like this: our fully-expanded argument will contain lots of premises which are neither true nor false.

And if they are neither true nor false, they are not true. And if they are not true, the argument is not sound. This gives us the response to Galen we want.

So far this looks very plausible. When we use words like ‘tall,’ we don’t really care whether we get a clear verdict for every single object in existence. We use them instead to talk about objects that are clearly tall, and objects that are clearly not tall — we simply leave open whether objects in the grey area are tall, or not.

Unfortunately, this view faces a serious problem, which we can see if we re-examine our diagram.

We have gotten rid of the sharp cut-off point posited earlier; but one might worry that we have simply replaced it with two new sharp-cut off points: one between the clear non-heaps and the grey area, and the other between the grey area, and the clear heaps.



1 grain of sand is not a heap.
2 grains of sand is not a heap.
3 grains of sand is not a heap.
...
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999 grains of sand is not a heap.
1000 grains of sand is not a heap.

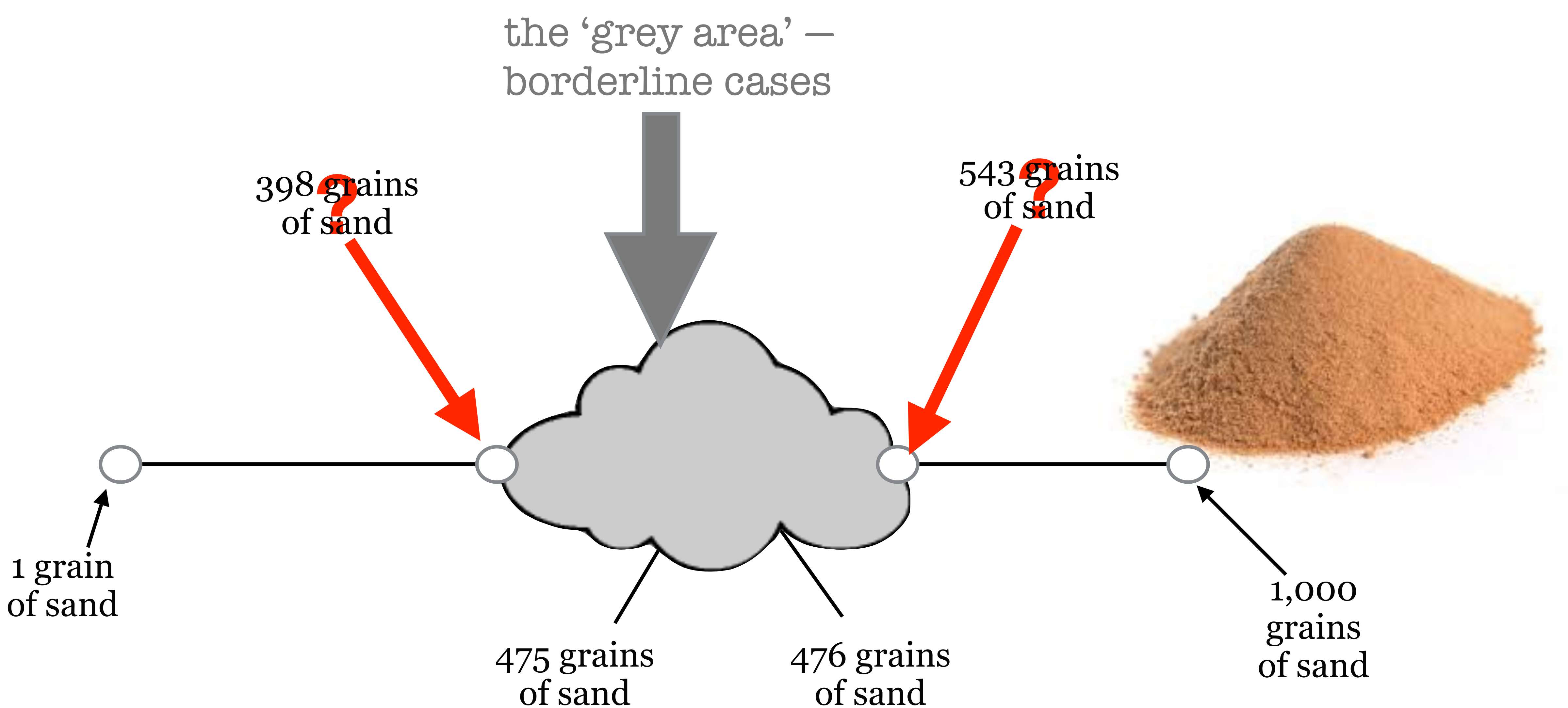
If we look at this list of sentences, the sharp cut-off view says that there is some sentence on the list which is true, and is followed by a sentence which is false. The sharp cut-off point is between the numbers mentioned in those two sentences.

The truth-value gap theorist finds this implausible, says that some sentences on the list are neither true nor false, but instead are undefined, or indeterminate.

But on this view, the first sentences on the list are still true, and the last sentences on the list are still false. So it looks like there must be some true sentence on the list which is followed by an undefined sentence, and some undefined sentence on the list which is followed by a false sentence.

So it looks like our two sharp cut-off points must have precise locations.

And it looks like we can raise the same sorts of worries about these cut-off peoples as we could about the cut-off point posited earlier. It seems in principle impossible to know where they are, and seems to invest individual grains of sand with undue importance.



There is thus a sense in which one might think that truth-value gap approaches don't avoid the problems of sharp cut-off theories, but instead multiply them.

This might incline you to simply accept the sharp cut-off view. Maybe it sounds weird to say that you can go from non-bald to bald by losing a single hair — but maybe this is not so bad.

I want to close by presenting a version of the sorites paradox which, many people think, makes this sort of view hard to sustain. This is a paradox sometimes called the 'phenomenal sorites.'

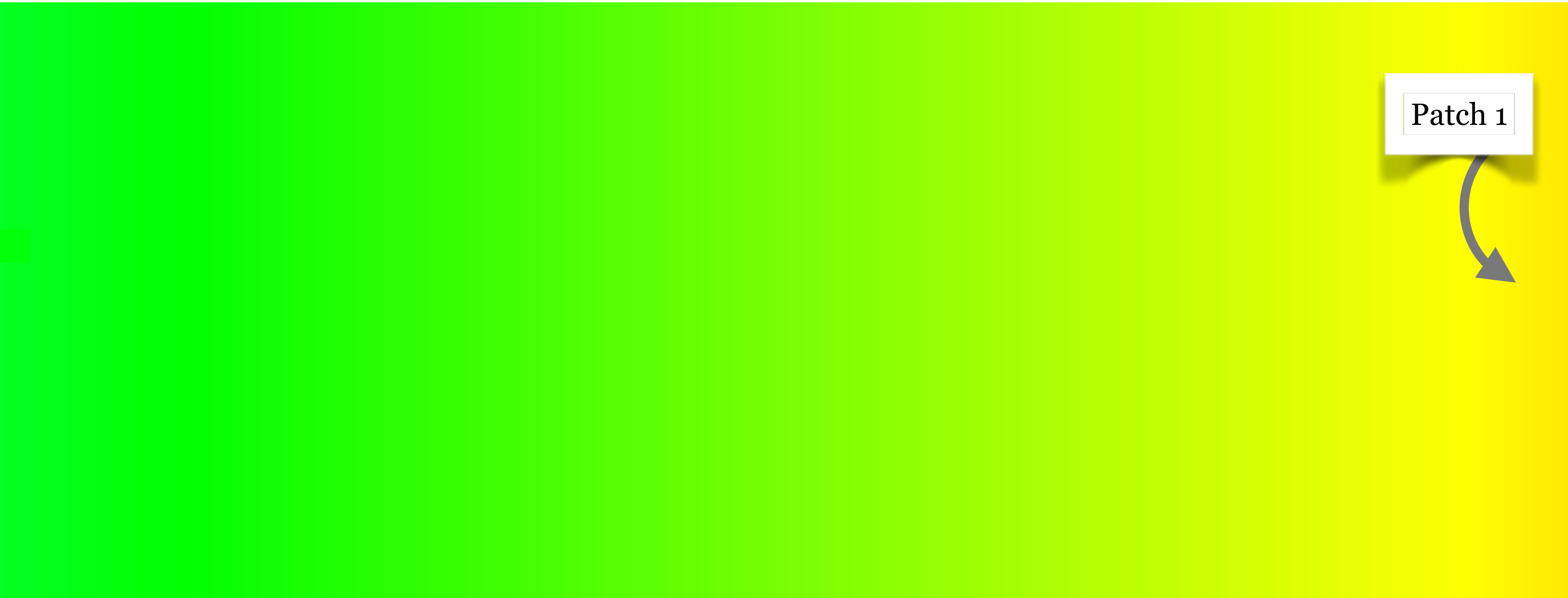
Look at the following color patch:

Is it green?

Call this ‘patch 1.’ I think that we can agree that the sentence ‘Patch 1 is not green’ is true.

But of course Patch 1 is just part of a continuous range of colors which stretch from yellow to green.

We can select a similarly sized patch from the green range. Call this ‘Patch X.’



We can select a similarly sized patch from the green range. Call this 'Patch X.'

Now consider the sentence, 'Patch X is green.' This is surely true.

But here's the problem. If we re-examine our spectrum, it seems like we could get, by very small movements, from Patch 1 to Patch X via series of patches each of which was indiscriminable from the last.

This gives us the following interesting sorites argument:

1. Patch 1 is not green.
2. For any patches x and y, if x is indiscriminable from y, then if x is not green, y is not green either.

C. Patch X is not green. (1,2)

Suppose that we follow the route suggested by the sharp cut-off theorist here. It appears that we must say that there is some pair of color patches which is such that, in one good sense, they look exactly the same — they are indiscriminable from each other — and yet one is green and one is not. Does this make sense?

The paradox of the heap, while in a way quite simple, is one of the most difficult and challenging paradoxes that there is. Much work on it is still being done today — some of it quite technical. But there is no consensus solution. And yet — if there are such things as tall people, bald people, and heaps of sand — as there really seem to be — there must be some solution.