# Intensionality

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## 1 Intensional contexts

We've been developing ways to analyze the semantic values of sentences, relative to a model and assignment, on the basis of the semantic values of their parts, relative to that model and assignment, plus the ways in which those parts are combined. But there are some sentences in English which seem not to be amenable to this sort of analysis.

Consider how we might try to compute:

$[\![\text{Pavarotti was hungry.}]\!]^{M,g}$

$[\![\text{It is possible that Pavarotti is hungry.}]\!]^{M,g}$

$[\![\text{If Pavarotti were hungry, he would be eating.}]\!]^{M,g}$

$[\![\text{Sophia Loren believes that Pavarotti is hungry.}]\!]^{M,g}$

Cases like the ones above are often called 'intensional contexts.' They are also called 'non-extensional contexts' and 'opaque contexts.' What they have in common is that they all involve a location – which you can just think of as a node – in the sentence with the following characteristic: you can remove one term with a given semantic value relative to $M$ and $g$, replace it with another term with exactly the same semantic value relative to $M$ and $g$, and yet change the truth conditions of the sentence.

Our theory so far does not give us the resources to handle sentences of any of these kinds. Our aim is to extend our theory so that it can do this. But to do that we'll need to change a bit the way we are thinking about valuations and semantic values.

## 2 Intensions and extensions

In a way, this involves making something explicit which was implicit in our earlier practice of deriving semantic values relative to a model. The meaning of an expression is not its semantic value; it is something like a rule for deriving a semantic value given some information about the world. So, for example, suppose that $[\![\text{is hungry}]\!]=\{\text{Pavarotti}\}$; Pavarotti is the only hungry person. One can clearly understand 'is hungry' without knowing this; what one knows is rather, very roughly, a rule for determining the semantic value of the term given information about the world – namely, about which people are hungry.

It is natural to think of this 'rule' as a function. But from what to what?

A first try is to say that it is a function from times to semantic values, since what we know when we understand 'is hungry' is not the members of the set of hungry people right now, but rather a rule which tells us what it takes, at any time, to be a member of that set.

But this won't quite work. One way to bring this out is to consider an expression, like 'the teacher of PHIL 43916 at Notre Dame in the fall of 2014', which has the same semantic value relative to every time. Suppose that 'Jeff Speaks' is another such expression (forget for now about complications involving other people having that name). Then the sentence

> Jeff Speaks is the teacher of PHIL 43916 at Notre Dame in the fall of 2014.

will be true at every time, just like

> Jeff Speaks is Jeff Speaks.

But now consider

> Necessarily, Jeff Speaks is the teacher of PHIL 43916 at Notre Dame in the fall of 2014.

> Necessarily, Jeff Speaks is Jeff Speaks.

These don't seem the same. To capture this difference in truth-value, we need to assign to each expression not just a function from times to semantic values, but a function from a time and a possible world to a semantic value. This is called an *intension* . We now call the semantic value of an expression – that is, the value delivered by an intension given a particular world and time as argument – its extension or reference.

What are possible worlds? What are times?

How would the intension of 'Jeff Speaks' differ from that of 'the teacher of PHIL 43916 at Notre Dame in the fall of 2014'?

What we need to think about next is how to integrate intensions into our semantics. Where before the valuation function delivered semantic values as its values, it will now deliver intensions. That means that we can now derive the semantic value (extension) of an expression not just relative to a model and assignment, but also relative to a time and world. It will not surprise you to learn that we express this using superscripts. The expression

$$[\![\text{e}]\!]^{M,w,i,g}$$

means: 'the semantic value of e relative to a model $M$, a world $w$, a time $i$, and an assignment $g$.'

Models in this new system differ in some ways from models in our old system. They now include a set of worlds, and a set of times ordered by the $<$ ('earlier than') relation. And the valuation function, which as earlier is part of a model, now has as its values not semantic values, but intensions.

In these terms, what does it take for a sentence to be simply true, rather than true with respect to some arbitrary world and time?

## 3 Basic modal and tense operators

How does this help us to understand the sentences with which we began? We will be spending a few classes on this topic, but a way to see the basic strategy is to introduce into our language four new sentence operators:

P: It was the case that

F: It will be the case that

$\Box$: It is necessary that

$\Diamond$: It is possible that

to add to 'it is not the case that.' We give the semantics for these operators as follows:

$[\![\text{P}\phi]\!]^{M,w,i,g} = 1$ iff for some i* $\in$ I, i*<i and $[\![\phi]\!]^{M,w,i*,g}$=1

$[\![\text{F}\phi]\!]^{M,w,i,g} = 1$ iff for some i* $\in$ I, i*>i and $[\![\phi]\!]^{M,w,i*,g}$=1

$[\![\Diamond\phi]\!]^{M,w,i,g} = 1$ iff for some i* $\in$ I and some w* $\in$ W, $[\![\phi]\!]^{M,w*,i*,g}$=1

$[\![\Box\phi]\!]^{M,w,i,g} = 1$ iff for every i* $\in$ I and every w* $\in$ W, $[\![\phi]\!]^{M,w*,i*,g}$=1

Now consider what this tells us about:

Jeff Speaks was the teacher of PHIL 43916.

Necessarily, Jeff Speaks is Jeff Speaks.

Necessarily, Jeff Speaks is the teacher of PHIL 43916.

Necessarily, the teacher of PHIL 43916 is the teacher of PHIL 43916.

Is the last of these ambiguous?

Consider now the false sentence

It is always true that Jeff Speaks is the teacher of PHIL 43916.

Can you define the 'it is always true that' operator in terms of P and F?