Pascal's Wager

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1 Pascal's presentation of 'The Wager'

Pascal poses the problem of belief in God's existence as follows:

> "Let us then examine this point, and let us say: 'Either God is or he is not.' But to which view shall we be inclined? Reason cannot decide this question. Infinite chaos separates us. At the far end of this infinite distance a coin is being spun which will come down heads or tails. How will you wager? Reason cannot make you choose either, reason cannot prove either wrong."

Pascal is here expressing skepticism about the ability of philosophy to either prove or disprove the existence of God. But, he says, the lack of proofs does not remove the requirement that we make a choice on this point:

> "Yes, but you must wager. There is no choice, you are already committed. Which will you choose then? Let us see: since a choice must be made, let us see which offers you the least interest. You have two things to lose: the true and the good; and two things to stake: your reason and your will, your knowledge and your happiness. . . . Since you must necessarily choose, your reason is no more affronted by choosing one rather than the other. . . . But your happiness? Let us weigh up the gain and the loss involved in calling heads that God exists.

> Let us assess the two cases: if you win you win everything, if you lose you lose nothing. Do not hesitate then; wager that he does exist."

A difference in kind between this argument and the arguments for the existence of God we have considered. Pascal does not provide us any evidence for thinking that God exists. He gives us *prudential* rather than *theoretical* reasons for forming a belief that God exists. The distinction between these two kinds of reasons.

Pascal goes on to spell out more explicitly his reasoning for thinking that it is rational to believe in God, using an analogy with gambling:

> "... since there is an equal chance of gain and loss, if you stood to win only two lives for one you could still wager, but supposing you stood to win three?
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> ... it would be unwise of you, since you are obliged to play, not to risk your life in order to win three lives at a game in which there is an equal chance of winning and losing. ... But here there is an infinity of happy life to be won, one chance of winning against a finite number of chances of losing, and what you are staking is finite. That leaves no choice; wherever there is infinity, and where there are not infinite chances of losing against that of winning, there is no room for hesitation, you must give everything. And thus, since you are obliged to play, you must be renouncing reason if you hoard your life rather than risk it for an infinite gain, just as likely to occur as a loss amounting to nothing."

Clearly Pascal thinks that there is some analogy between believing in God and making an even-odds bet in which you stand to win three times as much as you stand to lose; to be more precise about what this analogy is supposed to be, we can introduce some concepts from decision theory, the study of the principles which govern rational decision-making.

2 The wager and decision theory

Pascal was one of the first thinkers to systematically investigate what we now call 'decision theory', and elements of his thought on this topic clearly guide his presentation of the wager.

Suppose that we have two courses of action between which we must choose, and the consequences of each choice depend on some unknown fact. E.g., it might be the case that we have to bet on whether a coin comes up heads or tails, and what the result of our bet is depends on whether the coin actually does come up heads or tails. Imagine first a simple bet in which if you guess correctly, you win \$1, and if you guess incorrectly, you lose \$1. We could represent the choice like this:

Courses of action	**Possibility 1: Coin comes up heads**	**Possibility 2: Coin comes up tails**
Choose 'heads'	Win \$1	Lose \$1
Chose 'tails'	Lose \$1	Win \$1

Should you choose heads, or tails? It seems that neither option is better; each gives you the same odds of winning and losing, and the relevant amounts are the same in each case.

Now suppose that you are given a slightly stranger and more complicated bet: if you choose heads, and the coin comes up heads, you win §10; if you choose heads, and the coin comes up tails, you win $5. However, if you choose tails, and the coin comes up heads, you win §0; but if you choose tails, and the coin comes up tails, you win $5. Your choices can then be represented as follows:

Courses of action	**Possibility 1: Coin comes up heads**	**Possibility 2: Coin comes up tails**
Choose ‘heads’	Win $10	Win $5
Chose ‘tails’	Win $2	Win $5

If given this bet, should you choose heads, or choose tails? Unlike the simpler bet, it seems that here there is an obvious answer to this question: you should choose heads. The reason why is clear. There are only two relevant ways that things could turn out: the coin could come up heads, or come up tails. If it comes up heads, then you are better off if you chose ‘heads.’ If it comes up tails, then it doesn’t matter which option you chose. One way of putting this scenario is that the worst outcome of choosing ‘heads’ is as good as the best outcome of choosing ‘tails’, and the best outcome of choosing ‘heads’ is better than the best outcome of choosing ‘tails’. When this is true, we say that choosing ‘heads’ *superdominates* choosing ‘tails.’

It seems clear that if you have just two courses of action, and one superdominates the other, you should choose that one. Superdominance is the first important concept from decision theory to keep in mind.

The next important concept is *expected utility.* The expected utility of a decision is the amount of utility (i.e., reward) that you should expect that decision to yield. Recall the more complicated coin bet above. If you choose ‘heads’, the bet went, there were two possible outcomes: either the coin comes up heads, and you win $10, or the coin comes up tails, and you win $5. Obviously, this is a pretty good bet, since you win either way. Now suppose that you are offered the chance to take this bet on a fair coin toss, but have to pay $7 to make the bet. Supposing that you want to maximize your money, should you take the bet?

The answer here may not seem obvious — it is certainly not as obvious as the fact that you should, if given the choice, choose ‘heads’ rather than ‘tails.’ But this is the kind of question that calculations of expected utility are constructed to answer. To answer the question:

> Should I pay $7 to have the chance to bet ‘heads’?

we ask

> Which is higher: the expected utility of paying $7 and betting ‘heads’, or the expected utility of not paying, and not betting?

To simplify, we treat 'expected utility' here as equivalent to 'expected dollars earned.'

Well, what is the expected utility of not paying to take the bet? You don't have to pay to take the bet; but you also won't win anything from the bet. So the expected outcome is that you will not win any money, and will not lose any money — \$0, or just 0.

What is the expected utility of paying \$7 and betting heads? Plausibly, we have the following:

> The expected utility of paying \$7 and betting heads = the expected utility of paying \$7 + the expected utility of betting 'heads.'

and

> The expected utility of paying \$7 = -7

so

> The expected utility of paying \$7 and betting heads = (-7) + the expected utility of betting 'heads.'

To figure out the expected utility of betting heads, we reason as follows:

> There are two possibilities: the coin could come up heads, or could come up tails. There is a 1/2 probability that either will happen, since we know that the coin is fair. If the coin comes up heads, I get \$10; if it comes up tails, I get \$5. Since there is a 1/2 chance of each, what I should expect from the bet is
>
> $(1/2)*10 + (1/2)*5 = 7.5$

I should expect the bet to yield \$7.5. Since I only have to pay \$7 to take the bet, and \$7.5 > \$7, it seems like a rational gambler should pay to take the bet.

This seems to be a good way to reason about this situation. The method we employed seems to be summed up as follows:

> If deciding between a number of options, choose the option which has the highest expected utility.

This is a plausible principle, and one which is employed in some versions of Pascal's wager.

3 Three versions of the wager

Using this terminology, we can, following Ian Hacking and Alan Hájek, distinguish three different versions of the wager which seem to be present in Pascal's text.

3.1 The argument from superdominance

One version of Pascal's argument is that the decision to believe in God superdominates the decision not to believe in God, in the above sense. He seems to have this in mind when he writes,

> "...if you win, you win everything, if you lose you lose nothing."

This indicates that, at least at this point in the next, he sees the decision to believe or not believe in God as follows:

Courses of action	**Possibility 1: God exists**	**Possibility 2: God does not exist**
Believe in God	Infinite reward	Lose nothing, gain nothing
Do not believe in God	Infinite loss	Lose nothing, gain nothing

How should you respond to the choice of either believing, or not believing, in God? It seems easy: just as in the above case you should choose 'heads', so in this case you should choose belief in God. After all, belief in God superdominates non-belief: the worst case scenario of believing in God is as good as the best case scenario of non-belief, and the best case scenario of believing in God is better than the best case scenario of non-belief.

Pascal, however, seems to recognize that there is an objection to this way of representing the choice of whether or not one should believe in God: one might think that if one decides to believe in God, and it turns out that God does not exist, there has been some loss: you are then worse off than if you had not believed all along. Why might this be?

If this is right, then it looks like the following is a better representation of our choice:

Courses of action	**Possibility 1: God exists**	**Possibility 2: God does not exist**
Believe in God	Infinite reward	Loss
Do not believe in God	Infinite loss	Gain

If this is a better representation of the choice, then it is *not* true that believing in God superdominates non-belief.

3.2 The argument from expected utility

How, then, should we decide what to do? One method was already suggested earlier: you should see which of the two courses of action has the higher expected utility.

But, to figure this out, we have to know what probabilities we should assign to the possibilities that God exists, and that God does not exist. Pascal suggests when setting up the argument that there is "an equal chance of gain and loss", which would put the probabilities of each at 1/2.

We also need to figure out how to measure the utility of each of the two outcomes, given either of the two choices. Below is one way to do that:

Courses of action	Possibility 1: God exists	Possibility 2: God does not exist
Believe in God	Positive infinity (∞)	Finite loss (-n, where n is some finite number)
Do not believe in God	Negative infinity (-∞)	Finite gain (+m, where m is some finite number)

Then, to figure out whether belief or disbelief have higher expected utilities (and therefore which is the rational course of action), we reason as follows, beginning with the case of belief in God:

$$\begin{aligned}\text{Expected utility of belief in God} &= \text{(1/2)*(Utility of belief in God, given that God exists) + (1/2)*(Utility of belief in God, given that God does not exist)}\\ &= (1/2)*(\infty) + (1/2)*(-n)\\ &= \infty\end{aligned}$$

We perform analogous calculations for the expected utility of disbelief in God:

$$\begin{aligned}\text{Expected utility of non-belief in God} &= \text{(1/2)*(Utility of non-belief in God, given that God exists) + (1/2)*(Utility of non-belief in God, given that God does not exist)}\\ &= (1/2)*(-\infty) + (1/2)*(m)\\ &= -\infty\end{aligned}$$

The case is, apparently, clear cut. The expected utility of belief in God is infinite, and the expected utility of non-belief is negative infinity. So, given that it is rational to act so as to maximize expected utility, one clearly ought to believe in God.

3.3 The argument from expected utility generalized

A response: deny that we should assign equal probabilities to the possibilities that God does, and does not exist. Perhaps it is rational to think that there is only a very remote chance that God exists, and hence that we should assign a very low probability to the possibility that God exists.

A reply to the response. Why the assignment of probability 1/2 is dispensable.

4 Objections to Pascal's wager

4.1 The impossibility of believing at will

The difference between deciding to believe and deciding to pursue some ordinary course of action. The intuition behind the thought that it is impossible to decide to believe.

Pascal seems to consider this reply to his argument when he imagines someone replying as follows:

> "...is there really no way of seeing what the cards are? ...I am being forced to wager and I am not free; I am begin held fast and I am so made that I cannot believe. What do you want me to do then?"

Pascal's reply:

> "That is true, but at least get it into your head that, if you are unable to believe, it is because of your passions, since reason impels you to believe and yet you cannot do so. Concentrate then not on convincing yourself by multiplying proofs of God's existence, but by diminishing your passions. ..."

4.2 Rationality does not require maximizing expected utility

The St. Petersburg paradox; the counterintuitive consequences which result from (i) the requirement that we should act so as to maximize expected utility, and (ii) the possibility of infinite expected utilities.

Why the result that we should sometimes fail to maximize expected utility is puzzling.

4.3 We should assign 0 probability to God's existence

How this blocks the argument.

The case against assignment of 0 probability to the possibility that God exists.

4.4 The 'many gods' objection

(For more detail, and a list of relevant further readings, see the excellent entry "Pascal's Wager" in the Stanford Encyclopedia of Philosophy by Alan Hájek, from which much of the above is drawn.)