Ayer and Quine on the a priori

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1 The problem of a priori knowledge

Ayer's book is a defense of a thoroughgoing empiricism, not only about what is required for a belief to be justified or count as knowledge, but also about what is required for a sentence to have a meaning at all. But a priori knowledge seems to pose a problem for the view that all knowledge, and thought, is based in experience:

> "Having admitted that we are empiricists, we must now deal with the objection that is commonly brought against all forms of empiricism; the objection, namely, that it is impossible on empiricist principles to account for our knowledge of necessary truths. . . .
>
> . . . whereas a scientific generalization is readily admitted to be fallible, the truths of mathematics and logic appear to everyone to be necessary and certain. But if empiricism is correct no proposition which has a factual content can be necessary or certain. Accordingly the empiricist must deal with the truths of logic and mathematics in one of the following two ways: he must say either that they are not necessary truths, in which case he must account for the universal conviction that they are; or he must say that they have no factual content, and then he must explain how a proposition which is empty of all factual content can be true and useful and surprising." (72-3)

There are a number of hidden premises behind the formulation of this dilemma. Recall that for the empiricist of Ayer's kind, a proposition has meaning (factual content) only by being associated with certain sense experiences/observation sentences. But such a proposition can only be known by knowing the truth of these observation sentences; and such knowledge is always a posteriori. So it looks like any proposition with factual content will be a posteriori and contingent; thus the dilemma.

As Ayer notes, this dilemma has important consequences:

> "If neither of these courses proves satisfactory, we shall be obliged to give way to rationalism. We shall be obliged to admit that there are some truths about the world which we can know independently of experience ... [a]nd we shall have to accept it as a mysterious inexplicable fact that our thought has this power to to reveal to us authoritatively the nature of objects which we have never observed." (73)

This state of affairs would be unsatisfactory; we would then have a kind of failure of philosophy to explain some of the most interesting facts about our mental lives. But it would also have strategic consequences for Ayer's attack on metaphysics:

> "It is clear that any such concession to rationalism would upset the main argument of this book. For the admission that there are some facts about the world which could be know independently of experience would be incompatible with our fundamental contention that a sentence says nothing unless it is empirically verifiable." (73)

The idea here is that if we admit the possibility of non-empirical knowledge, we thereby admit that we can have some non-experiential access to facts about the world. But, once this is admitted, there seems no reason to believe that a sentence can have meaning *only* by bearing a certain relation to observation sentences.

2 Necessity and the a priori

Ayer in this chapter constantly switches back and forth between talking about which propositions are knowable a priori and which propositions are necessary. Though this was not widely recognized in Ayer's time, it is important to note that these two categories are at least conceptually distinct. To say that a sentence expresses a necessary truth is to say that, no matter how the world had turned out, what that sentence says could not have been false. To say that a sentence expresses an a priori truth is to say that one can know what the sentence says to be true without relying for one's justification on any experience of the world.

Ayer pretty clearly assumes that a claim is necessary if and only if it is a priori. There are a few intuitively appealing arguments that make this position plausible:

If a proposition is a priori, it must be necessary. If a proposition is a priori, then one can know it to be true without any experience of the world. But if one can know a proposition to be true without any experience of the world, then the truth of that proposition must not depend on any contingent features of the world – for, if it did, one would have to check whether those contingent features of the world in fact obtained. But in that case it would be a posteriori.

If a proposition is necessary, it must be a posteriori. If a proposition is necessary, then it is true independently of the way the world happens to be. But then how can it be necessary for experience – which only delivers information about how the world happens to be – to play any role in explaining how we can know that proposition?

There are grounds for doubting both of these arguments. But we will for now take the plausibility of these arguments at face value, and follow Ayer in accepting them. The important point at present is that, strictly, Ayer has two distinct facts to explain: (i) our ability to know the propositions of logic and mathematics a priori, and (ii) the necessity of the propositions of logic and mathematics.

3 Mill's radical empiricism

An example of a philosopher who took the first horn of Ayer's dilemma for the empiricist was John Stuart Mill, who (at least on Ayer's interpretation) regarded the truths of logic and mathematics to be both a posteriori and contingent. On this interpretation, Mill thought of these propositions as being empirical generalizations of which we could be fairly certain because of the large number of observed instances which confirm them. But they are not necessary, since they could in principle be false; and they are not a priori, since we know them to be true on the basis of observation. (In the case of mathematics, the observation in question might be observation of quantities of things.)

Ayer argues that Mill mistakes the nature of propositions of mathematics. These are, according to Ayer, special propositions; we do not confirm them to be true by observation, but rather *stipulate* that they are true. He says,

> "The best way to substantiate our assertion that the truths of formal logic and pure mathematics are necessarily true is to examine cases in which they might seem to be confuted. . . . [In such cases] one would adopt as an explanation whatever empirical hypothesis fitted in best with the accredited facts. The one explanation which would in no circumstances be adopted is that ten is not always the product of two and five. . . . And this is our procedure in every case in which a mathematical truth might appear to be confuted. We always preserve its validity by adopting some other explanation of its occurrence.
>
> . . . The principles of mathematics and logic are true universally simply because we never allow them to be anything else."

This indicates that such principles are different in kind than simple empirical generalizations. The thought suggested by this passage is that we simply stipulate that these claims are true: we say that they are to mean whatever is required for them to be true.

Ayer tried to capture this by saying that the truths of logic and mathematics were *analytic*, in a sense which could explain their status as a priori. Our next task is to understand this explanation of the a priori.

4 Ayer's linguistic explanation of the a priori

4.1 Analyticity as truth by definition

Ayer defines analyticity as follows:

> "...a proposition is analytic when its validity depends solely on the definitions of the symbols it contains, and synthetic when its validity is determined by the facts of experience." (79)

Immediately after this, though, Ayer seems to define analyticity in terms of a prioricity; he says "the proposition 'Either some ants are parasitic or none are' is an analytic proposition. For one need not resort to observation to discover that there are or are not ants which are parasitic" (79). But it seems that the most charitable reading is to regard this as a mis-step: it is analyticity that is brought in to explain the a priori, not the other way around. We should regard Ayer's account of analyticity as truth by definition as the fundamental one.

4.2 How the analyticity of a propositon can explain its a prioricity

Suppose that Ayer is right, and that all truths of mathematics are true by definition. How could this explain their a prioricity?

The idea is that to understand a proposition which is true by definition, one must know the definitions of the relevant terms. And, in the case of analytic sentences which are true by definition, this knowledge of the definitions of terms is enough to show that they are true. Ayer seems to give this kind of explanation when he says:

> "If one knows what is the function of the words 'either,' 'or,' and 'not,' then one can see that any proposition of the form 'Either *p* is true or *p* is not true' is valid." (79)

The basic idea here seems to be that knowing the function of words – in particular, knowing their definitions – can, in the case of analytic propositions, be enough to know the truth of a sentence.

4.3 How can analytic truths be surprising?

One of the intuitive facts which stands in the way of a treatment of all mathematical and logical propositions as having no factual content is the fact that these propositions can often be surprising. How can we account for this, if to learn the truth of a mathematical proposition is not to learn about some new and surprising fact?

Ayer says:

> "When we say that analytic propositions are devoid of factual content, and consequently that they say nothing, we are not suggesting that they are senseless in the way that metaphysical utterances are senseless. For, although they give us no information about any empirical situation, they do enlighten us by illustrating the way in which we use certain symbols. ...there is a sense in which analytic propositions do give us new knowledge. They call attention to linguistic usages, of which we might not otherwise be conscious, and they reveal unsuspected implications in our assertions and beliefs." (79-80)

Ayer is suggesting that, since analytic truths are true in virtue of certain linguistic facts – the definitions of expressions in analytic sentences – coming to know an analytic truth can bring us to awareness of these linguistic facts.

But, one might ask, even if this is so, how can definitions surprise us? Aren't the linguistic facts in question trivial ones that everyone knows? In the end of this passage, Ayer offers an answer to this question: even if we know the definitions in question, the definitions might have consequences which we do not immediately recognize. Ayer expands on this point later:

> "The power of logic and mathematics to surprise us depends, like their usefulness, on the limitations of our reason. A being whose intellect was infinitely powerful would take no interest in logic and mathematics. For he would be able to see at a glance everything that his definitions implied, and, accordingly, could never learn anything from logical inference which he was no fully conscious of already. But our intellects are not of this order." (85-6)

4.4 Sentences about linguistic rules and sentences true in virtue of linguistic rules

This doctrine gives rise to a puzzle, though. Analytic sentences are supposed to be necessary ('universally valid'); but facts about linguistic rules are contingent. After all, we could have decided to use expressions in our language differently and, in particular, could have defined various expressions differently. So if analytic sentences are about linguistic rules, how can they be necessary (as they must be, if mathematical and logical truths are to be analytic)?

Ayer gives his answer to this puzzle in the Introduction to the 2d edition of *Language, Truth, & Logic*:

> "It has, indeed, been suggested that my treatment of *a priori* propositions makes them into a subclass of empirical propositions. For I sometimes seem to imply that they describe the way in which certain symbols are used, and it is undoubtedly an empirical fact that people use symbols in the way that they do. This is not, however, the position that I wish to hold ... For although I say that the validity of *a priori* proposition depends upon certain facts about verbal usage, I do not think that this is equivalent to saying that they describe these facts ...
>
> ... [An analytic] proposition gives no information in the sense in which an empirical proposition may be said to give information, nor does it itself prescribe how [the terms in question are] to be used. What it does is to elucidate the proper use of [these terms]; and it is in this way that it is informative." (16-17)

One might say that, in Wittgenstein's terminology, analytic propositions show the way that certain symbols are used, but do not say that they are used that way. They are informative in virtue of showing this.

We can also understand Ayer's point in terms of the distinction between what sentences mean, and the information that those sentences can be used to convey on certain occasions of use. Ayer's point can be seen as a special case of the general point that the latter often exceeds the former.

5 Quine's first critique: "Truth by convention"

One of the puzzling aspects of Ayer's discussion is that although he seems to lay great weight on the notion of truth in virtue of definitions and knowability in virtue of knowledge of definitions, he says very little about what definitions are. Quine's article "Truth by Convention" is an attack on the explanation of a prioricity in terms of analyticity which takes as its starting point the nature of definition. Indeed, Quine claims that this view of mathematics and logic hardly makes sense:

> "... developments of the last few decades have led to a widespread conviction that logic and mathematics are purely analytic or conventional. It is less the purpose of the present inquiry to question the validity of this contrast than to question its sense." (70)

Quine begins by explaining one clear sense in which a sentence may be true by definition:

> "A definition, strictly, is a convention of notational abbreviation. . . . Functionally a definition is not a premise to a theory, but a license for rewriting theory by putting definiens for definiendum or vice versa. By allowing such replacements definition transmits truth: it allows true statements to be translated into new statements which are true by the same token." (71)

This shows us one way to define truth by definition in a relative sense. One sentence S can be true by definition relative to another sentence $S*$ if (i) $S*$ is true and (ii) S can be obtained from $S*$ by putting definiens for definiendum or vice versa. As Quine suggests, perhaps we could view truths of mathematics as true by definition *relative to truths of logic*. This would give us an explanation of both the necessity of mathematical truths and the fact that such truths can be known a priori given the fact that logical truths are necessary and knowable a priori.

But this raises an immediate problem. Ayer and the other positivists claimed, on the basis of their empiricism, that *all* a priori truths and necessary truths may be explained on the basis of their analyticity. But this means that if analyticity is truth by definition, we'll have to come up with some non-relative sense of 'truth by definition.' This is the problem that Quine has in mind at the end of §I when he writes,

> "If for the moment we grant that all mathematics is thus definitionally constructible from logic, then mathematics becomes true by convention in a relative sense: mathematical truths become conventional transcriptions of logical truths. Perhaps this is all that many of us mean to assert when we assert that mathematics is true by convention . . . But in strictness we cannot regard mathematics as true purely by convention unless all those logical principles to which mathematics is supposed to reduce are likewise true by convention. And the doctrine that mathematics is *analytic* accomplishes a less fundamental simplification for philosophy than would at first appear, if it asserts only that mathematics is a conventional transcription of logic and not that logic is convention in turn: for if in the end we are to countenance any a priori principles at all which are independent of convention, we should not scruple to admit a few more . . .
>
> But if we are to construe logic also as true by convention, we must rest logic ultimately upon some manner of convention other than definition: for it was noted earlier that definitions are available only for transforming truths, not for founding them." (80-1)

The question, then, is if we can make sense of the idea that logic is true by convention in some non-relative sense which would explain its status as necessary and a priori. In §II of 'Truth by Convention', Quine tries to do just this.

We arrived at definitional truths by giving the meaning of one expression in terms of another expression. This course will not be available for giving an account of the meanings of logical expressions, since these are supposed to be true by convention in an absolute rather than a relative sense. Quine's idea is that we can make sense of

this absolute sense of truth by convention if we can imagine logical expressions being given their meaning, not by definition, but by stipulations of the following kind *Let 'x' have whatever meaning is required to make sentences of the form 'AxB' true.* Just as someone who understands an expression defined in terms of another might know its definition, so someone who understands the imagined logical expression 'x' might know the stipulation which determines its meaning. So, one might think, we would then, simply on the basis of this linguistic knowledge, be in a position to know a priori that any sentence we might encounter of the form 'AxB' is true; after all, we know that the meaning of 'x' was determined by a stipulation that it mean *whatever* is it must for sentences of this form to be true.

In practice, then, one would want to define all of mathematics in terms of truths essentially involving some small set of logical constants; Quine imagines that we have defined mathematics in terms of the universal quantifier, negation, and if-then. The next step would be to give stipulations for each of these constants from which all of the logical truths could be derived. Quine lays out some of these stipulations in detail; here we can just focus on one example, from p. 85:

(II) *Let any expression be true which yields a truth when put for 'q' in the result of putting a truth for 'p' in 'If p then q.'*

This would be one of the stipulations used to define 'if-then.' How might this explain our a priori knowledge of some logical truths? Suppose we are given that 'x' and 'if x, then y' are true. It seems that we can deduce a priori from this that 'y' is true as well. The idea is that our ability to carry out this a priori deduction might be explained by our knowledge of the linguistic stipulation (II). For, after all, (II) tells us that 'if-then' sentences are to have that meaning which guarantees that any expression 'q' be true whenever the expressions 'p' and 'if p then q' are true. We might then be able to go on to give similar stipulations which would provide similar explanations of our ability to know truths of logic a priori.

About this way of explaining our a priori knowledge of logic, Quine says

> "In the adoption of the very conventions ... whereby logic itself is set up, however, a difficulty remains to be faced. Each of these conventions is general, announcing the truth of every one of an infinity of statements conforming to a certain description; derivation of the truth of any specific statement from the general convention thus requires a logical inference, and this involves us in an infinite regress." (96)

We can see the point Quine is making here by laying out the above line of reasoning more explicitly. We are given as premises the following two claims:

P1. x

P2. If x then y

from which we can derive a priori the conclusion

C. y

The aim is to explain this bit of a priori knowledge; the suggestion is that we do so by appealing to knowledge of the linguistic stipulation (II); this is equivalent to adding (II) as a premise to the argument, so that we have the following chain of reasoning:

P1. x
P2. If x then y
P3. Any expression is true which yields a truth when put for 'q' in the result of putting a truth for 'p' in 'If p then q.'

C. y

This inference is, as Quine notes, sound. The problem is that this is still, to put it bluntly, a logical inference. We were trying to explain how we were able to derive C from P1 and P2 a priori; we tried to do this by adding our knowledge of P3. But now we just have a new bit of a priori knowledge to explain: the inference from P1, P2, and P3 to C. This is Quine's regress. He states it succinctly as follows:

> "In a word, the difficulty is that if logic is to proceed *mediately* from conventions, logic is needed for inferring logic from the conventions. " (97)

There are an infinite number of logical truths; our stipulations, if such there be, do not concern each of this infinity of truths, but rather general claims about these truths. But then to derive a truth from these stipulations, we will always need a logical inference which cannot itself be explained by stipulation, even if the inference is the trivial one from 'If S is a sentence of such-and-such form then S is true' and 'S is a sentence of such-and-such form' to 'S is true.' The moral of the story is that logic cannot all be true by convention.

The similarity of Quine's argument to Carroll's 'What the tortoise said to Achilles' (as Quine notes in fn. 21).

As Quine also notes, the same regress can be restated as a problem about the definition of logical constants. We try to make logic true by convention by saying that we *assign* meanings to its expressions by stipulating that certain forms of sentences should be true. But

> "the difficulty which appears thus as a self-presupposition of doctrine can be framed as turning upon a self-presupposition of primitives. If is supposed that the *if*-idiom, the *not*-idiom, the *every*-idiom, and so oon, mean nothing to us initially, and that we adopt conventions . . . by way of circumscribing their meaning; and the difficulty is that [these conventions] themselves depend upon free use of those very idioms which we are attempting to circumscribe, and can succeed only if we are already conversant with the idioms."

The examples of defining ‘and’ using a truth table, or defining the universal quantifier.

Quine’s moral is that we can make no sense of the claims of positivists to explain the necessity and a prioricity of logic in terms of convention. If he is right, then Ayer’s attempt to make mathematics and logic safe for empiricism fails.

6 Quine’s second critique: “Two dogmas of empiricism”

In 1951, Quine presented another argument against Ayer’s (and other verificationists’) use of analyticity to explain the a priori. Unlike his first argument, Quine’s second critique applied not only to Ayer’s attempt to give a non-relative sense in which a sentence could be true by convention or definition, but also to the idea that a sentence could be knowable a priori because it is definable in terms of another a priori truth. So Quine’s second critique, if successful, would rule out Ayer-style explanations of the a prioricity of mathematics, even if we could give some independent account of the a priori nature of propositions of logic.

The basic premise underlying Quine’s argument is a simple one: if analyticity is to be used to explain both a prioricity and necessity, then we should be able to explain what analyticity is without using facts about what is a priori and what is necessary in the explanation. Quine argues in “Two dogmas of empiricism” that this cannot be done.

Philosophers often say that analytic truths are *true by definition* or *true in virtue of meaning alone*. But it is not entirely clear what these slogans mean. Quine, plausibly, says that what these philosophers have in mind is the idea that a sentence is analytic if and only if it can be turned into a logical truth by replacing synonyms with synonyms (or, equivalently, definiens with definiendum). This leads to a first attempt to define analyticity:

Definition of analyticity. S is analytic $\equiv_{df}$ S can be turned into a logical truth by replacing synonyms with synonyms.

So then in order to explain analyticity, we need to explain two notions without presupposing any facts about the necessary or the a priori: synonymy and logical truth. For purposes of this article, Quine in effect grants that the notion of logical truth is unproblematic. He asks instead: what is it for two expressions to be synonymous? Consider the following attempt:

1st Definition of synonymy. Two expressions e_1 and e_2 are synonymous $\equiv_{df}$ e_1 can be substituted for e_2 in any simple (extensional) sentence without changing its truth value.

The problem is that it does not look as though this is true. Consider, for example, the following two pairs of sentences: ‘is a creature with a heart’/‘is a creature with a kidney’; ‘the first Prime Minister of Canada’/‘John MacDonald.’ These do not seem to be synonyms, as is seen by the fact that by replacing one with the other we can move from analytic and a priori sentences like

Every creature with a heart is a creature with a heart.

The first Prime Minister of Canada is the first Prime Minister of Canada.

to sentences which seem neither analytic nor a priori like

Every creature with a heart is a creature with a kidney.

The first Prime Minister of Canada is John MacDonald.

Nonetheless, it seems as though our first definition of synonymy yields the result that these expressions *are* synonyms. Hence this first definition must be rejected.

A natural next step is to abandon the restriction in the first definition to simple sentences, and to move to the following definition:

2nd Definition of synonymy. Two expressions e_1 and e_2 are synonymous $\equiv_{df}$ e_1 can be substituted for e_2 in any (intensional) sentence without changing its truth value.

This seems an improvement, as is shown by the fact that this second definition correctly counts the above pairs of expressions as non-synonymous. To show this, note that although the following sentences seem to be true

Necessarily, every creature with a heart is a creature with a heart.

Necessarily, the first Prime Minister of Canada is the first Prime Minister of Canada.

the sentences obtained by replacing alleged synonyms with synonyms are false:

Necessarily, every creature with a heart is a creature with a kidney.

Necessarily, the first Prime Minister of Canada is John MacDonald.

But here we run into a problem. We began by trying to explain necessity and a prioricity in terms of analyticity; but, in our attempt to define analyticity we have now had to make use of facts about what is necessary. This is Quine's 'circle argument.' It's moral seems to be that we can give no account of what analyticity is which makes it fit to explain a prioricity or necessity (which were held at this time to amount to the same thing).

If we can give no non-circular definition of analyticity, does Quine's argument show that there is no analytic/synthetic distinction?

Can we give a non-circular definition of analyticity in terms of sentences about the psychology of agents?