Propositions as sets of possible worlds

Jeff Speaks
PHIL 93503

October 23, 2023

What kind of question are we asking when we ask what propositions are?

Why primitivism about propositions can seem less plausible than primitivism about properties.

Two conceptions of propositions: sets of worlds vs. structured propositions. What does structure talk amount to here?

1 Arguments for the possible worlds theory of propositions

1.1 They do what sentence meanings are supposed to do (and no more)

From Lewis (1970):

> In order to say what a meaning *is*, we may first ask what a meaning *does*, and then find something that does that. ...
>
> We call the truth-value of a sentence the extension of that sentence; we call the thing named by a name the extension of that name; we call the set of things to which a common noun applies the extension of that common noun. The extension of something in one of these three categories depends on its meaning and, in general, on other things as well: on facts about the world, on the time of utterance, on the place of utterance, on the speaker, on the surrounding discourse, etc. It is the meaning which determines how the extension depends upon the combination of other relevant factors. What sort of things determine how something depends on something else? Functions, of course; functions in the most general set-theoretic sense, in which the domain of arguments and the range of values may consist of entities of any sort whatever, and in which it is not required that the function be specifiable by any simple rule. We have now found something to do at least part of what a meaning for a sentence, name, or common noun does: a function which yields as output an appropriate extension when given as input a package of the various factors on which the extension may depend.

Possible worlds semantics as the 'default' conception of meaning operative in contemporary semantics.

1.2 From the nature of inquiry

Contrast the view of propositions as sets of worlds with a view according to which they are structured sentence-like entities.

The first suggests a 'negative' theory of inquiry, according to which the basic thing is your 'belief set:' the set of worlds compatible with what you believe. Inquiry is then the process of eliminating worlds from this belief set.

The second suggests a 'positive' theory of inquiry, according to which the basic thing is an individual belief. Inquiry is then the process of adding more beliefs to your stockpile.

An argument in favor of the negative theory: it makes sense of tacit belief, e.g. Russell's belief that Big Ben is bigger than Frege's earlobe. On the negative conception, this is just a matter of Russell's belief state including no worlds in which Big Ben is not bigger than Frege's earlobe — so the proposition can be literally believed without every having been considered. Related virtue: it makes sense of reasoning using premises which are never explicitly called to mind.

Some replies:

1. The fan of the sentence-like propositions can adopt the negative theory, so long as those sentence-like propositions determine sets of worlds. But: doesn't it look like the sets of worlds are doing all of the work?

2. The fan of sentence-like propositions can say that you believe not just the propositions you have considered and accepted, but also the consequences of those propositions. But: does 'consequences of' just mean 'propositions entailed by'? Then we again get the result that for any alleged distinct but equivalent propositions p, q, one believes p iff one believes q. So it again looks like the relevant sets of worlds are doing all of the work.

3. The fan of sentence-like propositions can say that you believe not just the propositions you have considered and accepted, but also the ones you are disposed to accept. But there are tricky cases — I can consistently act as if p is false even if, were I to explicitly consider p, I would see that it is true. Not obvious that it would be true to say that I believed p before considering it.

1.3 From the nature of conversation

Some features of conversations seem to require a conception of the information taken for granted, and mutually known to be taken for granted, by participants in the conversation. This is called the *common ground* of the conversation.

Example: presupposition. Some sentences (in contexts) presuppose a proposition p in the sense that an utterance of S is infelicitous unless (i) p is part of the common ground or (ii) participants are willing to add p to the common ground without objection.

The following all seem to presuppose, in this sense, that Daniel hates puppies:

> Jane knows that Daniel hates puppies.
>
> Jane doesn't know that Daniel hates puppies.
>
> Does Jane know that Daniel hates puppies?

We can raise a question about the common ground which is parallel to the question we raised about inquiry: is it best understood as a set of worlds — in which case the purpose of conversation is to eliminate worlds from the set — or as a collection of structured sentence-like propositions — in which case the purpose of conversation is to add worlds to the set?

Stalnaker argues for the former option, and calls the common ground the 'context set' (by parallel with 'belief set'). One advantage of this way of viewing things: propositions can be part of the common ground even if they have not been made explicit in the conversation. Example: suppose that I'm at a meeting of parents at my child's school, and it is common ground that every person there has children in the school. An utterance of

> Is John's kid the troublemaker?

seems fine, even though it presupposes that John has kids and this proposition has not been explicitly uttered or entertained till now.

There are worries here about overgeneration. Some presupposition triggers are anaphoric: 'too,' 'again.' These don't seem to fit the simple model. Even if it is common ground in our conversation that some people or other will eat dinner at North Dining Hall tonight, if I say out of the blue

> I'm going to NDH tonight, too.

it sounds bizarre. Possible reply: we distinguish between 'active' and 'passive' context sets, with different presupposition triggers sensitive to different sets. (This gets discussed in Kripke (2009).)

Residual worry about overgeneration: suppose that it is part of the active context set that John just walked around the lakes. Then it will also be a part of the active context set that John's feet were recently touching the ground (since this is entailed by his having gone for a walk). Still, in this context

> My feet are touching the ground, too.

sounds bizarre. Fans of structured views of propositions take this as evidence that we need a more fine-grained conception of the active common ground.

1.4 From the nature of belief and desire

We have beliefs and desires. A common thought: this cannot be a basic fact about us; there must be facts about us in virtue of which we have the beliefs and desires we have.

Some plausible attempts to give a 'theory of mental content' seem to imply that propositions are no more fine-grained than sets of possible worlds.

Begin with Stalnaker calls the 'pragmatic' theory of belief and desire: to believe p_1, p_2, ... and desire q_1, q_2, ... is to be disposed to act in ways which would satisfy q_1, q_2, ... in a world in which p_1, p_2, ... are true.

If this theory of belief and desire were true, then if there were distinct propositions true in just the same possible worlds, one would believe/desire one of them iff one believed/desired the other. Natural thought: the objects of belief and desire just are sets of worlds.

Basic problem for the pragmatic theory: entails wild indeterminacy of belief and desire. But: some extra constraints which we might add seem to preserve the features of the account which suggest that the objects of belief and desire can be no more fine-grained than sets of worlds.

1.5 From Fido

Stich gives the example of Fido, who sees his master bury a meaty bone in teh back yard. If Fido goes out and starts digging, we are inclined to say things like 'Fido believes that there is a bone in the backyard.'

Suppose that the objects of belief are sentence-like structured propositions. Then it is hard to see how we could truly attribute a belief in one of those to Fido, who has no capacity to distinguish between real bones and various types of bone-simulacra.

Are things any better on the 'sets of worlds' view? That view would still seem to force us to decide, e.g., whether Fido's belief is true in a world where the bone is fake, or whatever. Reply: the contents of Fido's beliefs are defined over an impoverished set of worlds. Nothing stops the 'there is a bone in the backyard' out of my mouth exactly agreeing with Fido's belief with respect to truth in that set of worlds.

2 The problem of necessary equivalence

The central problem for this conception of propositions is its inability to distinguish between necessarily equivalent propositions. Proponents (as we have seen) regard this as a feature rather than a bug. But there are two different ways to argue that it is a bug.

Both modes of argument make use of (what seem to be) necessary but non-obvious propositions, and impossible but not-obviously-false propositions. Some widely accepted examples of the former:

> Hesperus is Phosphorus.
>
> Water is H_2O.
>
> That desk is made of wood.
>
> There are infinitely many primes.

Negations of these are examples of the latter.

One kind of argument focuses on propositional attitudes like belief and assertion; the second focuses on sentences which ascribe these attitudes to subjects.

2.1 The problem of the attitudes

Imagine that I am told for the first time that water is H_2O. I seem to acquire a new belief, which I can use to make a variety of inferences. I already believed that water comes out of the faucet; I can now form the new belief that H_2O comes out of the faucet, etc.

This does not seem puzzling. But it looks puzzling on the view of inquiry sketched above. After all, the proposition that I 'learn' is true in every world. So it does not help me to narrow down my belief set at all. The puzzle: given this, how can we explain why it seemed as though I formed a new belief?

A parallel problem arises in conversation. Imagine that I am at a philosophy conference talking with an admirer of Daniel's work on impossible worlds. If Daniel walks into the room, I might point to him and say 'That is Daniel Nolan.'

Again, this looks like an ordinary unproblematic assertion. But it looks puzzling on the view of conversation sketched above. After all, the proposition that I assert is true in every world (or at least every world in the context set, since we are assuming in the conversation that Daniel exists.) So it does not narrow down the context set at all; it is as if I said nothing. The puzzle: given this, how can we explain why it seems as though I made an informative assertion?

2.2 The problem of attitude ascriptions

Related but distinct problems arise when we consider sentences which ascribe propositional attitudes like belief and assertion.

The above comments about belief sets suggest something like the following view of the truth conditions of attitude ascriptions:

> 'A believes that S' is true iff S is true in every world of A's belief set.

But every necessary truth is true in every world of A's belief set; so is every truth entailed by something A believes. So any attitude ascription involving one of these truths will come out true.

The stuff about belief sets is not mandatory for the possible worlds view of content. Suppose that we ditch it and adopt something like the following view of the truth conditions of attitude ascriptions:

> 'A believes that S' is true iff A stands in the belief relation to the proposition which is expressed by S in the context.

This is sometimes called the 'naive relational theory' of attitude ascriptions. One way to make trouble for the conjunction of the naive relational theory with the possible worlds conception of content makes use of the principle that belief distributes over conjunction:

> If 'A believes that S and T' is true, then 'A believes that S' is true and 'A believes that T' is true.

Now note that if the possible worlds conception of propositions is true, any sentence S which expresses some proposition p will express the same proposition as the conjunction of S with any sentence expressing a proposition entailed by p. (The two will be true in just the same possible worlds.)

So if we start with some garden variety attitude ascription

> Jeff believes that USC was overrated.

this will entail (given the naive relational theory and the possible worlds view of propositions)

> Jeff believes that USC was overrated and there are infinitely many primes.

which given the distribution of belief over conjunction gives us

Jeff believes that there are infinitely many primes.

So it looks as though any ascription of a belief whose complement expresses a necessary truth (or anything entailed by a proposition the subject believes) will be true. But that is absurd.

One can run basically the same argument in the other direction. Suppose for reductio that there was a true ascription whose complement expressed a necessary falsehood. Then the conjunction of the complement with any sentence will express the same proposition as the complement. So any attitude ascription regarding that subject will be true. But that is absurd. So it has to be that there can never be a true ascription whose complement expressed a necessary falsehood. But that too looks absurd; sometimes sentences like

Bob believes that there are only finitely many primes.

Bob believes that he [pointing at Daniel] is Timothy Williamson.

can be true.

Some regard this pair of problems as a decisive refutation of the possible worlds conception of propositions. Proponents of the view agree that these are problems which need solving, but regard the framework as sufficiently fruitful that solutions within the framework should be explored.

3 Two-dimensionalism

3.1 Stalnaker's two-dimensionalism

Stalnaker's response to these problems is best presented starting with the case of assertion. Recall the case where I say 'That is Daniel Nolan.' Given that 'that' and 'Daniel Nolan' rigidly designate Daniel in this context, the proposition expressed by this sentence is a (the) necessary proposition.

But there are worlds in the context set in which the person whom I demonstrate is not Daniel. Perhaps in one such world it is Tim Williamson, and in another it is David Chalmers. The sentence would express different propositions if uttered in these different worlds. If the purpose of the conversation is to narrow the context set, this leads to a problem: parties to the conversation will not know how to update.

The solution: consider not the proposition actually expressed by the sentence, but the whole propositional concept, and diagonalize. When we do this, we get a contingent proposition. That is the proposition asserted, and it (unlike the one semantically expressed by the sentence in the context) does narrow the context set.

These kinds of rules of interpretation are somewhat ad hoc. They are meant to be pragmatic rules of the sort emphasized by Grice. A classic example: 'He has excellent penmanship.'

How would this work for our other examples of necessary truths?

Stalnaker thinks that this story about assertion will also help us to understand what subjects are doing when they (under one description) form a belief in a necessary proposition. Just as in the previous cases the sentence accepted will be false if uttered in some worlds in the context set, so in the belief case it will be false if considered in some worlds in the belief set. So the new belief will be contingent.

A worry: this makes all mathematical ignorance meta-linguistic ignorance.

Belief ascriptions are in some ways the trickiest case. Suppose that I say to Daniel (in a variant of the above example)

> Bob believes that I am Daniel Nolan.

What belief am I asserting that the subject of the ascription has?

- The diagonal proposition corresponding to Bob's utterance of 'I am Daniel Nolan' relative to my belief set — no, because my belief set rules out me being identical to Daniel
- The diagonal proposition corresponding to Bob's utterance of 'I am Daniel Nolan' relative to Bob's belief set — no, because Bob could not use 'I' to refer to Jeff Speaks
- The diagonal proposition corresponding to my utterance of 'I am Daniel Nolan' if I had Bob's belief set — something like this is what Stalnaker suggests.

3.2 Generalized two-dimensionalism

Stalnaker does not take the diagonal proposition associated with a sentence in a context to be part of, or derivable from, the sentence's meaning. More ambitious versions of two-dimensionalism assign all expressions of the language two dimensions of meaning.

A good place to start: Kaplan's logic of indexicals. Examples of truths of the logic of indexicals; the idea that sentences with necessary diagonal intensions are a priori and analytic. 'I am here now', 'Actually S iff S', 'I am Jeff Speaks.'

One way to think of some more ambitious forms of two-dimensionalism is as generalizing Kaplan's picture. Much as 'I' and the like are associated with characters — functions from contexts of utterance to contents — so other expressions are associated with reference-fixing rules of a certain sort. These rules (which may or may not be statable using short descriptions) tell us what would be in the extension of the expression were the actual world different in various ways. The distinction between considering a world as actual and considering it as counterfactual.

How this works for Kripke's examples of the necessary a posteriori and contingent a priori.

The diagonal proposition corresponding to a sentence is sometimes called its A-intension (Jackson) or primary intension (older Chalmers); the horizontal proposition corresponding to the actual world is sometimes called its C-intension (Jackson) or secondary intension (Chalmers).

How might this help with the puzzles about attitude ascriptions? One the distinction between primary and secondary intensions is on the table, the two-dimensionalist will want to complicate the naive relational analysis to incorporate this distinction between two different kinds of meaning.

A first natural thought:

> 'A believes that S' is true iff A has a belief with the same primary and secondary intension as S in the context.

But that would make it impossible to report beliefs using indexicals, and in general would place too-strong constraints on the truth of attitude ascriptions. So instead something like:

> 'A believes that S' is true iff A has a belief with the same secondary intension as S in the context and a similar enough primary intension.

where what is 'similar enough' may depend on context. Why this blocks the argument against the simple possible worlds treatment sketched above.

The worry that (as with Stalnaker) this sort of two-dimensionalist view mislocates our ignorance about the truth of some (alleged) necessary truths, like 'This is made of wood.' The distinction between the two-dimensionalist treatment of this case and the view of someone who admits epistemically possible but metaphysically impossible worlds.

The remaining problem of mathematical beliefs.

Chalmers' worry: 'Language exists' and the like seem to come out a priori on this model. The distinction between contextual and epistemic two-dimensionalism. Epistemic intensions are functions from scenarios to extensions. Scenarios as complete 'PQTI' descriptions of possible worlds. A sentence is true at a scenario iff it is a priori that if the PQTI description is true, then the sentence is true. We reason a priori about what would be true if the scenario were actual rather than inserting ourselves into the scenario and asking whether the sentence would be true if the scenario were actual.

Later Chalmers ends up going for a structured view which incorporates two-dimensionalist elements.

References

Saul A. Kripke, 2009. Presupposition and Anaphora: Remarks on the Formulation of the Projection Problem. *Linguistic Inquiry* 40(3):367–386.

David Lewis, 1970. General Semantics. *Synthese* 22:18–67.