Basic semantics for quantification in English

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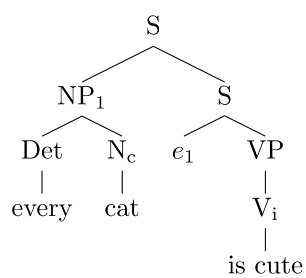
1 Models and assignments

Our language now contains trees like



What we need to know now is how to interpret trees like this: what does it take for them to be true or false?

Two answer this question we need to introduce two new concepts: the concept of a *model* and the concept of an *assignment*.

A model M is a pair of two things: a valuation function V and a domain (or universe) U of discourse. These ideas are already familiar, in somewhat different form, from our simpler language. Recall that we needed to talk not just about, for example, $[\![\text{is boring}]\!]$, but also $[\![\text{is boring}]\!]^v$ – where the latter is $[\![\text{boring}]\!]$ relative to some circumstance of evaluation. A valuation function V_1 is a function from the expressions of our language to their semantic value in a situation v_1.

A domain U_1 is just the list of things that exist in v_1. The semantic values assigned to expressions by V_1 must be built up from elements of U_1. In the examples we've been discussing, the domain was {Pavarotti, James Bond, Sophia Loren}.

So we can talk about the semantic value of "is boring" relative to a model in much the way we before talked about $[\![\text{is boring}]\!]^v$. This is a first step in making more precise what we meant by 'relative to a state of the world' before.

An assignment g is a function from traces to elements of the domain. Recall that we have infinitely many traces e_1, e_2, ... in our language. One assignment, given the above domain of discourse, might be

> [e_1 → James Bond
> e_2 → Sophia Loren
> e_n → Pavarotti] for any $n > 2$.

Another might be

> [e_1 → James Bond
> e_2, 3_3 → Pavarotti
> e_n → Sophia Loren] for any $n > 3$.

A very simple one is

> [e_n → James Bond] for any n

All that's required is that the assignment g be a function from the variables to members of the domain.

Rather than talking about semantic values relative to v, we can now talk about the semantic value of an expression relative to a model and an assignment (written as, e.g., $[\![\text{is boring}]\!]^{M_1,g_1}$). This notion is defined (for the lexicon described above) as follows:

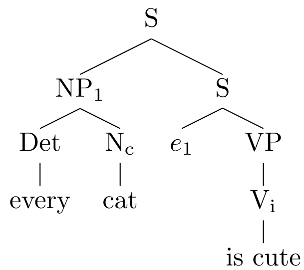
For a model $M_1 = \langle U_1, V_1 \rangle$, if A is an expression which is not a trace, then $[\![\text{A}]\!]^{M_1,g_1} = V_1(\text{A})$; and if A is a trace, then $[\![\text{A}]\!]^{M_1,g_1} = g_1(\text{A})$.

What does every valuation function return when given 'Pavarotti' as argument?

What is $g_2(e_4)$?

2 Interpretation of sentences containing 'every'

Let's return to the sentence above:



What does it take for this sentence to be true or false? We know that $[\![\text{cat}]\!]^{M,g}$ will be a set of things – the set of cats in the relevant model. So our question boils down to the question of how we should understand the contribution of 'every' to the truth conditions of sentences in which it occurs.

The basic idea is that we start with an assignment g of values to all of the traces, and we then consider every assignment function which agrees with g on every trace other than e_1. If our sentence is true in M with respect to every such assignment, then our quantified sentence is true relative to M and g.

We use

> ${g_1}^{[u/e_1]}$

to mean

> the assignment function which differs from g_1 only in assigning u as the value of e_1

Using this notation, we can state the lexical entry for 'every' as follows:

> $[\![[[\text{every } \beta]_{\text{i}} \text{ S}]]\!]^{M,g}$=1 iff for all $u \in U$, if $u \in [\![\beta]\!]^{M,g}$, then $[\![\text{S}]\!]^{M,g^{[u/e_i]}}$=1

What does this say?

Can you use this rule to derive truth conditions for 'Every cat is cute'?

If this sentence is true relative to one assignment g_1, can it be false relative to another assignment g_2 (assuming that we hold fixed the model)?

3 'A' and 'the'

Sentences involving 'a' and 'the' don't differ in their structure from sentences containing 'every'; so what we need to be able to interpret these sentences are just the lexical entries for 'a' and 'the.' They are as follows:

$[\![[[\text{a } \beta]_\text{i} \text{ S}]]\!]^{M,g}$=1 iff for some $u \in U$, $u \in [\![\beta]\!]^{M,g}$ and $[\![\text{S}]\!]^{M,g^{[u/e_i]}}$=1

$[\![[[\text{the } \beta]_\text{i} \text{ S}]]\!]^{M,g}$=1 iff for some $u \in U$, $[\![\beta]\!]^{M,g}$={u} and $[\![\text{S}]\!]^{M,g^{[u/e_i]}}$=1

What does the clause for 'the' say?

What does this lexical entry tell us about the truth conditions of 'The cat is cute'? Is this surprising?

Not all quantifier phrases are as simple as 'every cat.' In particular, some quantifier phrases contain others, as in 'every cat next to a dog.' Do our semantic rules tell us how to handle quantifier phrases of this sort? What would happen if you tried to apply the rules above to this phrase?