Plurals

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1 Plurality and number

Consider the sentence

> John ate pancakes.

This sentence seems quite closely related to

> John ate a pancake.

One might think that just as the second sentence is true iff there is at least one pancake such that John at it, so the first sentence is true iff there is more than one pancake which John ate.

Could we treat 'more than one' as a determiner, on par with 'a'? How would you state its semantic value?

But this intuitive view of negation doesn't fit very well with our intuitions about the negations of these sentences. To most people, it seems that the following have the same truth conditions:

> John didn't eat pancakes.
> John didn't eat a pancake.

But this is inconsistent with the view that plurals mean 'more than one.' Another example: one can't say 'I don't have children' if you have exactly one child.

We seem forced to accommodate either the intuition that our first two sentences differ in truth condition, or the intuition that our second two sentences have the same truth condition, at the level of pragmatics.

2 Pluralities and sets

Consider the sentence

> All the students gathered at the window.

Suppose first that we just treated 'all the students' as a quantified noun phrase of the same sort as 'every dog.' What would be odd about this interpretation of the sentence?

It seems that the above sentence is different than normal examples of universally quantified sentences. It seems to, not just say something about every member of a certain collection, but rather to say something about the collection, or plurality, itself. But what is a plurality, and what does it mean to say something about one?

A natural idea is that a plurality is a set. If so, then a natural thought is that the sentence above says something about a set. A first pass at an analysis of this sort of the sentence might be:

> $\exists$S ($\forall x$ ($x \in$S iff x is a student) & S gathered)

The basic idea is that sentences of this form say something about a set, where the set includes all and only the members of the relevant plurality. A set is an individual thing; so, in a way, this approach reduces talk about pluralities to talk about individuals. This approach gives rise to two puzzles.

The first is that the above analysis has us saying that a set gathered; but this makes no sense. Sets are individuals, and hence (like an individual student) cannot, on their own, gather. (And even pluralities of sets are obviously the wrong sorts of things to gather.) We might try to get around this problem by analyzing 'gathered' as a property of sets explicable in terms of relations holding between the members of the set.

A second puzzle can be brought out by considering the sentence

> Some things aren't members of themselves.

This seems to be true; for example, you and I are not sets, and hence have no members, and hence are not members of ourselves. So the following also seems true:

> The things which aren't members of themselves include you and me.

It looks like the above model should have us analyze this sentence as

> $\exists$S ($\forall x$ ($x \in$S iff x is not a member of x) & S includes you and me)

But this implies the existence of a set which includes all and only the non-self-membered things, which implies a contradiction.

One response to this problem is to claim that unrestricted quantification is impossible.

A second response, which has been increasingly popular since the 1980's, is to say that we should not analyze sentences like the above in terms of sets. Instead, we might introduce a new kind of quantification – plural quantification.

Just as

$$\exists x \text{ dog}(x)$$

means 'there is a dog,' so

$$\exists xx \text{ dog}(xx)$$

means 'there are some dogs.' On this sort of view, plural quantification of this sort is irreducible to ordinary 'singular' quantification. (For a classic paper developing this view, see Boolos (1984), which is one of the optional readings on the web site.)

3 Collective vs distributive uses

Suppose that we opt for plural quantification. This leaves us with the question of how to account for two different uses of plurals, called distributive and collective. This is the difference between, on the one hand,

> The students ate lunch.
>
> The students took a nap.

and

> The men carried the piano.
>
> All the students gathered at the window.

The difference is that the first (distributive) sentences entail, for each person in the plurality, that they did the thing indicated by the verb; so, if the students ate lunch, it follows that each individual student individually ate lunch. But it does not follow from the above that each individual man carried a piano. (One way to see this is that might well be true to say of each of the men, 'He can't carry anything more than X pounds', where 'X' is some amount less than the weight of a piano.)

A central question is whether plural constructions are ambiguous between distributive and collective readings — perhaps the former can be analyzed using some sort of restricted universal singular quantification, and the latter are analyzed using irreducibly plural quantification. On the one hand, it looks a bit implausible to posit an ambiguity here. On the other hand, if we don't posit an ambiguity of this sort, our theory will not explain the fact that the distributive sentences above entail the corresponding singular claims. And it will be hard to capture the intuitive datum that a sentence like

The students finished the project.

has two interpretations, depending on whether the project was a group or individual project. One view is that we can explain this difference via the presence or absence of an unpronounced 'distributive operator' (which intuitively means something like 'for each of them') which requires that the predicate be true of every member of the relevant plurality.

4 Some remaining puzzles

4.1 Dependent plurals

The problem for compositionality posed by the difference in truth conditions between

Unicycles have wheels. (Or: All unicycles have wheels.)

My unicycle has wheels.

4.2 More problems with 'and'

We've already discussed the problem of giving a unified treatment of 'and' when used in predicate conjunctions and when used as a sentence connective. In both of these uses, one can think of it as taking the intersection of the intensions of the expressions it connects. But the following seems quite different:

Bob, Jim, and Kate gathered by the window.

Here the function of 'and' seems to be more like union than intersection; and yet it seems very odd to say that this is simple ambiguity, and that this use of 'and' simply has an entirely different meaning than the other uses.

Collective plurals show that we can't eliminate these uses of 'and' in the obvious way in terms of uses of the expression as a sentence connective, and if we try to turn this into predicate conjunction ('the things which are Bob and are Jim and are Kate') we get exactly the wrong results.

We'll return to this problem when we discuss generalized quantifiers.

References

George Boolos, 1984. To Be is to Be a Value of a Variable (or to Be Some Values of Some Variables). *Journal of Philosophy* 81(8):430–449.