Wittgenstein's *Tractatus Logico-Philosophicus*

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The *Tractatus* can be seen as breaking down in roughly the following way: sections 1
2 lay out the basic elements of Wittgenstein's metaphysics; sections 3-5 concern his
theory of representation, which is closely tied to his metaphysics; and sections 6 and
7 draw out some of the consequences of the preceding sections for science, value, and
the nature of philosophy.

[If you would like to read more about the *Tractatus*, I would recommend the fol-
lowing three works, some of the points of which are incorporated below: Fogelin,
Wittgenstein; Anscombe, *An Introduction to Wittgenstein's Tractatus*; and Soames,
Philosophical Analysis in the 20th Century, v. 1. Ray Monk's biography of Wittgen-
stein, entitled *Wittgenstein: The Duty of Genius* is also very good.]

1   The metaphysics of the *Tractatus*

1.1   A world of facts (1-1.21)

The basic theses here are reminiscent of Russell's logical atomism. When Wittgen-
stein says

> 1.1   The world is the totality of facts, not of things.

the claim is similar to Russell's claim that claims about what facts there are not
reducible to claims about what objects there are.

But he also adds two further claims this claim about facts. First, a claim about the
nature of the world:

> 1.13   The facts in logical space are the world.

The world, on this view, consists of facts — and consists of nothing but facts. He
also adds a claim about the independence of facts from each other:

> 1.21   Each item can be the case or not the case while everything else
>   remains the same.

We'll understand the reason for this later.

As with much in the *Tractatus*, it is initially hard to see the motivations behind these cryptic remarks. Often (though not always) later places in the text will make clearer why Wittgenstein says what he does. For now, we'll continue to lay out his system.

1.2   Objects and states of affairs (2.01-2.0141)

Wittgenstein's next move is to tell us a bit about what facts are. He says:

> 2   What is the case — a fact — is the existence of states of affairs.

A point of terminology: though they are not the same, 'states of affairs' plays for Wittgenstein much the same role as 'atomic facts' played for Russell. States of affairs are the more basic facts from which facts in general are constructed.

But this does not yet say much about what these states of affairs, or atomic facts, are. Wittgenstein then tells us

> 2.01   A state of affairs (a state of things) is a combination of objects
>   (things).

We will examine this later, but this seems to differ from Russell's conception of atomic facts. Russell though that every fact contained at least one universal; but here we have Wittgenstein saying that states of affairs are combinations of objects, without saying anything about properties or relations. He later expresses another thought which indicates that states of affairs contain only objects, and not properties:

> 2.03   In a state of affairs objects fit into one another like the links of a
>   chain.

The idea seems to be that, at least in the case of states of affairs, there is no need for properties to join objects into states of affairs; rather, states of affairs are composed entirely of objects.

The natural next question is: what are these objects which make up states of affairs? (It will soon become clear that they are not the ordinary objects with which we are acquainted.)

The first thing Wittgenstein tells us about them has to do with their relationship to states of affairs:

> 2.0122 Things are independent in so far as they can occur in all *possible*
>   situations, but this form of independence is a form of connexion with
>   states of affairs, a form of dependence. ...

The idea seems to be that objects always have the possibility of occurring in a number of different states of affairs; but it is written into the nature of objects that they be

a part of *some* state of affairs or other.

He makes a comparison, though it is only a comparison, to the case of color and material objects (§2.0131): "a speck in the visual field, though it need not be red, must have some colour..."

1.3 The simplicity and subsistence of objects (2.02-2.034)

Wittgenstein then moves from talking about the relationship between objects and states of affairs to the natures of the objects themselves. He begins with the claim

> 2.02   Objects are simple.

What does it mean to say that an object is simple? One thing Wittgenstein seems to mean is that it cannot be analyzed as a complex of other objects. This seems to indicate that if objects are simple, they cannot have any parts; for, if they did, they would be analyzable as a complex of those parts.

It is not obvious that there are objects which are simple in this sense. But Wittgenstein presents an argument for their existence here:

> 2.021   Objects make up the substance of the world. That is why they cannot be composite.

> 2.0211 If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.

> 2.0212 In that case we could not sketch any picture of the world (true or false).

Wittgenstein's idea here seems to be that without there existing a class of simple objects, it would be impossible to picture the world — that is, to represent it. But we can represent the world; so there must be such a class of objects. The question is: why does the possibility of representation require the existence of objects which are simple in Wittgenstein's sense?

In trying to understand this argument, we need to understand two things: (i) why would the absence of simple objects make whether a proposition has sense depend on whether another proposition was true? and (ii) why would this situation mean that we could not sketch any picture of the world?

To answer these questions, we need to consider some doctrines which are only clearly stated later in the *Tractatus*. The first of these has to do with simple names:

> 3.203   A name means an object. The object is its meaning. . . .

Suppose that every object were complex. Then every name would stand for a complex object (there being no simple ones). Now, we know from §3.203 that a name is meaningful only if the object it purports to refer to exists. This means that a proposition containing the name will be meaningful only if the object the name purports

to refer to exists. But the existence of a complex object depends upon its parts being arranged in a certain way; and its parts will be arranged in a that way only if some proposition, which says that they are arranged that way, is true. So we get the thesis stated in §2.0211: if all objects were complex, then propositions would be meaningful only if other propositions were true.

There are two routes from this to the conclusion stated in §2.0212.

The regress argument. A sentence can be true only if it is meaningful. But now consider the general principle that for every proposition S, S is meaningful only if there is some other proposition S^* which is true. This seems to lead to an infinite regress. For if S^* is true, then it must be meaningful. But by our general principle, if S^* is meaningful, there must be some other proposition S^{**} which is true. And so on, and so on.

Objection to the argument: it is not obvious that this infinite regress is a vicious one; it is also not obvious that there is anything more problematic here than in the supposition that all objects are complex itself.

The argument from understanding. A slightly different form of argument can also be given for the conclusion here, by employing a thought Wittgenstein expresses later about the relative independence of the meaning and truth of a sentence:

> 4.024 To understand a proposition means to know what is the case if it is true. (One can understand it, therefore, without knowing whether it is true.)

Using this principle, one might argue as follows: one can always understand a proposition without knowing its truth value. Indeed, one can understand *every* proposition without knowing *any* of their truth values. But one can understand a proposition only if it is meaningful; so to understand a proposition one must know that it is meaningful. But if the meaningfulness of a proposition depended on the truth of some other proposition, then one could only understand a proposition if one knew that some other proposition was true. But this contradicts our initial supposition that one can understand every proposition without knowing which of them is true. (See also §3.24.)

Objection to the argument: it has several questionable steps; but one is the seeming inference from (i) I know p and (ii) p depends on q to (iii) I know q. This seems not in general to be true.

Objects are not only simple, they are also, in a certain sense, unchangeable. After saying that objects are the substance of the world, Wittgenstein says

> 2.024 Substance is what subsists independently of what is the case.

and later

> 2.0271 Objects are what is unalterable and subsistent; their configuration is what is changing and unstable.

The idea here is that objects do not themselves change, but rather are that which explains change. Change is a matter of states of affairs coming into and going out of existence, and states of affairs are produced by arrangements of objects:

> 2.0272 The configuration of objects produces states of affairs.

There are two senses in which the simple objects underlie change. First, they underlie change in the actual world over time: this is, I think, the sort of change alluded to in §2.0271. But they are also what is held in common between the actual world and possible, or imaginary worlds:

> 2.022   It is obvious that an imagined world, however different it may be from the real one, must have *something* — a form — in common with it.

> 2.023   Objects are just what constitute this unalterable form.

Just as there must be something which underlies change over time, so there must be something which underlies 'modal' change: the differences between the actual and various 'imaginary' worlds.

So far we have noted two aspects of Wittgenstein's view of objects: they must be simple, and they must be what subsists through all change. One question we should ask is: how are these two aspects of objects related (if at all)?

One line of reasoning which suggests itself (though is not explicitly in the text) is the following: suppose that simple objects were susceptible to change: that they themselves could come into and go out of existence, rather than simply causing states of affairs to come into and go out of existence. Then it would be a contingent fact whether a given object existed. But every contingent fact is a matter of simple objects combining in a certain way. So an object's coming into and going out of existence would be a matter of other objects combining or ceasing to be combined in certain ways. But then the object would be a complex made out of the objects whose combining brought it into existence.

(Objection to the suggested argument: it is not (to say the least) obvious that any object created by the recombination of other objects would have those objects as parts.)

1.4   The world, facts, and reality (2.04-2.063)

So far, we've introduced a number of Wittgenstein's metaphysical theses concerning objects, states of affairs, facts, and the relation between the three. Now Wittgenstein returns to the theme with which he began the *Tractatus*: the relationship between these three and the world, and reality, as a whole.

The most important claim Wittgenstein makes here concerns the relationship between states of affairs and the world:

> 2.04   The totality of existing states of affairs is the world.

The claim here is that if we had a list of all the states of affairs (i.e., all the atomic facts) we would thereby have given a complete account of the world: the world is nothing over and above the states of affairs that exist. Here we have a point of contrast with Russell's logical atomism, which held that, in addition to atomic facts, both negative and general facts made up the world. Wittgenstein states his disagreement with Russell over negative facts in his next claim:

> 2.05   The totality of existing states of affairs also determined which states
>   of affairs do not exist.

All there are are states of affairs: these determine the whole world, including the 'facts' about which states of affairs do not exist.

But, if you recall, Russell had a kind of convincing argument for the existence of negative facts: suppose that we have a list of atomic facts $f_1 \ldots f_n$. Now consider some true sentence 'not-S.' Is the truth of 'not-S' determined by $f_1 \ldots f_n$? It seems not. For $f_1 \ldots f_n$ are atomic facts, and there is nothing to prevent a series of atomic facts from being consistent both with the truth of S, the falsity of S, or even S lacking a truth-value. Hence, Russell concluded, true negations of atomic propositions must correspond to negative facts. How can Wittgenstein avoid this argument?

I think that his ideas about objects provide him a way out. Recall that, for Wittgenstein, objects are not only what underlie change over time, but also what underlie necessity and possibility: all possible changes to the world are just a matter of the recombination of simple objects. As he puts it,

> 2.0124 If all objects are given, then at the same time all *possible* states of
>   affairs are also given.

If there are a fixed number of objects, then a list of all the states of affairs (i.e., atomic facts) will *not* be consistent with both the truth and falsity of a sentence S.

A worry about this view: the intuition that all objects exist only contingently.

Wittgenstein often discusses the *world* or *reality*. How are these two notions related? (This question is made especially difficult by the fact that Wittgenstein seems to say contradictory things in §§2.04, 2.06, 2.063.) I think that the basic idea can be stated as follows: the world consists of all the existing states of affairs, whereas reality consists of the world plus all possible but non-actual states of affairs. Wittgenstein's claim is then that the world determines reality: once we know everything about what states of affairs exist, we know everything about what states of affairs could exist as well. (Indeed, as Fogelin points out, this follows from the claims that the world consists of states of affairs, that all objects must be in some state of affairs, and the passage from §2.0124 cited above.)

2   The picture theory of representation

At this point in the text, around §2.1, Wittgenstein switches from talking about facts in general to talking about one particular kind of fact: a fact which is a picture, or representation, of another. He begin by talking about the kind of representation which he takes to be most basic: a picture. His idea is that by understanding how pictures represent the world, we can also understand how propositions of different kinds represent the world.

2.1   Pictures of facts (2.1-2.19)

Wittgenstein's discussion of pictures divides them into two parts: the elements of the picture, and the form (or structure) of the pictures.

About the elements, he writes:

> 2.131   In a picture the elements of the picture are the representatives of objects.

Elements of the picture stand for objects in something like the way that names stands for their referents. But the form of the picture also has a role to play in determining what the picture represents:

> 2.15   The fact that the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way.
> Let us call this connection of its elements the structure of the picture.. . .

It seems that just as elements of the picture represent objects, so the structure of the picture represents the way in which those objects are combined.

One question in which Wittgenstein is interested here is the question: How is representation possible? As applied to the case of pictures, this comes to: How do elements of the picture come to represent objects, and how does the structure of a picture come to represent a way that those objects could be combined? Wittgenstein does not address the first part of this question here; but he does say something about how the structure of a picture is related to the structure of the fact that it represents:

> 2.161   There must be something identical in a picture and what it depicts, to enable the one to be a picture of the other at all.

> 2.17   What a picture must have in common with reality, in order to be able to depict it - correctly or incorrectly - in the way it does, is its pictorial form.

The example of model cars in a traffic court. The idea that the structure of a picture is identical with the structure of the fact that it represents. This is a crucial idea in

Wittgenstein's thought about representation. He does not argue for it here, but it is an attractive idea. For, you might wonder, how *else* could a picture represent a fact?

We also get here the first mention of a theme which will recur in a more important context later: the idea that a picture cannot represent its own structure, but can only *show* it:

> 2.172   A picture cannot, however, depict its pictorial form: it displays it.

This is actually a kind of intuitive thought; some justification is given by Wittgenstein's point that

> 2.174   A picture cannot, however, place itself outside its representational form.

The idea here seems to be that pictures have a structure which is identical to the structure of some (possible) fact. But suppose that we were to supplement the picture by adding an element which represents its form. This would change the form of the picture so that the added element no longer represents the structure of the picture of which it is an element, but rather that of some other picture.

(Possible objection: if the structure of a picture represents the structure of a state of affairs, then why doesn't the structure of every picture represent its own structure? One idea is that what he has in mind here is a distinction between the way that the structure of a picture represents and the way that the elements of the picture represent.)

2.2   Pictures, sense, and truth conditions (2.2-2.25)

Wittgenstein's next main point about pictures, presented in the 2.2's, follows naturally on the first. Given that a picture consists of elements and a structure which respectively represent objects and a way in which those objects can be combined, and given that states of affairs are combinations of objects, it is no surprise that

> 2.201   A picture depicts reality by representing a possibility of existence and non-existence of states of affairs.

The meaning, or sense, of a picture is the possible state of affairs that it represents. This can be seen as a version of the view that the meaning of a representation is its truth conditions: the conditions which would have to obtain for it to be an accurate representation of reality.

2.3   Thoughts, possibility, and the a priori (3-3.05)

The final step in Wittgenstein's discussion of pictures is to note a connection between what can be pictured and what is necessary and knowable a priori. Wittgenstein's idea is that whatever state of affairs can be pictured must also be possible. To see the plausibility of this, recall the analogy with the model cars in the traffic court. Could

we so arrange the cars that the model was a representation of a state of affairs which not only failed to obtain, but *could not* obtain? Wittgenstein thinks, plausibly, not.

One confusing part of his discussion here is that he expresses this conclusion in terms of what is thinkable, rather than in terms of what can be pictured:

> 3.02   A thought contains the possibility of the situation of which it is the thought. What is thinkable is possible too.

The idea here, I think, is that we can only think what we can picture to ourselves; so we can link the things which can be pictured with the things that can be thought. As Wittgenstein says:

> 3.001   'A state of affairs is thinkable': what this means is that we can picture it to ourselves.

This tells us something about what is impossible; but we also get a claim about what is necessary. But this comes via a claim about a priori knowability. Wittgenstein's first move is to link these two notions:

> 3.04   If a thought were correct a priori, it would be a thought whose possibility ensured its truth.

The idea is that any picture knowable a priori would also have to be necessary: for, if it were contingent, we would need empirical evidence to know whether the contingent state of affairs that it represents obtains. And a priori propositions, Wittgenstein claims (§3.05) would have to be recognizable from the form of the proposition itself. (More on this later.)

3   Propositions as pictures

At this point, Wittgenstein moves from a discussion of pictures to a discussion of sentences. One thought in the background seems to be that both pictures and sentences (or, as he says, 'propositions') represent facts; but it is easy to see how pictures do this, whereas it is more difficult to see how propositions can do it. Wittgenstein's idea was that the two work in roughly the same way: propositions are just disguised pictures of facts.

Wittgenstein was not the first philosopher to raise the question of how it is possible for us to represent the world. But he was one of the first to raise this question in the linguistic form which has received most attention this century: how can linguistic items represent the world?

3.1   Propositional signs, propositions, and the 'projective relation' (3.1-3.144)

Wittgenstein begins by saying a bit about some of the terms which will be central to his discussion of propositions. It is clear that the discussion of propositions is meant to parallel the discussion of pictures. Sometimes, this is represented in his numbering scheme, as in the following two claims:

> 2.14   What constitutes a picture is that its elements are related to one another in a determinate way.

> 3.14   What constitutes a propositional sign is that in it its elements (the words) stand in a determinate relation to one another. . . .

This expresses the central analogy between pictures and propositional signs: just as elements of a picture stand for objects, so the words in a propositional sign stand for objects. And just as the structure in which the elements of a picture are combined represent the structure of the state of affairs represented, so the structure in which the words in a propositional sign are combined represent the structure of the state of affairs represented.

This should cause some initial puzzlement. Recall the claim of §2.161 that a picture must have its form in common with the fact that it represents. This is perhaps not so difficult to understand in the case of a picture in which the spatial arrangement of elements of the picture mirrors the spatial arrangements of the objects represented by those elements. But how can a propositional sign share its form, or structure, with what it represents? We'll come back to this.

We can also now set aside a simplifying assumption we've been making so far: the idea that propositions are just sentences, or meaningful series of words. Wittgenstein distinguishes between a propositional sign and a proposition:

> 3.12   I call the sign with which we express a thought a propositional sign.
> - And a proposition is a propositional sign in its projective relation to the world.

So propositional signs, not propositions, are sentences. But what we have to under-stand in order to understand propositions, and teh way that they represent facts, is what this projective relation is supposed to be.

Wittgenstein never says explicitly what he has in mind here. I think that the closest he comes is a bit later in the text, when he says:

> 4.0141 There is a general rule by means of which the musician can obtain the symphony from the score, and which makes it possible to derive the symphony from the groove on the gramophone record, and, using the first rule, to derive the score again. That is what constitutes the inner similarity between these things which seem to be constructed in such entirely different ways. And that rule is the *law of projection* which projects the symphony into the language of musical notation. ...

(Emphasis mine.) A projective relation is something like a rule for moving from one kind of thing to another - from a score to a symphony or, in the case of propositions, from a propositional sign to a fact. It is worth pointing out an important difference here between Wittgenstein's discussion of pictures and his discussion of propositional signs. Both propositional signs and pictures are a certain kind of fact: facts which represent other facts. In the case of pictures, Wittgenstein discusses the structure and elements of pictures and their relation to the facts the picture represent; he never mentions projective relations in this discussion. And there is a good reason for this. Whereas in the case of pictures Wittgenstein could rely on an identity between the form of the picture and the form of the fact represented, there is clearly no such identity in the case of propositional signs; it is unclear what it would even mean to say that a propositional sign has the same form as a fact. This is why Wittgenstein must invoke the notion of a projective relation, and why he talks about the logical forms of propositions (which are propositional signs + their projective relations) rather than just about the logical forms of propositional signs.

3.2   Names (3.2-3.263)

Recall that in the case of pictures, representation was a matter of elements of the pic-ture standing for objects arranged according to the structure of the picture. Wittgen-stein's claim about identity of structure gives us some grip on how half of this picture of representation works; but it does not explain how elements of a picture get cor-related with their objects. Just so, in the case of propositions, we have as yet no explanation of how names are correlated with their objects. It seems as though this too must be explained in terms of the projective relation. This may be the point of the following claim:

> 3.2   In a proposition a thought can be expressed in such a way that el-ements of the propositional sign correspond to the objects of the thought.

The idea here is that the projective relation is a certain way of using or thinking the elements of (names in) the propositional sign in such a way that they correspond to

objects. Wittgenstein does seem to think that names become correlated with objects in virtue of the use of propositional signs in which those names occur:

> 3.3 Only propositions have sense; only in the nexus of a proposition does a name have a meaning.

This may go some distance toward explaining the asymmetric relation that holds between names and objects, since objects are never used in propositions. But, by itself, this is not completely satisfying: it seems to leave the relationship between names and their objects mostly unexplained. What we want to know is: in virtue of what is a given propositional sign associated with a given rule of projection?

I think that Wittgenstein later came to see that this is a problem. In his later work, he spends a lot of time addressing the question of how a given rule gets associated with a certain representation. (See *Philosophical Investigations* §§138-242; for an important contemporary discussion, see Kripke, *Wittgenstein on Rules and Private Language.*

The outlines of Wittgenstein's discussion here are, I think, somewhat plausible. But for more detail on how propositional signs and propositions represent facts, we'll have to wait for the discussion of the theory of elementary propositions.

4 Logical form as unsayable

But at this stage in the text Wittgenstein turns away from the topic of propositions in general to a discussion of logical form before returning later (beginning roughly in §4.2) to the crucial topic of elementary propositions.

4.1 Logical form, notation, and self-reference (3.3-3.5)

Somewhat puzzlingly, Wittgenstein begins this discussion by discussing logical notation, or logical symbols. (He distinguishes between a sign and a symbol; this seems to be roughly the same as the distinction between a propositional sign and a proposition.)

Wittgenstein here draws a sharp distinction between everyday language and a logical language. Everyday language contains ambiguities; logical language, to avoid the confusions this causes, will not (§3.323). This amounts to the suggestion that we avoid the fundamental confusions of which philosophy is allegedly full (§3.324) by constructing a new notation for the expression of our thoughts:

> 3.325 In order to avoid such errors we must make use of a sign-language that excludes them ...

A background thought here is that we cannot trust ordinary language. Just as we cannot trust the fact that the same sign occurs in 'x is good' and 'x is x' and 'x

is' so we cannot trust the fact that certain kinds of propositions seem, in ordinary language, to make sense.

Among these are certain paradoxical sentences which, in a certain way, make reference to themselves. Consider, for example, the following claim:

> There is a set which contains all the sets which are not members of themselves.

This is one way of expressing the claim which leads to 'Russell's paradox.'

One way to block the paradox, which Wittgenstein seemed to favor, was to restrict our notation in a certain way. He says:

> 3.332   No proposition can make a statement about itself, because a propositional sign cannot be contained in itself. . . .

The next proposition (§3.333) expresses more directly the kind of restriction Wittgenstein has in mind: no function can be part of its own argument. Application of this to the theory of propositions. The phrase 'the set of all non-self-membered sets' as violating this rule. This phrase as being, like ambiguities, a confusing artifact of our everyday language: it is a phrase which seems to make sense, but in fact does not.

A question about Wittgenstein's view here: is this a prescriptive claim about how our notation should be modified, or a claim about what it is possible for any logical notation to express?

4.2   Everyday language and philosophical problems (4-4.0031)

Here Wittgenstein just touches on a theme to which he will return later, both in the *Tractatus* and in his later work: the idea that philosophical problems are linguistic confusions to be dissolved by the analysis of language. Comparison to Russell's treatment of the problem of negative existentials and Wittgenstein's treatment of Russell's paradox.

4.3   Pictures, propositions, and projection (4.01-4.0641)

[We'll be skipping this section; it is partly a spelling out of the idea of a projective relation which we discussed earlier, and partly a discussion of issues about logic to which Wittgenstein returns later.]

4.4   Science vs. philosophy and saying vs. showing (4.1-4.128)

Again in this section, Wittgenstein introduces a theme to which he returns near the end of the book: the distinction between saying and showing, and the related distinction between science and philosophy.

It emerges in this section that Wittgenstein thinks of philosophy as a kind of process of clarification, whose subject matter (insofar as it has one) is the logical form of our propositions and thoughts, rather than their subject matter:

> 4.112   Philosophy aims at the logical clarification of thoughts.

The idea here is that philosophy is not interested in which facts about the world happen to obtain, but only about the form, or nature of those facts (including the forms of those facts by which we represent the world). This is in some ways a traditional view of philosophy. To know which facts about the world obtain, one would have to engage in some sort of a posteriori investigation of the world. But philosophy is an a priori discipline whose results, therefore, must concern the form of any possible fact rather than the particulars of actual facts.

Given this conception of philosophy, Wittgenstein sets out a thesis with which many philosophers now would disagree: science is not only distinct from philosophy, but completely irrelevant to it. Wittgenstein states this thesis in no uncertain terms, using as examples two kinds of scientific theories which many have thought *do* have importance for philosophy:

> 4.1121 Psychology is no more closely related to philosophy than any other natural science. . . .

> 4.1122 Darwin's theory has no more to do with philosophy than any other hypothesis in natural science.

These are not just bald assertions, but follow from Wittgenstein's view of philosophy's

aim. If philosophy does really aim at giving the form of any possible fact, then what use can it have of the results of scientific theories which do no more than enumerate and systematize actual facts?

But here a kind of paradox arises. Wittgenstein thinks that it is the job of philosophy to tell us what the logical forms of our thoughts (and so of all possible facts) are. But doesn't this just mean that philosophy is interested in a particular kind of fact: facts about the logical forms of facts? And in this sense, isn't philosophy just like a science (even if its methods are quite different)?

Wittgenstein thinks not:

> 4.12 Propositions can represent the whole of reality, but they cannot represent what they must have in common with reality in order to be able to represent it - logical form.
> In order to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world.

Recall that Wittgenstein thinks that there is a very tight connection between language and the world; indeed, he seems to think that the following *fact-proposition equivalence* holds:

> x is a possible fact if and only if there is some proposition which has x as its sense.

So when Wittgenstein says here that there is no proposition which can represent logical form, we can take him as also committed to the claim that there are no facts about logical form. And, if philosophy is concerned with logical form, it follows that there is no class of facts which philosophy studies. So there is a tight connection between Wittgenstein's view of the status of logical forms and his view of the relationship between philosophy and science.

But, one wants to know: what is the basis for Wittgenstein's claim that there are no facts about logical form? The following are two possible arguments which may be extracted from the text for this conclusion.

The argument from the impossibility of self-reference. §4.12 makes it seem as though part of the story is that there is a kind of incoherence in a certain kind of self-reference – this is a thought which we already encountered above, in Wittgenstein's brief discussion of Russell's paradox.

But even if we grant this point about self-reference, this does not show that we could not state any propositions about logical form. The point about self-reference would get us the conclusion that no proposition can state its own logical form; but wouldn't it still be possible for there to be some propositions that could state the logical forms of other propositions?

Perhaps the idea is this: if we grant the point about self-reference, then there would have to, on pain of infinite regress, be *some* propositions such that no proposition

stated their logical form. But it would be absurd to say that there are facts about the logical forms of some propositions but not about the logical forms of others. So it follows from the fact-proposition equivalence that, if there are some propositions whose logical forms are not stated by any other proposition, there are no facts about the logical forms of any propositions.

The argument from internal properties. A different kind of argument for the view that there are no facts about the logical forms of propositions is suggested by some of Wittgenstein's difficult remarks about internal and external properties in this section. Consider especially the following:

> 4.122 In a certain sense we can talk about formal properties of objects and states of affairs, or, in the case of facts, about structural properties: and in the same sense about formal relations and structural relations. (Instead of 'strutural property' I also say 'internal property'; instead of 'structural relation', 'internal relation.')
> . . .
> It is impossible, however, to assert by means of propositions that such internal properties and relations obtain: rather, this makes itself manifest in the propositions that represent the relevant states of affairs and are concerned with the relevant objects.

This indicates that the logical forms of propositions and other facts, and the relations between propositions and the facts they represent, are, respectively, internal properties and relations. A bit later Wittgenstein adds:

> 4.123 A property is internal if it is unthinkable that its object should not possess it. . . .

This means that, if an object has an internal property, it is a necessary truth that it has that property (and also that, if two objects or facts stand in a certain internal relation, that it is a necessary truth that they stand in that relation). But, as will become clearer later, there is a sense in which Wittgenstein does not believe that there are any *facts* which are necessary. Indeed, one of the explanatory ambitions of the *Tractatus* seems to be to explain a host of metaphysical notions, including necessity and possibility, in terms of the sparse metaphysical resources provided by a set of mutually independent contingent states of affairs. To admit that there are facts about logical forms (while holding to the plausible thesis that these facts are internal in Wittgenstein's sense) would be to give up that ambition.

Suppose that we grant that there are no facts about logical forms, and that it is the business of philosophy to investigate logical forms. Given this, it seems to follow that, since the *Tractatus* is a philosophical work, it must not contain any propositions which state facts. But then what are all the sentences in the book which seem to state facts doing?

The short answer is that these sentences do not, strictly speaking, say anything: their function is to *show* us what we need to know about logical forms. As Wittgenstein puts it:

> 4.114　[Philosophy] will signify what cannot be said, by presenting clearly
> what can be said.

This touches on the central paradox of the *Tractatus*. We will return to it at the end of the book.

5　The theory of elementary propositions

The heart of the *Tractatus* is Wittgenstein's theory of propositions; and the heart of his theory of propositions is his theory of elementary propositions. Beginning in §4.2, Wittgenstein turns to this topic.

5.1　Elementary propositions and states of affairs (4.2-4.28)

Given Wittgenstein's repeated reliance on the existence of a correspondence between language and the world, it should be no surprise that his philosophy of language – i.e., his theory of propositions and their constituents – mirrors his metaphysics – i.e., his theory of facts and their constituents. Just as the facts are all determined in some sense by a class of basic facts – states of affairs – so propositions are all, in a sense to be explained, determined by a class of basic propositions – the elementary propositions.

Elementary propositions stand for states of affairs:

> 4.21　The simplest kind of proposition, an elementary proposition, asserts
> the existence of a state of affairs.

So just as all states of affairs are independent of all other states of affairs, all elementary propositions are independent of all other elementary propositions:

> 4.211　It is a sign of a proposition's being elementary that there can be no
> elementary proposition contradicting it.

This is already enough to show that elementary propositions are not the sorts of propositions with which we are familiar, and are certainly not the same kinds of things as Russell's atomic propositions. Consider, for example, two apparently simple claims:

> That is red.
>
> That is exactly 6 feet tall.

Claims like these about the colors and sizes of objects cannot be elementary, since they are contradicted by other propositions about the colors and sizes of the same objects:

That is green.

That is exactly 5 feet tall.

Another clue that elementary propositions are different than the propositions with which we are familiar is Wittgenstein's claim that

> 4.22   An elementary proposition consists of names. It is a nexus, a concatenation, of names.

This obviously parallels Wittgenstein's thesis about the nature of states of affairs:

> 2.03   In a state of affairs objects fit into one another like the links of a chain.

The idea seems to be that names stand for objects, and so that, since a state of affairs is just an arrangement of objects and elementary propositions stand for states of affairs, an elementary proposition will be an arrangement of names. Relevance of the picture theory of representation – do we have methods of projection here?

We know that elementary propositions are correlated with states of affairs; it follows that

> 4.3   truth possibilities of elementary propositions mean possibilities of existence and non-existence of states of affairs.

There is also a connection between Wittgenstein's view about the relationship between elementary propositions and other propositions, and his view about the relationship between states of affairs and other facts. If elementary propositions are matched up one-to-one with states of affairs, then a list of all the true elementary propositions will state all there is to know about the states of affairs. And if the states of affairs determine all the facts, then it seems that all the true elementary propositions will say all that there is to be said about the facts. But we know further that every proposition has as its sense some (possible) fact – and whether a proposition is true depends on whether the fact is actual. So it must be that the list of all the true elementary propositions determine the truth values of all the propositions.

Wittgenstein expresses this conclusion when he says,

> 4.41   Truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions.

5.3   Tautologies and contradictions (4.46-4.4661)

But this leaves open the question of what we should say about those propositions which are true, or false, no matter what is the case. Wittgenstein calls the former 'tautologies' and the latter 'contradictions' (§4.46).

There is a sense in which these kinds of claims do not say anything; as Wittgenstein rightly says, "...I know nothing about the weather when I know that it is either raining or not raining" (§4.461). But neither are they nonsensical:

> 4.4611 Tautologies and contradiction are not, however, nonsensical. They are part of the symbolism, much as '0' is part of the symbolism of arithmetic.

It is not transparent what the analogy here is supposed to be. But the basic point is that tautologies and contradictions are a kind of artifact of the ways in which we can combine propositions - they are the kind of limiting case in which the propositions combined cancel each other out.

Connection of this with the doctrine that meanings are truth conditions, and the claim of §4.465 that "the logical product of a tautology and a proposition says the same thing as the proposition."

How the identification of meanings with truth conditions ca begin to explain the sense in which all propositions can be analyzed in terms of elementary propositions.

6   Elementary propositions as the foundation of all propositions

6.1   Propositions as 'generalizations of elementary propositions' (4.5-5.02)

6.2   Elementary propositions and the foundations of probability (5.1-5.156)

Here we come across an interesting case study: propositions about the probabilities of events. One of Wittgenstein's explanatory ambitions is to explain the nature of probability: to say what it is that we are saying when we say that an event has a certain probability.

He begins with the claim

> 5.153   In itself a proposition is neither probable nor improbable. Either an event occurs or it does not: there is no middle way.

It is difficult to see what the second half of this claim does to justify the first half. Here's one idea: states of affairs either obtain or do not obtain. So when we say that a certain event has a certain probability, we are not making a claim that a certain state of affairs obtains: the state of affairs of such and such having x chance of happening. Rather it must be the case that "probability is a generalization" (§5.156).

How the apparatus of elementary propositions may be used to support this claim.

6.3   Internal relations between propositions are the result of truth-functions (5.2-5.32)

We know that elementary propositions are all independent of each other – so we know that there are no internal relations between elementary propositions. But some propositions do have internal relations to each other – and this is a phenomenon that Wittgenstein thinks that we can explain in terms of the ways that elementary propositions are combined.

6.4   Logical constants (5.4-5.476)

6.5   Joint negation (5.5-5.5151)

We know by now that Wittgenstein thinks that all propositions are truth-functions of elementary propositions. In these sections he points out that from this we can get the result that all propositions are the result of repeated applications of a certain operation, which Wittgenstein symbolizes with the letter 'N', and which is sometimes called 'joint negation.' Wittgenstein explains joint negation as follows:

> 5.51   If $\bar{\xi}$ has only one value, then $N(\xi) = \neg p$ (not p); if it has two values,
> then $N(\xi) = \neg p.\neg q$ (neither p nor q).

Here 'ξ' is a variable which has propositions as its value; the expression '$N(\bar{\xi})$' signifies the complex formula obtained by negating each proposition which can be substituted for 'ξ' and conjoining them. If there is one such proposition, the result is its negation; if there are two such propositions, the result is the negation of the first conjoined with the negation of the second; and so on.

To get an idea of how one can define other truth-functions in terms of joint negation, consider the following translations;

Table 1: Wittgenstein's operator 'N'

$\neg p$	$N(p)$
$p \ \& \ q$	$N[N(p), N(q)]$
$p \lor q$	$N(N[p, q])$

6.6   The general form of the proposition: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ (6-6.031)

Wittgenstein then uses joint negation to state the result toward which his theory of propositions has been working: his statement of the general form of the proposition.

> 6   The general form of a truth-function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$.
> This is the general form of a proposition.

> 6.001   What this says is that every proposition is a result of successive ap-
> plications to elementary propositions of the operation $N(\xi)$.

This is just a succinct way of stating the combination of the two theses that all propositions are truth-functions of elementary propositions, and that all truth-functions can be defined in terms of joint negation. What the formula in §6 says is that every proposition may be generated by the following procedure: begin with the class of elementary propositions; apply joint negation to some subset of those propositions to arrive at a new proposition; apply join negation again to some subset of the set of propositions consisting of the elementary propositions plus the one just obtained; apply joint negation again . . . and so on and so on.

7 Challenges to the view that all propositions are truth-functions of elementary propositions

We have interrupted our sequential exposition the *Tractatus* to jump ahead to the statement of the general form of the proposition. On the way to this conclusion, Wittgenstein considers three kinds of propositions which challenge his view that all propositions are truth-functions of elementary propositions.

7.1 Generality (5.52-5.5262)

One initially problematic class of propositions is the class of general propositions, including those that we would symbolize as

$$\forall x \; Fx$$

Such propositions are problematic because there is an intuitive problem in representing them as truth-functions of propositions about particular objects. The only plausible way to analyze such general propositions seems to be as long conjunctions:

$$Fa \; \& \; Fb \; \& \ldots \& \; Fn$$

where a-n collectively name each object. But it seems as though the conjunction is not equivalent to the general claim. After all, it seems that there could have been some other object which doesn't actually exist and so is not named by any of the terms in the long conjunction - the general statement, but not the conjunction, will then imply that this object is F.

Nonetheless, Wittgenstein claims that general claims are truth-functions of elementary propositions:

> 5.52 If ξ has as its values all the values of a function fx for all values of x, then $N(\bar{\xi}) = \neg(\exists x).fx$.

Remember the definition of 'N'. The idea is that if we consider the set of all propositions of the form 'fx', then the joint negation of that set – the conjunction of the negations of every member of the set – will be equivalent to the claim that there is no object which is 'f'. The importance of this point for general propositions follows from the fact that the negation of an existential claim like this one is a general proposition: the claim that everything is 'not-f.' So here Wittgenstein claims that we can analyze this general claim as, in effect, a long conjunction of the negations of attributions of 'f' to particular objects.

From this it is but a short step to the claim that we can analyze all general claims in similar terms. If we can analyze

$$\forall x \neg Fx$$

as the joint negation of a set S of propositions, then we can analyze

$$\forall xFx$$

as the joint negation of the set of propositions consisting of the negations of every member of S. This means that Wittgenstein seems to basically endorse the strategy of analyzing general claims in terms of conjunctions which we found wanting above. How does he deal with this problem?

The answer to this problem comes from another part of Wittgenstein's system. The problem for the conjunctive analysis of general claims seemed to be tied to the possibility of the truth conditions of the general claim and its analysis diverging in cases where one possible situation contains an object which does not exist in the other possible situation. But recall that for Wittgenstein, possibility and necessity are alike explained in terms of a stable supply of objects. So, according to Wittgenstein's metaphysics, the problematic possibility seems to be impossible.

7.2 Identity (5.53-5.5352)

A second class of claims which pose a problem for Wittgenstein's theory of propositions are identity statements like

$$a = b$$

and statements of non-identity like

$$a \neq b$$

To see why these claims are problematic, it's important to note that there is no place in Wittgenstein's system for claims about how many objects exist. Wittgenstein realized this:

> 4.1272 . . .
>> So one cannot say, for example, 'There are objects', as one might say 'There are books.' And it is just as impossible to say, 'There are 100 objects' . . .
>> And it is nonsensical to speak of the *total number of objects*.

Recall that Wittgenstein thought that all propositions were truth-functions of elementary propositions. Further, he thought that elementary propositions were all contingent, and independent of all other elementary propositions. This seems to indicate that every truth-function of elementary propositions – and hence every proposition

– must fall into one of three categories: (i) contingent claim, (ii) tautology (logical truth), or (iii) contradiction.

The problem is that claims about how many objects there are do not fit comfortably into any of (i)-(iii). First, they seem to be neither tautologies nor contradictions, for two reasons: they seem to be of the same form as paradigmatically non-logical claims like 'There are 100 books', and, while tautologies like 'Either it is raining or it is not raining.' say nothing, existence claims like 'There are 100 objects' do seem to make a claim about the world.'

So if claims like 'There are 100 objects are to fit into Wittgenstein's system, they must be contingent claims. But there are two reasons why this does not work, either. First, the same objects exist across all possible ways the world could be; so the claim that there are 100 objects, if true, is necessarily rather than contingently true. Second, all contingent claims are, according to Wittgenstein, made true by objects combining in certain ways; but the claim that there are 100 objects does not seem to require anything about the *combination* of objects.

The problem with statements of identity and non-identity is that they provide a way to formulate claims about how many objects there are. Consider, for example, the following claims about how many objects there are and their translations into claims of identity and non-identity:

There is at least one object.	$\exists x(x = x)$
There are at least two objects.	$\exists x \exists y(x \neq y)$
There is exactly one object.	$\exists x \forall y(x = y)$
There are exactly two objects.	$\exists x \exists y \forall z(x \neq y \ \& \ (z = x \lor z = y))$

Each of these claims would, for the above reasons, be problematic from the perspective of the *Tractatus*.

Wittgenstein's response is to claim that propositions about identity and non-identity are only pseudo-propositions:

> 5.533 The identity-sign, therefore, is not an essential constituent of conceptual notation.

> 5.534 And now we see that in a correct conceptual notation pseudo-propositions like '$a = a$' …cannot even be written down.

> 5.535 This also disposes of all the problems that were connected with such pseudo-propositions. …

It is one thing to say this; but it does seem as though there are propositions which are *not* pseudo-propositions but which we would naturally formalize using the identity sign. Consider, for example,

'Someone loves no one besides herself.'

which would naturally be analyzed as

$$\exists x \forall y \ (x \text{ loves } y \to x = y)$$

If Wittgenstein's rejection of statements of identity is to be plausible, he must find some other way of analyzing such claims. His suggestion is contained in the following:

> 5.53 Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs.

Wittgenstein's suggestion is, in effect, that we reform our logical vocabulary, so that we use different variables only for different objects. So we could analyze the above claim about as

$$\exists x \forall y \ \neg \ (x \text{ loves } y)$$

But there is no way, it may seem, to give an analysis of 'There are at least two objects' which does not make use of the identity sign. So it may seem that Wittgenstein's notation does exactly what we would want it to.

How about the claim that there is at least one object – isn't this a necessary truth, and can't we express it without use of the identity sign as follows:

$$\exists x \ (x \text{ is an object})$$

Wittgenstein's response to this kind of claim is similar: 'object' does not express a real concept, but only a pseudo-concept.

One might still wonder whether every claim which we should be able to express will be analyzable using this logical notation. Consider, for example, the sentence 'Someone loves someone.' Intuitively, this sentence could be true either because someone loves himself or because someone loves someone else – the sentence is simply noncommittal on the relation between the person doing the loving and the person loved. The problem is that there seems no way to analyze this claim which remains similarly noncommittal. If, in Wittgenstein's system, we analyze the sentence as

$$\exists x \exists y \ (x \text{ loves } y)$$

then we are committed to someone loving someone *else* - and this falsifies the meaning of the original sentence. If we analyze it as "$\exists x \ (x \text{ loves } x)$" then we face the opposite problem. The best we seem to be able to do is the slightly unnatural

$$\exists x \exists y \ (x \text{ loves } y) \lor \exists z (z \text{ loves } z)$$

But there is a further worry: even granting that claims about how many objects there are are pseudo-propositions, can we generate other claims which will pose the same problem — of being necessary truths which are not plausibly regarded as logical truths?

Consider, e.g.:

Quantification over properties:

$\exists F \exists x (Fx)$

(This says, intuitively, 'Something is some way.' This is not plausibly a logical truth, but it may not seem to be contingent either.)

7.3   Psychology (5.54-5.5423)

The third and final challenge to Wittgenstein's view that all propositions are truth-functions of elementary propositions which we will consider are propositions about psychological facts, like

"John believes that grass is green."

It looks as though the proposition "Grass is green." is a part of this proposition about John. If this is so, as it seems, then the proposition about John must be a truth-function of this proposition about grass; this is, after all, the only way for one proposition to be a part of another.

However, if a proposition p is a truth-function of another proposition q (along with, perhaps, other propositions) it should be possible to substitute another proposition q* for q without changing the truth-value of p, so long as q and q* have the same truth-value. (This is just what it means to say that one proposition is a *truth*-function of another.)

But in the case of propositions about an agent's psychology, we cannot perform this kind of substitution. For consider: even though 'Grass is green.' and 'There are an infinity of prime numbers.' both have the same truth-value (true), it does not follow from

"John believes that grass is green."

that

"John believes that there are an infinity of prime numbers."

It follows that, in general, propositions of the form 'A believes that S' are not truth-functions of the propositions substituted for 'S.' But those poses a problem for

Wittgenstein's view that the only way for one proposition to be a part of the analysis of another is via truth-functional combination.

Wittgenstein saw that this was a problem:

> 5.541   At first sight it looks as if it were also possible for one proposition to occur within another in a different way.
> Particularly with certain forms of proposition in psychology, such as 'A believes that p is the case' and 'A has the thought p', etc.
> For if these are considered superficially, it looks as if the proposition p stood in some kind of relation to an object A.

Wittgenstein was inclined to solve this problem by rejecting this analysis – by rejecting the thought that 'p' is a part of these propositions about psychology.

> 5.542   It is clear, however, that 'A believes that p', 'A has the thought p' and 'A says p' are of the form '"p" says p': and this does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.

It is not easy to see what Wittgenstein is saying here. Here's one interpretation: recalling his previous claim that propositions are facts, when he denies that claims about psychology express correlations of a fact with an object, he is denying that such claims correlate an agent with a proposition. This expresses the denial of the view that the propositions which (seem to) come after 'that' in such claims are parts of such claims.

His positive suggestion is harder to understand: that these claims ar correlations of facts by means of the correlation of their objects. Perhaps the idea is that claims like 'A believes that S' can be thought of as analyzable into two parts: 'A says 'S'' and '"S' says that S.' The problem here is that the second claim would be a claim about representation which is, according to Wittgenstein, unsayable.