1 Four puzzles for Millian-Russellian semantics

A Millian-Russellian semantic theory is one according to which the meanings of proper names are the objects for which they stand, and the meanings of predicates are the properties (or relations) they express. Given a compositionality principle (which I will assume), the Millian-Russellian must hold that sentences which differ only in the substitution of proper names which have the same reference (relative to the relevant context) must express the same proposition.

A Fregean semantic theory, which identifies meanings with modes of presentation of, or ways of thinking about, Russellian contents, will make distinctions in meaning that the Millian-Russellian does not. The following four puzzles for Millian-Russellian theories are among the central motivations for Fregean semantics:

Puzzle 1: the a priori

The proposition expressed by [1] is knowable a priori, whereas the proposition expressed by [2] is not.

[1] If Hesperus exists, then Hesperus is Hesperus.
[2] If Hesperus exists, then Hesperus is Phosphorus.

Puzzle 2: cognitive significance

The proposition expressed by [1] is trivial (‘lacks cognitive significance’), whereas the proposition expressed by [2] is non-trivial (‘has cognitive significance’), and might be something that one discovers to be true.
Puzzle 3: substitution failures

It is easy to imagine cases in which the proposition expressed by [3] is true whereas the proposition expressed by [4] is false; so they must be distinct.


Puzzle 4: the problem of the essential indexical

The following sentences can’t express the same proposition, since we can imagine circumstances in which my coming to believe the proposition expressed by the first would lead to immediate action on my part, whereas my coming to believe the proposition expressed by the second might not.

[5] I am on fire.
[6] Jeff Speaks is on fire.

Constraints on Fregean explanations of this linguistic data:

1. Extensional adequacy. The Fregean should provide distinctions in content corresponding to the intuitive differences above.
2. Non-circularity. The Fregean should not define her explanans — the relevant Fregean contents — in terms of the linguistic data to be explained.

It is intuitively plausible that there should be some sort of non-circularity constraint on explanations, but it is not easy to give an uncontroversial statement of what, exactly, this constraint should be. A plausible example of what this constraint should rule out: a simple version of the memory theory of personal identity, according to which what it is for \( x \) to be the same person as \( y \) is for \( x \) to have memories of \( y \), and part of what it is for \( x \) to have memories of \( y \) is for \( x \) to be the same person as \( y \).

However, we don’t want our non-circularity constraint to be so strong that it rules out the possibility of an illuminating, but jointly circular, series of explanations. Plausibly, what the example of the memory theory shows is that our constraint should at least rule out cases in which the circle is ‘too small’ — in which we explain \( A \) in terms of \( B \), and then explain \( B \) in terms of \( A \). (So our constraint should not rule out the possibility that the Fregean might define the relevant class of Fregean contents in terms of one of the explananda above, and use the contents so defined to solve another of the four puzzles.)

## 2 Epistemic two-dimensionalism

### 2.1 Two-dimensional semantics

The newest, and perhaps most currently popular, forms of Fregeanism are various forms of two-dimensionalist semantics.

For our purposes, we can think of two-dimensionalist semantics as the view that a sentence has, relative to a context, two sorts of meaning, one of which — its primary intension — is more closely related to the sentence’s epistemic or cognitive properties, and one of which — its secondary intension — is more closely linked to the sentence’s (metaphysical) modal profile. The two-dimensionalists I will be interested in also endorse the following two theses:
A sentence is a priori (i.e., epistemically necessary) iff it has a necessary primary intension.

A sentence is metaphysically necessary iff it has a necessary secondary intension.

Because secondary intension just is (for our purposes) Millian-Russellian content, the second thesis is common ground between two-dimensionalists and their opponents. The first thesis is what David Chalmers calls the ‘Core Thesis’ of two-dimensionalism; different versions of two-dimensional semantics are defined by their different views of the nature of primary intensions.

2.2 Epistemic intensions

_Epistemic two-dimensionalism_ is a version of two-dimensionalism which Chalmers has defended in an important recent series of papers, and it is, I think, probably the most widely endorsed version of two-dimensionalist semantics.

According to epistemic two-dimensionalism, primary intensions are _epistemic intensions_. Epistemic intensions, like all intensions, are functions from indices of a certain sort to extensions. Chalmers calls the relevant indices _scenarios_ and the relevant function _epistemic necessitation_.

We can think of a scenario as a conjunction of sentences which is both epistemically possible — in the sense that its negation is not a priori — and epistemically complete — in the sense that for any sentence, the scenario epistemically necessitates either it or its negation.

The extension of an expression with respect to a scenario is what we would come to know its extension to be, were we to learn that this scenario is actual. So a sentence is epistemically necessitated by a scenario iff learning that that scenario is actual would put us in a position to know, without use of any further information, that the sentence is true.

Chalmers discusses several ways of making the notion of epistemic necessitation precise but, given the Core Thesis, it turns out that epistemic necessitation must be equivalent to a priori consequence, in the following sense (proof at the end for anyone interested):

\[ e \rightarrow S \]

2.3 Excluded vocabulary and the construction of scenarios

We are thinking of scenarios as long interpreted sentences. But, for two reasons, certain sorts of vocabulary must be systematically excluded from the construction of scenarios:

- The problem of trivializing epistemic intensions.

- The need to preserve a link between conceivability and possibility by guaranteeing that every scenario describes a metaphysically possible world.

For our purposes, the most important class of excluded vocabulary will be proper names; the necessary a posterioricity of non-identities involving names, together with the Core Thesis, entails that if names were included in the construction of scenarios, at least some scenarios would not describe a metaphysically possible world.

But since names have epistemic intensions, they at least sometimes must have a reference with respect to a scenario. As Chalmers (2006), §3.7 suggests, a natural idea about how this might
work is that for a \( n \) to have a reference in a scenario is for there to be some description ‘the \( F \)’ such that the identity statement ‘\( n = \text{the } F \)’ is epistemically necessitated by the scenario.

2.4 A toy example

How learning the name ‘Hesperus’ the old fashioned way, by having it demonstrated as the brightest body in a certain part of the night sky, and ‘Phosphorus’ in the same way, but with the demonstration being to a certain part of the morning sky, might lead you to associate different epistemic intensions with the names; and how this might lead to there being at least one scenario with respect to which [2] is true even though there is no scenario at which [1] is false — which, because it entails a difference in epistemic intension between [1] and [2], is just what the epistemic two-dimensionalist should want.

3 Epistemic two-dimensionalism and the four puzzles

3.1 The a priori

Objection 1: the epistemic two-dimensionalist is forced to make even more shocking claims about the a priori than the Millian-Russellian.

Consider my use of the name ‘Mick Jagger.’ Suppose that the properties I attribute to Mick Jagger can be divided into two conjunctions of properties, \( F \) and \( G \):

\[ F: \] the lead singer on *Sympathy for the Devil* & member of the greatest band of the 1960's & once enrolled in the London School of Economics & the star of *Freecajk* & …

\[ G: \] member of the greatest band of the 1960’s & one of the Glimmer Twins & born in Kent in 1943 & helped to organize the infamous concert at Altamont & …

What matters is that there is a rough parity between the two as regards the number of properties in the two conjunctions and the centrality of those properties to my conception of Mick Jagger.

The following seems to be epistemically possible, and hence true at some scenario:

[7] Something is the \( F \), and something else is the \( G \).

For any scenario which epistemically necessitates [7], will there be a description ‘the \( D \)’ such that ‘Mick Jagger is the \( D \)’ is epistemically necessitated by the scenario? Parity reasoning indicates not. There are two candidate references for ‘Mick Jagger’ in the scenario; nothing will favor one over the other, and so it will not be epistemically necessary that either is the referent of the name. So, in scenarios of this sort, the epistemic intension of ‘Mick Jagger’ (for the relevant person) will deliver no reference. So, presumably,

Mick Jagger exists.

will be false at these scenarios. But then it looks like the material conditional

[8] Mick Jagger exists \( \rightarrow \) \( \neg \) (something is the \( F \), and something else is the \( G \))
will have an epistemic intension which is true at every scenario, since any scenario at which the antecedent is true will be one in which ‘Mick Jagger’ has a reference and so will not be one in which something is $F$ and something else is $G$, and hence will be one in which the consequent of the conditional is true. So, by the Core Thesis, this conditional will be a priori. But this is clearly incorrect; given knowledge that Mick Jagger exists, one cannot deduce a priori that it is not the case that one thing is $F$, and something else is $G$.

Nor is the objection tied to any view about the semantics of existence claims; another way of putting the objection is that the sentence

\[ [9] \text{ Mick Jagger is the } F, \text{ and there is someone else who is the } G. \]

will never be true at a scenario, from which it follows that [9] is not epistemically possible, and hence that its negation is a priori:

\[ [10] (\text{Mick Jagger is the } F) \rightarrow \neg (\text{the } x: Gx \neq \text{Mick Jagger}) \]

But would coming to know that Mick is the $F$ really put you in a position to deduce a priori that nothing else is the $G$?

Further oddities result if we consider my use of the name ‘Keith Richards.’ As it happens, we can also split the properties I associate with Keith’s name into two complex properties, one of which happens to be the property $G$ discussed in connection with Mick Jagger above, while the other is a distinct property $H$ of the same sort. Suppose we know

\[ [11] \text{ Keith Richards is the } G \lor \text{ Keith Richards is the } H. \]

This is presumably a posteriori, as is

\[ [12] \text{ Mick Jagger is the } F. \]

But, given the above argument that [10] is a priori, we can know a priori that if [12] is true, nothing which is not Mick Jagger exists which is $G$. But then it follows that if we know just [11] and [12] along with the proposition that Keith Richards is not Mick Jagger, we are in position to deduce a priori that Keith Richards is $H$. That is, the following is a priori:

\[ [13] (\text{Mick Jagger is the } F) \rightarrow ((\text{Keith Richards is the } G \lor \text{Keith Richards is the } H) \rightarrow \text{Keith Richards is the } H) \]

But this seems odd; how could knowing some stuff about Mick Jagger put me in a position to decide, a priori, between two otherwise epistemically possible hypotheses about what Keith Richards is like?

A connection between this argument and Kripke’s epistemic argument against descriptivism.

\textit{Objection 2: Even if there is some way of blocking this argument, the epistemic two-dimensionalist’s explanation of the epistemic difference between [1] and [2] fails the noncircularity condition.}

One of the notions used in defining epistemic intensions was epistemic necessitation. But we already know from our discussion above that, given the Core Thesis, the following biconditional must hold:

\[ e \text{ epistemically necessitates } S \text{ iff } (e \rightarrow S) \text{ is a priori.} \]

This points up the difficulty in giving an account of epistemic intensions which does not presuppose facts about the a priori: it is equivalent to giving an account of a priori entailment which
does not presuppose facts about the a priori. But this means that epistemic two-dimensionalism offers an explanation of the difference in epistemic profile between [1] and [2] if and only if it is packaged with an independent account of a priori entailment — which, given that a prioricity can be defined in terms of a priori entailment, is an explanatory virtue which any theory has.

This of course needn’t rule out the possibility that epistemic intensions so defined might help with the other three puzzles; but it does indicate that the epistemic two-dimensionalist can claim no special advantage with respect to the explanation of speaker intuitions about a prioricity.

3.2 Cognitive significance

As noted above, one version of Frege’s puzzle which is challenging for the Millian-Russellian is to explain the difference in cognitive significance between sentences like [1] and [2]:

[1] If Hesperus exists, then Hesperus is Hesperus.

[2] If Hesperus exists, then Hesperus is Phosphorus.

Epistemic two-dimensionalism promises to explain the difference in cognitive significance between [1] and [2] in terms of the difference in epistemic intension between the two names.

However, any two expressions which can be known a priori to be coreferential will have the same epistemic intension, and this poses a problem for the proposed explanation of differences in cognitive significance, because we can generate a difference in cognitive significance via the substitution of two such terms, as in the following:


[14] and [15] seem to differ in cognitive significance in just the same way as do [1] and [2]. For this reason, epistemic intensions do not by themselves offer a solution to Frege’s puzzle.

Chalmers has suggested two kinds of responses to this problem.

3.2.1 Three-dimensionalism?

One idea, suggested in Chalmers (ms.), §7, is that just as we defined epistemic intensions to track the epistemic properties of sentences, so we can define a third dimension of meaning which will more closely correspond to Fregean senses in tracking differences in cognitive significance.

Epistemic intensions are defined in terms of a kind of idealized a priori knowability; but we could define Frege-intensions in terms of some other property more closely tied to the differences in cognitive significance that Frege’s puzzle tracks. Let’s introduce ‘obvious’ as a term of art for the property of lacking cognitive significance, in the sense sketched above. We could then use this property to define Frege-scenarios (all those possibilities which are such that it is not obvious that they are not actual), Frege-necessitation (obvious entailment), and thereby a new kind of semantic value, Frege-intensions. Perhaps we can explain differences in cognitive significance using Frege-intensions, rather than epistemic intensions.

*Objection 1: this fails non-circularity, for the same reason as the epistemic two-dimensionalist explanation of the apparent epistemic difference between [1] and [2].*
Objection 2: it is not even clear that we have succeeded in defining a coherent notion.

It seems clear that the property of being an ‘obvious’ entailment in the above sense will not be transitive; there can be a trio of sentences $p, q, r$ such that $p \rightarrow q \lnot$ lacks cognitive significance, $q \rightarrow r \lnot$ lacks cognitive significance, but $p \rightarrow r \lnot$ has cognitive significance, and so is not obvious.

But this, plus the definition of Frege-intensions, seems to lead to incoherence. If $p \rightarrow q \lnot$ lacks cognitive significance, then every scenario at which the Frege-intension of $p$ determines truth is one at which the Frege-intension of $q$ does as well; and if $q \rightarrow r \lnot$ lacks cognitive significance, then every scenario at which the Frege-intension of $q$ determines truth is one at which the Frege-intension of $r$ does as well. Put these together, and what you get is that every scenario at which the Frege-intension of $p$ is true is one at which the Frege-intension of $r$ is true; but this contradicts our supposition that $p \rightarrow r \lnot$ could have cognitive significance, and hence be false at at least one Frege-scenario.

3.2.2 Structured epistemic intensions

An alternative is to explain cognitive significance by moving from epistemic intensions as defined above to more fine-grained structured epistemic intensions, thereby finding a difference between pairs of expressions, like ‘49’ and ‘the positive square root of 2401’, which can be known a priori to be coreferential, on the grounds that the latter but not the former is semantically complex (see Chalmers forthcoming, §3). This is a fair point about the above example, but does not in the end help very much. The problem is that the explanation of cognitive significance in terms of semantic complexity and simplicity seems to overgenerate. Consider the following sentence:

\[ [16] \, 49 = \text{the number } 49. \]

This seems to lack cognitive significance, and to be in that respect on par with [14]. But on the proposal that we explain cognitive significance not by difference in epistemic intension but by difference in structured epistemic intension, [16] gets lumped in with [15]. This seems to be a mistake. The notion of ‘cognitive significance’ is already notoriously difficult to pin down; if we are to claim that [14] but not [16] lacks cognitive significance, and that [16] like [15] has cognitive significance, it is far from clear that any intuitive distinction at all is being captured.

3.3 Substitution failures

Let’s now examine epistemic two-dimensionalism’s capacity to explain the apparent differences in truth-value between sentences like [3] and [4]:


The epistemic two-dimensionalist’s strategy for providing a difference in truth value between these two sentences will be to provide a semantics for attitude ascriptions which makes their truth conditions sensitive to the epistemic intensions, as well as the secondary intensions, of expressions in the complement of the ascription.

3.3.1 A simple semantics for attitude ascriptions

The simplest way to do this is as follows:
\( \forall a \) believes that \( S \) is true in a context \( C \) iff \( a \) has a belief which

(i) has the same secondary intension as \( S \) in \( C \), and
(ii) has the same epistemic intension as \( S \) in \( C \)

Why this is a non-starter.

3.3.2 The semantics once-complicated

A more plausible view is that our semantics for ascriptions should be an instance of the following schema:

\( \forall a \) believes that \( S \) is true in a context \( C \) iff \( a \) has a belief which

(i) has the same secondary intension as \( S \) in \( C \), and
(ii) has a structured epistemic intension which bears \( R \) to the structured epistemic intension of \( S \) in \( C \)

instances of which are obtained by replacing ‘\( R \)’ with a term for some similarity relation, which might depend on context, between epistemic intensions.

The problem is that we can generate substitution failures using singular terms which can be known a priori to be coreferential, and so must have the same epistemic intension:

[17] John believes that 49 is odd.

[18] John believes that the positive square root of 2401 is odd.

3.3.3 The semantics twice-complicated

But as above, we can perhaps get around this by appeal to structured epistemic intensions, giving a semantics of the following sort:

\( \forall a \) believes that \( S \) is true in a context \( C \) iff \( a \) has a belief which

(i) has the same secondary intension as \( S \) in \( C \), and
(ii) has a structured epistemic intension which bears \( R \) to the structured epistemic intension of \( S \) in \( C \)

A problem for any account of this sort arises from the fact that the epistemic intensions of names will vary widely between speakers.

Sadly, I can never remember which name is supposed to be linked to the description ‘the morning star’ and which is supposed to be linked to ‘the evening star.’ In this case, it is plausible to think that the epistemic intensions of the names converge for me: given any scenario, I will identify the denotation of a description ‘the \( F \)’ as ‘Hesperus’ if and only if I identify it as ‘Phosphorus’. Since the two names have the same primary and secondary intensions out of my mouth, then, given that some version of the semantics for attitude ascriptions sketched above is true, then, out of my mouth, the two names are substitutable salva veritate in attitude ascriptions.

1. Since the two names have the same epistemic intensions out of my mouth, all of the following must be true, as uttered by me:
The earliest astronomers believed that Hesperus is Phosphorus.
Anyone who has ever believed that Hesperus is visible in the evening has also
believed that Phosphorus is visible in the evening.
It is knowable a priori that if Hesperus exists, Hesperus is Phosphorus.

But these are the kind of counterintuitive assignments of truth-values which epistemic
intensions were meant to block.

2. The two-dimensionalist might reply that epistemic intensions don’t explain why these sentences seem false when uttered by me, but they do at least explain why sentences of this sort seem false when uttered by you, for whom the epistemic intensions of the names diverge.
But any plausible semantics for attitude ascriptions should not, given the above description of the case, rule out my being able to report the thoughts you express using the word ‘Hesperus’ using the same word, and the same for ‘Phosphorus.’ So let’s suppose that the epistemic intensions we assign to the word are ‘similar enough’ to allow for this.
Now let’s imagine that a friend of ours uses the two terms with the same epistemic intensions as you, and, furthermore, that this friend has quite radical astronomic views. Suppose he turns to us and says, ‘Hesperus is not Phosphorus!’ Since we know that our friend assigns these names the same epistemic intension you do, and that our epistemic intensions are similar enough for me to report your speech, I can truly say of our friend,

[19] He said that Hesperus is not Phosphorus.

and hence, since the names have the same epistemic intension for me, if [19] is true out of my mouth, I can also truly say

[20] He said that Hesperus is not Hesperus.

But given that you can report my speech, you can report my utterance of [20] by saying

[21] JS said that he said that Hesperus is not Hesperus.

Moreover, since you might know that my utterance was true, you might expand upon [21] by saying

[22] JS said that he said that Hesperus is not Hesperus, and what JS said was true.

and it seems that your utterance of [22] will be true. But if you can truly utter [22], then you must also be able to truly utter [20]. But [20] certainly seems to be false. So it seems that the epistemic two-dimensionalist has to assign [19] and [20] the same truth-value out of both of our mouths.

3. Another way to see the oddness here is by imagining our friend with radical astronomic views uttering

[23] I said that Hesperus is not Hesperus.

This certainly seems as though it should be false; the problem with Millianism is supposed to be that it is forced to say that utterances such as this are, even though the speaker takes them to be false, really true. But the epistemic two-dimensionalist is forced into saying that [23] is true as well — after all, she takes your utterance of [20] to be true, and you and our friend with radical astronomic views assign ‘Hesperus’ the same epistemic intension.

Three replies to these arguments:

A. The counterexample is not possible, because it is not possible for me to associate exactly the same epistemic intension with ‘Hesperus’ and ‘Phosphorus’; the epistemic intensions of names always include metalinguistic information. But: (i) the same point could likely be made with ‘similar enough’ epistemic intensions, and (ii) this reliance on meta-linguistic information threatens to make propositions like the following knowable a priori:
If Hesperus ≠ Phosphorus, then language users exist.

B. The context-dependent relation $R$ can shift during conversation; for example, our friend’s utterance may introduce a context in which it is impossible for me to report his speech with [19]. Perhaps. But: (i) it really seems like [19] as uttered by me is true; and (ii) we need to know more about $R$; it seems plausible that any way of defining this relation will lead to other counterintuitive consequences. And one can’t let $R$ be defined by our intuitions about attitude ascriptions without running afoot of the noncircularity constraint.

C. We might adopt a thrice-complicated semantics which lets the truth-values of ascriptions be sensitive not just to epistemic and secondary intensions but also to the words used in the ascription:

\[ \text{⌜a believes that S⌝ is true in a context } C \text{ iff a has a belief which} \]
\[ \text{(i) has the same secondary intension as S in } C, \]
\[ \text{(ii) has a structured epistemic intension which bears R to the structured epistemic intension of S in } C, \text{ and} \]
\[ \text{(iii) S bears some relation } R^* \text{ to the sentence that the subject of the belief uses to express the belief.} \]

Same worry as above about violation of non-circularity, absent an account of the relation $R^*$.

3.3.4 A fallback position

Many Millian-Russellians try to explain away speaker intuitions about substitution failures in terms of (i) a pragmatic theory which generates pragmatically conveyed propositions which have the truth-values which speakers take the sentences to have and (ii) a claim to the effect that speaker intuitions about the truth-value of $S$ often track the truth-value what would typically be pragmatically conveyed by an utterance of $S$ rather than the truth-value of the proposition $S$ semantically expresses (in the relevant context).

One might (as suggested in Chalmers (2002), p. 631 n. 23) think that epistemic intensions might play a role in developing (i); perhaps when one utters a belief ascription, one pragmatically conveys that the subject of the ascription has a belief with the epistemic intension of the complement sentence. Why this is not very plausible.

3.4 The problem of the essential indexical

The problem of the essential indexical is often thought of as the problem of explaining the apparent difference between sentences like [5] and [6]. So put, there are really at least three different versions of the problem, depending on which apparent difference between these sentences we focus on:

1. I can believe that Jeff Speaks is $F$ without believing that I am $F$, even though I am Jeff Speaks.
2. I can believe that Jeff Speaks is $F$ without being in a position to deduce a priori that I am $F$, even though I am Jeff Speaks.
3. My beliefs that I have various properties have a kind of direct effect on my actions which my beliefs that Jeff Speaks has various properties do not.
Versions (1) and (2) of the problem are just special cases of the problems of explaining the a priori and apparent substitution failures in attitude ascriptions; so it remains to discuss only version (3).

Let’s suppose that the following are both epistemic possibilities:

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>The $F$ is on fire.</td>
<td>The $F$ is not on fire.</td>
</tr>
<tr>
<td>I am not on fire.</td>
<td>I am on fire.</td>
</tr>
</tbody>
</table>

and suppose that each scenario epistemically necessitates the identity claim ‘The $F$ is Jeff Speaks.’ Then the epistemic intensions of ‘Jeff Speaks is on fire’ and ‘I am on fire’ will differ — which is, from the epistemic two-dimensionalist’s point of view, all to the good.

But to make room for these two sentences to have different epistemic intensions is not to explain why ‘I am on fire’ has a kind of built-in direct connection to action which ‘Jeff Speaks is on fire’ does not. Within the framework of epistemic two-dimensionalism, ‘I’ is just one term among others which can appear in scenarios; nothing in the framework seems to explain why ‘I am on fire’ should have any closer connection to action than ‘$n$ is on fire’.

The two-dimensionalist might reply as follows:

> If I learned that Scenario 2 is actual, then I would be disposed to douse myself with water; if I learned that Scenario 1 is actual, then I wouldn’t be. But (to oversimplify a bit) epistemic contents just are sets of scenarios; so the difference between the epistemic intensions of ‘I am on fire’ and ‘Jeff Speaks is on fire’ is the difference between Scenarios 2 and 1. But then the fact that Scenario 2 but not Scenario 1 would dispose me to douse myself with water shows that the epistemic intension of ‘I am on fire’ but not ‘Jeff Speaks is on fire’ has an immediate connection to action.

But suppose we ask: ‘Why would learning that Scenario 2 is actual lead me to douse myself with water?’ Presumably, because this scenario includes the claim, ‘I am on fire.’ Given this, we can hardly turn around and answer ‘Why does “I am on fire” have a direct connection to action?’ by saying ‘Because its epistemic intension rules out Scenario 1 in favor of Scenario 2.’

Epistemic two-dimensionalism can’t solve any of the four puzzles; the solutions oscillate between violations of extensional adequacy and of non-circularity, and sometimes violate both constraints.

Some concluding thoughts on ‘the golden triangle of constitutive connections between meaning, reason, and modality.’

References

- David Chalmers, ms. The Nature of Epistemic Space. [consc.net/papers/espace/html](consc.net/papers/espace/html).
Proof of equivalence of epistemic necessitation and a priori consequence

Given the Core Thesis, epistemic necessitation must be some relation $R$ such that for any sentence $S$, $S$ is a priori iff for all scenarios $e$, $R(e, S)$. What we want to show is that, given this,

$$R(e, S) \text{ iff } \uparrow e \rightarrow S^\top \text{ is a priori.}$$

One way to show this is to begin with the following Lemma:

**Lemma.** $R(e, S) \text{ iff } R(e, e \rightarrow S)$

This Lemma may be intuitively obvious, but we can also give the following argument for it, assuming only that the Core Thesis is true.

Left to right: $\uparrow S \rightarrow (e \rightarrow S)^\top$ is a priori, so, given the Core Thesis, it must be true at every scenario. So any scenario which epistemically necessitates $S$ must also epistemically necessitate $\uparrow e \rightarrow S^\top$.

Right to left: if $e$ epistemically necessitates $S$, then knowing that $e$ is the case puts one in a position to know that $\uparrow e \rightarrow S^\top$ to be true: for $e$ to be false, or for $S$ to be true. But knowing that $e$ is the case can never put one in a position to know that $e$ is false; so if $e$ epistemically necessitates $\uparrow e \rightarrow S^\top$, $e$ must put one in a position to know that $S$ is true. But then $e$ epistemically necessitates $S$.

We then argue for the equivalence of a priori entailment and epistemic necessitation, i.e. the truth of the biconditional

$$R(e, S) \text{ iff } \uparrow e \rightarrow S^\top \text{ is a priori.}$$

as follows:

Left to right: suppose that $\uparrow e \rightarrow S^\top$ is not a priori. Then, by the Core Thesis, there must be some scenario $e^*$ such that $\neg R(e^*, e \rightarrow S)$. Either $e^* = e$ or $e^* \neq e$. But the latter is impossible, since, scenarios being complete epistemic possibilities, any scenario must epistemically necessitate the negation of any other scenario; and if $e^*$ did epistemically necessitate $\neg e$, then, given that $\uparrow \neg e \rightarrow (e \rightarrow S)^\top$ is a priori, $e^*$ would epistemically necessitate $\uparrow e \rightarrow S^\top$, which is contrary to our supposition. So $e^* = e$, and $\neg R(e, e \rightarrow S)$. But then it follows from our Lemma that $\neg R(e, S)$.

Right to left: suppose that $\uparrow e \rightarrow S^\top$ is a priori. Then, by the Core Thesis, for any scenario $e^*$, $R(e^*, e \rightarrow S)$; so, in particular, $R(e, e \rightarrow S)$. But then, by our Lemma, $R(e, S)$.