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The fine-tuning argument for the existence of God

Suppose that I have an opaque jar with some number of ping pong balls in it. You know that the jar either has 10 ping pong balls in it or 1000 ping pong balls in it. In either case the balls are numbered sequentially. You think that the two possibilities are equally likely.

Suppose that we pick a ball at random. It is #3. Does this favor one hypothesis over the other?

It seems to favor the 10 ping pong ball hypothesis, since that hypothesis makes it more likely that #3 will be picked (rather than some number higher than 10).

But how much? How likely should I think that it is that the jar has 10 ping pong balls in it, vs. 1000?

Suppose someone offered you 40:1 odds that it is the 10-ball jar — so you win $40 if it is the 1000 ball jar, and lose $1 if it is the 10 ball jar. Should you take the bet?

Before we drew the ball, we thought that each theory had a 0.5 probability of being correct. This is called the theory's **prior probability.**

We also know something else. We know how likely each theory predicts it to be that we draw the #3 ball.

We know that the 10-ball theory says that there is a 0.1 probability that the #3 ball will be drawn. The 1000-ball theory says that there is 0.001 probability that this ball will be drawn.

This is a claim about what is called **conditional probability**. This is the probability that something will happen if something else is true. What we seem to know here is the probability of the evidence (e) conditional on the hypothesis (h). We write that like this:

$$Pr(h \mid e)$$

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So we know the prior probability of the hypotheses, and the probability of the evidence given the hypothesis. What we **want** to know is this:

$$Pr(h \mid e)$$

This is the probability of the hypothesis given the evidence. It tells us how likely we should take the hypothesis to be, given the evidence we have observed.

How can we do this?

How can we do this?

One way to answer these questions employs a widely accepted rule of reasoning called "Bayes' theorem," named after Thomas Bayes, an 18th century English mathematician and Presbyterian minister.

To arrive at the theorem, we begin with the following definition of conditional probability:

$$P(a|b) = \frac{P(a\&b)}{P(b)}$$

This says that the probability of *a* given *b* is the chance that *a* and *b* are both true divided by the chance that *b* is true.

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Why might this be true? Intuitively, the idea is this. Take all of the chances that *b* could be true, and look at how many of them *a* is also true in. Divide the situations in which both *a* and *b* are true by the total number of situations in which *b* is true — and that will tell you how likely it is that *a* will be true if *b* is. And that just is $Pr(a\,|\,b)$.

$$P(a|b) = \frac{P(a\&b)}{P(b)}$$

Using this definition of conditional probability, we can prove the following:

Bayes' theorem

$$P(h|e) = \frac{P(h)*P(e|h)}{P(e)}$$

(You can see the proof here.)

And this does **exactly** what we wanted in the case of the urns and the ping pong balls. It tells us how to take some prior probabilities and the probability of the evidence given the hypothesis and lets us figure out how likely the hypothesis is, given the evidence.

Bayes' theorem

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Let *h* be the 10 ball hypothesis, and let *e* be the event of ball #3 being picked. Then it looks like

$$P(h) = 0.5$$
$$P(e\,|\,h) = 0.1$$

How about $P(e)$? Well, we know that the probability of the #3 ball being picked on one hypothesis is 0.1, and on the other hypothesis it is 0.001; and we thought that these two hypotheses were equally likely to be true. So it seems that we should take the average, which gives us

$$P(e) = 0.0505$$

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This gives us all the information we need to plug into Bayes' theorem:

$$P(h\,|\,e) = \frac{0.5 * 0.1}{0.0505} \approx 0.99$$

Bayes' theorem

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So, there is about a 99% chance that the urn has 10 balls in it. You should not have taken the 40-to-1 bet.

This illustrates the fact that peoples' intuitive judgements about probability are often incorrect. Bayes' theorem is enormously useful in helping us to figure out what the chances are of some hypothesis being true, given some new evidence.

Here is a more real life example. Suppose that you go to the doctor for a routine cancer screening. You know that 1% of people in the population have cancer. You know that the test detects cancer 80% if the time that it is present (and so misses it 20% of the time). You also know that the test sometimes throws up false positives; in 10% of cases in which the person does not have cancer, the test gives an incorrect positive result (and in the other 90% it gets it right).

You test positive. What are the chances you have cancer?

Again, Bayes theorem tells us:

Bayes' theorem

$$P(h|e) = \frac{P(h)*P(e|h)}{P(e)}$$

$$P(h\,|\,e) = \frac{0.01 * 0.8}{0.107} \approx 0.075$$

So, about a 7.5% chance. Why is this?

In a population of 1000, 8 people will get a correct positive test for cancer. (2 will have cancer, but the test will miss it.) Of the 990 people who do not have cancer, 99 will get false negatives. So many more people get false negatives than are correctly diagnosed with cancer. Most likely, you are one of them!

Enough about probability (for now). Let's turn now to some results from contemporary physics which will be important to the argument which follows.

A first question: what does the theory provided by physics include?

> "The standard model of physics presents a theory of the electromagnetic, weak, and strong forces, and a classification of all known elementary particles. The standard model specifies numerous physical laws, but that's not all it does. According to the standard model there are roughly two dozen dimensionless constants that characterize fundamental physical quantities." (Hawthorne & Isaacs, "Fine-tuning fine-tuning")

These "dimensionless constants" will be our focus.

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One such constant is the cosmological constant, which measures the energy density of empty space.

The cosmological constant is just a number which (as far as we know) could have different values consistent with the laws of physics.

One thing that the standard model of physics gives us is a measure of how likely it is, given the laws of the nature, that the fundamental constants (like the cosmological constant) would fall in a certain range.

We can make certain plausible assumptions about what it would take for life to have evolved. For example, if there were nothing but hydrogen, it is hard to see how life could have evolved. If there were no planets, it is hard to see how life could have evolved.

Given assumptions such as these, we can look at what the standard model of physics tells us about how likely it is that, for example, the cosmological constant has a value which would permit the evolution of life.

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> "Physicists have determined the (approximate) values of the fundamental constants by measurement. (There's no way to derive the values of the fundamental constants from other aspects of the standard model. Any quantities that could be so derived wouldn't be fundamental.) Still, the underlying theory favored some sorts of parameter-values over others. ... Physicists made the startling discovery that -- given antecedently plausibly assumptions about the nature of the physical world -- the probability that a universe with general laws like ours would be habitable was staggeringly low."

This is the claim that that the cosmological constant having a life-supporting value, given the laws of nature, is very low.

This is the claim that that the cosmological constant having a life-supporting value, given the laws of nature, is very low.

> We should emphasize just how small we take the life-permitting parameter values to be according to the physically-respectable measures. "Small" here doesn't mean "1 in 10,000" or "1 in 1,000,000". It means the kind of fraction that one would resort to exponents to describe, as in "1 in 10 to the 120". The kind of package that we have in mind tells us that only a fantastically small range is life permitting.

The claim is that, according to current physics, the probability of the cosmological constant falling in a life-supporting range, given the laws of nature, is $\frac{1}{10^{120}}$.

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Let's now ask how to turn these facts about current physics and probability theory into an argument for the existence of God.

Much as we considered two different hypotheses about the number of balls in the urn, so we can consider two different hypotheses about the origins of the universe.

Let the **design hypothesis** be the hypothesis that the universe was created by an intelligent designer. Let the **non-design** hypotheses be the hypothesis that it was not.

Let our **evidence** (*e*) be the fact that the cosmological constant has a value in the life-permitting range.

Let the **design hypothesis** be the hypothesis that the universe was created by an intelligent designer. Let the **non-design** hypotheses be the hypothesis that it was not.

Let our **evidence** (*e*) be the fact that the cosmological constant has a value in the life-permitting range.

Then what do we know about the relevant probabilities?

$$Pr(e\,|\,\text{non-design}) = \frac{1}{10^{120}}$$

It seems at least reasonably plausible that, if the universe were created by an intelligent designer, that designer would make it the case that the fundamental constants are such as to permit life. Just to pick a number, let's suppose that

$$Pr(e\,|\,\text{design}) = 0.5$$

What we want to know is: what are the probabilities of our two hypotheses, conditional on our evidence?

$$Pr(e\,|\,\text{non-design}) = \frac{1}{10^{120}}$$

$$Pr(e\,|\,\text{design}) = 0.5$$

What we want to know is: what are the probabilities of our two hypotheses, conditional on our evidence?

Bayes' theorem

$$P(h|e) = \frac{P(h)*P(e|h)}{P(e)}$$

This is what Bayes' theorem is for. But to apply that theorem, we need two other pieces of information: the probability of the evidence, and the prior probability of the two hypotheses.

Let us suppose that we are perfect agnostics, so that

$$Pr(\text{non-design}) = 0.5$$

$$Pr(\text{design}) = 0.5$$

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$$Pr(e\,|\,\text{non-design}) = \frac{1}{10^{120}}$$

$$Pr(e\,|\,\text{design}) = 0.5$$

$$Pr(\text{non-design}) = 0.5$$

$$Pr(\text{design}) = 0.5$$

Bayes' theorem

$$P(h|e) = \frac{P(h)*P(e|h)}{P(e)}$$

How about Pr(e)?

Well, we know that e is close to 0 conditional on non-design, and 0.5 conditional on design. So let's approximate and say that

$$Pr(e) = 0.25$$

(Really, this number should be very, very slightly larger than 0.25.)

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$$Pr(\text{non-design}) = 0.5$$

$$Pr(e) = 0.25$$

$$Pr(e\,|\,\text{non-design}) = \frac{1}{10^{120}}$$

With this information in hand, figuring out the probability of the non-design hypothesis given our evidence is a matter of just plugging in the numbers.

$$Pr(\text{non-design}\,|\,e) = \frac{0.5 * \frac{1}{10^{120}}}{0.25} = \frac{2}{10^{120}}$$

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$$Pr(\text{non-design} \mid e) = \frac{0.5 * \frac{1}{10^{120}}}{0.25} = \frac{2}{10^{120}}$$

It is difficult to think about numbers as large as the denominator of this fraction. But to give you some idea: the odds of winning Powerball are about 1 in 300 million. Now consider the odds of winning Powerball one trillion times in a row. Call that a "super Powerball."

Now consider the odds of winning a super Powerball one trillion times in a row. Call that a "super duper Powerball."

Now consider the odds of winning a super duper Powerball one trillion times in a row. The odds of this happening are about $1 / 10^{44}$ — so much, much higher than the odds of the universe being life-permitting by chance.

This means that if you simply take the physics at face value, and begin by assigning a probability of 0.5 to the non-design hypothesis, you should think that the chances of the non-design hypothesis being true are vastly lower than the chances of winning a super duper powerball a trillion times in a row.

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By contrast, the probability of the design hypothesis is very close to 1. It is, in fact, 1 minus the very small number we were just discussing:

$$Pr(\text{design} \mid e) = 1 - \frac{2}{10^{120}}$$

This is about as close to certainty as it is possible to get.

So far we have not written this out as a premise/conclusion argument. But it would not be hard to do so.

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the fine-tuning argument

1. $Pr(e \,|\, \text{design}) = 0.5$
2. $Pr(e \,|\, \text{non-design}) = \dfrac{1}{10^{120}}$
3. Pr(design)=Pr(non-design)=0.5.
4. Bayes' theorem.
5. Pr(design) ≈ 1 (1,2,3)
6. If the universe was created by an intelligent designer, then God exists.

———————————————-

C. Pr(God exists) ≈ 1.

This looks like a powerful argument for the existence of God. It is notable that it seems to use no especially controversial philosophical assumptions, like the Principle of Sufficient Reason.

the fine-tuning argument

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How might someone object to this argument? I want to look at four different angles of attack.

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The convinced atheist objection

The hypothesis that an intelligent designer exists is really, really unlikely in the first place, so premise (3) is false.

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The convincved atheist objection

The hypothesis that an intelligent designer exists is really, really unlikely in the first place, so premise (3) is false.

Our initial statement of the argument assumed that there is a 0.5 prior probability of the claim that the universe was created by an intelligent designer. But of course not all will agree to that; imagine that an atheist claims that there is only a 1/1 million chance that the design hypothesis is true. Will that matter?

Not really. The problem is that the number 10^{120} is so **big**. Changing the initial probability from 1/2 to $1/10^{6}$ has no serious effect.

What if the atheist is **certain**, so that the prior probability of the design hypothesis is 0?

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Skepticism about physics

The physics is wrong (or at least might be).

What premise would this target?

Skepticism about physics

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What premise would this target?

This could make a significant difference to the probabilities. Suppose you thought that there is a 0.5 probability that recent physics is fundamentally mistaken, and that the probability of e given the laws is much, much higher. Then the probability of e conditional on non-design could be high enough to make non-design a viable hypothesis.

But it is worth noting that the cosmological constant is just one of the fundamental constants, and we could have used others. So this objection would have to rely on contemporary physics being confused about quite a lot of things.

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The anthropic objection

We could never observe the falsity of the claim that the constants permits life since, if it were false, we would not exist to observe it.

What premise does this target?

The anthropic objection

We could never observe the falsity of the claim that the constants permits life since, if it were false, we would not exist to observe it.

What premise does this target?

As it stands, this objection is a bit puzzling. It does not, by itself, seem to cast doubt on any of the premises of our argument.

One might turn it into an objection by saying that, if it is impossible for us to observe some fact, then the opposite of that fact can never be used as evidence for anything. This would show that there is something wrong with using the fact that the fundamental constants are life-life-permitting as evidence in our Bayesian argument.

But if we think about some examples, we can see that this principle is not very plausible.

But if we think about some examples, we can see that this principle is not very plausible.

Consider, for example, the following case:

The firing squad

A prisoner is standing in front of a firing squad of 10 gunmen, all of whom are excellent shots. The guns all fire at the same time and, to his surprise, the prisoner realizes that he is still alive, and without a scratch. He infers that the gunmen were not trying to kill him.

Could one object to the prisoner's reasoning by saying that, if the gunmen had shot him, he would not have been around to observe this? This does not seem very plausible; the prisoner's reasoning seems perfectly fine. But this seems to rule out the version of the anthropic objection we are considering.

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Let's look at one last objection:

The multiverse objection

There are very many — perhaps infinitely many — distinct universes, which can have different fundamental physical constants.

The multiverse objection

There are very many — perhaps infinitely many — distinct universes, which can have different fundamental physical constants.

Surely, this objection goes, if there were enough universes, then no matter how improbable it is that one of them would have constants that fall in the life-permitting range, it is not so improbable that some would.

Our situation would then be somewhat analogous to the position of someone who wins the lottery. The odds of **that** person winning the lottery were very small; but if enough people buy tickets, it is not so improbable that someone wins.

It would be unreasonable for the winner to infer that the lottery was rigged in her favor; just so, it would be unreasonable for us to assume that our universe was designed.

It would be unreasonable for the winner to infer that the lottery was rigged in her favor; just so, it would be unreasonable for us to assume that our universe was designed.

So the key question is: do we have good reason to think that the multiverse hypothesis is true?

A first point to note: it would be very surprising if this hypothesis were true. For, if it is, there are very many — perhaps infinitely many — other universes, each as real as ours, in which some near-duplicate of you exists. There is, for example, very likely one in which there exists some being with a qualitatively identical history to you who differs from you only in that she or he scratched his nose one second ago.

This does not show that the multiverse hypothesis is false; the universe might be strange, and science repeatedly shows us that it is. But it does suggest that the multiverse hypothesis is not one that we should believe without argument.

This does not show that the multiverse hypothesis is false; the universe might be strange, and science repeatedly shows us that it is. But it does suggest that the multiverse hypothesis is not one that we should believe without argument.

One might think that the very facts used in the fine-tuning argument can be used to support the multiverse hypothesis. For consider the following argument:

A Bayesian argument for the multiverse

It is very, very improbable that our universe is the only one and, just by chance, the constants came to be set in such a way as to make life possible. But if there were many many universes, it would not be very improbable that one would be life supporting. So, the fact that our universe is life-supporting is strong evidence in favor of the multiverse hypothesis.

But, while this reasoning sounds plausible, consideration of parallel cases shows that something has gone wrong.

But, while this reasoning sounds plausible, consideration of parallel cases shows that something has gone wrong.

A Bayesian argument for many dice rollers

I am sitting in my office, and I pick up 12 dice and decide to roll them. I roll all sixes. Amazed, I think to myself: there must be lots of people rolling dice in Malloy Hall right now. After all, what are the odds that someone rolls 12 sixes in Malloy in the case where there is just one person rolling dice?

This would be terrible reasoning; the fact that I rolled all sixes, however improbable, is not evidence for the existence of many rollers. What has gone wrong?

A Bayesian argument for many dice rollers

I am sitting in my office, and I pick up 12 dice and decide to roll them. I roll all sixes. Amazed, I think to myself: there must be lots of people rolling dice in Malloy Hall right now. After all, what are the odds that someone rolls 12 sixes in Malloy in the case where there is just one person rolling dice?

One diagnosis is that we need to distinguish between two pieces of evidence we might have:

E1. I rolled 12 6's.

E2. Someone in Malloy rolled 12 6's.

The existence of many rollers would make Evidence 2 more likely. Would it make Evidence 1 more likely?

If not, then it looks like Evidence 2, but not Evidence 1, provides evidence for the many rollers hypothesis. Since in our imagined scenario what I possess is Evidence 1, my inference that there must be many rollers was illegitimate.

But now compare this to the case of the multiverse.

E1. This universe has life-permitting constants.

E2. Some universe has life-permitting constants.

Which of these, if either, does the multiverse hypothesis make more likely? What does this show about the idea that the fact that the fundamental constants permits life supports the multiverse hypothesis?

Summing up: it appears that, if we have good reason to believe the multiverse hypothesis, this would be bad news for the fine-tuning argument. But it also seems that the fact that our universe is life-supporting is not itself evidence for the multiverse hypothesis. So the key remaining question is: do we have any good reason to believe in the multiverse?

This is a question very much in dispute — though the dispute is as much among physicists as philosophers. Some physicists think that there is physical evidence in favor of the multiverse hypothesis. Others think that the very idea of physical evidence about universes distinct from our own makes little sense.

Here — as in other cases — we have an example in which philosophical reasoning and scientific theory are intertwined.

What seems clear is that if (1) there is just one universe and (2) current thinking about the fundamental constants in physics is on the right track, then the fine-tuning version of the design argument is a powerful argument for the existence of a designer of the universe.

Appendix: Proof of Bayes' theorem

Remember our definition of conditional probability:

$$P(a|b) = \frac{P(a\&b)}{P(b)}$$

Using this definition, we can prove Bayes' theorem as follows:

Derivation of Bayes' theorem

1.	$P(a\|b) = \frac{P(a\&b)}{P(b)}$	def. of conditional probability
2.	$P(b\|a) = \frac{P(a\&b)}{P(a)}$	def. of conditional probability
3.	$P(a\|b) * P(b) = P(a\&b)$	(1), multiplication by ='; s
4.	$P(a\&b) = P(b\|a) * P(a)$	(2), multiplication by ='s
5.	$P(a\|b) * P(b) = P(b\|a) * P(a)$	(3),(4)
C.	$P(a\|b) = \frac{P(b\|a) * P(a)}{P(b)}$	(5), division by ='s

To go back to the main lecture, click here.