Further notes on Tarskian truth definitions

Jeff Speaks

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1 Truth theories for languages with quantification

In class last time, we discussed how to extend a Tarskian truth theory to cover sentences involving existential quantification. We first imagined that we had a name for every object in the domain of discourse (i.e., every object which the language can talk about). Then, I said, we could analyze truth for existentially quantified sentences in terms of truth for atomic sentences. E.g., we can say that the sentence

$\exists x$ x is red

is true if and only if there is some name 'n' such that the following atomic sentence is true:

n is red

I suggested, however, that this was not a general solution to the problem of giving a theory of truth for quantified sentences, since typically natural languages do not contain a name for every object in the domain.

We then considered a response to this objection: namely, that if you include definite descriptions as among the singular terms, it is not so implausible to imagine that we possess names for every object in the domain. (Even if we do not know how to formulate, for every object, a description which uniquely picks out that object, it is not implausible to think that our language contains the resources for doing so.)

We then considered several inconclusive objections to this suggestion.

The following is a better objection to the suggestion. If we allow definite descriptions to count as singular terms in our language, then it is clear that our language will contain (an infinite number of) empty singular terms: singular terms which do not refer to anything. But this poses a problem for the analysis of universally quantified sentences. Using the above strategy of analyzing quantified sentences in terms of the truth of atomic sentences, it would be natural to say that a sentence like

$\forall x$ x is red

is true if and only if for every name 'n', the following atomic sentence is true:

n is red

But now imagine that our language contains empty singular terms. This seems to imply that, given the above truth theory for universally quantified sentences, every universally quantified sentence of the form '$\forall x$ Fx' will be false, since for some name 'n' (one of the empty ones), 'Fn' will be false.

So it seems that the analysis of quantification in terms of names works only if (i) there is a name for every object and (ii) every name stands for an object. But even if English satisfies (i), it clearly does not satisfy (ii).

This might provide another source of the motivation for understanding the truth of quantified sentences in terms of assignments of values to variables rather than in terms of sentences involving names.

2 Infinitely long conjunctions and disjunctions

We began our discussion of the languages for which a Tarskian truth theory can be given by explaining how to give a 'list-like' definition of truth for a language with only finitely many sentences. Then I suggested that this would not work for a language with infinitely many sentences. However, one might think that it would work for a language with infinitely many sentences if we allowed infinite disjunctions into the language of the theory. This raises the question: what would be wrong with doing this?

Here I think that there are two related points.

1. It is not just that we would have to allow infinitely long strings of disjunctions; it is that we would have to allow such strings without being able to specify what the disjuncts were. There is thus a certain sense in which we would not even know what the theory was that we were proposing.

2. There are certain strange consequences of allowing an infinitely long string of disjunctions to be a well-formed sentence. For example, it seems (very) plausible that if we have a compound sentence formed only from atomic sentences and disjunction, then, if we assign truth values to each of the atomic sentences, we should have also thereby assigned a truth-value to the whole, compound sentence. E.g., if we assign truth-values to 'Fred is mean' and 'Sally is nice', we have thereby also assigned a truth-value to 'Fred is mean or Sally is nice.' The same, pretty clearly, should go for conjunction.

But now consider the sentences '0>0' and '1>0.' The former is false, the latter true. So this fact should suffice to fix the truth value of any compound sentence formed using these sentences and disjunction and conjunction. But, if we allow infinitely long formulae, this is not the case. Consider:

$$(((\ldots(((\ 0>0 \lor 1>0)\ \&\ 0>0) \lor 1>0)\ \&\ 0>0) \lor 1>0)\ \ldots)$$

True or false?