1) According to a study by Niccie McKay, PhD., the average cost per patient day for nursing homes in the US is

\[ C = A - .16X + .00137X^2 \]

Where X is the number of patient days per year (in thousands) and A is a constant that varies by region (but independent of X). How big should a nursing home be (in patient days per year) to minimize average costs. Show that your result minimizes rather than maximizes costs.

2) Suppose that you are currently selling magazine subscriptions for $36. You currently have 6,000 subscribers. You also know that for every $1 you lower your price, you will gain 200 new subscribers. Fine the price to maximize your revenues from magazine subscriptions.

3) Suppose that you are a petroleum company. Your profits depend on your production of heating oil and gasoline. Specifically, your profits as a function of gasoline (G) and heating oil (H) is given by:

\[ Profits = -60 + 140H + 100G - 10H^2 - 8G^2 - 6HG \]

Solve for the profit maximizing choices for gasoline and heating oil.

4) Consumer research for Kraft foods has discovered the following relationships between sales of TANG (Yes, the orange drink used by astronauts!) and advertising expenditures for two districts. Sales (S) and advertising (A) are in millions.

\[ S_1 = 10 + 5A_i - 1.5A_i^2 \]

\[ S_2 = 12 + 4A_i - .5A_i^2 \]

Solve Kraft’s maximization problem; maximize total sales across the two districts subject to a total advertising budget of $5M. How would a $1M increase in Kraft’s advertising budget influence sales?
5) You are designing a poster that has 2 inch margins at the top and along each side and a 3 inch margin at the bottom. To save on costs, you want the overall poster to be as small as possible, but you need the printed area inside the margins to be 180 square inches (the shaded area below). Solve for the dimensions of the poster.

6) Suppose that you are a livestock breeder. You have 1200 ft. of fencing to build a rectangular area divided into two pens. (see diagram below)

Find the dimensions to maximize the enclosed area. If you were given 10% more fencing, approximately how much bigger in area could your pens be?

7) You want to build a box with a square base and an open top that has a volume of 32 cubic inches. Find the dimensions that will minimize the surface area.
8) Stafford Rug Company produces wool rugs and cotton rugs. Total cost (in dollars) is given by

\[ C = 7X_1^2 + 9X_2^2 - 1.5X_1X_2 \]

Where \( X_1 \) represents the number of wool rugs per day and \( X_2 \) represents the number of cotton rugs per day. Stafford has commitments to deliver 10 rugs per day, but can deliver any combination of wool or cotton. Calculate the cost minimizing combination of rugs. What would the impact be on Stafford’s costs if their orders increased by 50%?

9) A firm’s inventory of a certain commodity is depleted at a constant rate per unit time. The firm reorders amount \( x \) whenever the firm’s inventory level drops to zero (new orders materialize instantaneously). The annual requirement for the commodity is 100 units and the firm orders \( n \) times per year.

The firm incurs two types of costs; a holding cost and an order cost. The holding cost is equal to .08 times the average amount of inventories in the warehouse (\( x/2 \)) while ordering costs are equal to .25 times the number of orders made per year. Therefore, total annual costs are equal to

\[ C = .08 \left( \frac{x}{2} \right) + .25n \]

Minimize inventory costs (by choice of \( x \) and \( n \)) subject to the constraint that total annual demand equals 100.

10) Suppose that you have the following technology for producing output.

\[ y = \frac{1}{k^2l^2} \]

The price of labor is $5 per hour and capital costs $200 per unit. You need to produce at least 500 units of output. Find \( k \) and \( l \) to minimize your total production costs.