1) Consider the following output table:

<table>
<thead>
<tr>
<th>Labor</th>
<th>Output</th>
<th>Marginal Product</th>
<th>Average Product</th>
<th>Elasticity of Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
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<td></td>
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</tr>
<tr>
<td>2</td>
<td>6</td>
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<td>3</td>
<td>16</td>
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<td>4</td>
<td>29</td>
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<td>5</td>
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<tr>
<td>6</td>
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<td>9</td>
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</tr>
<tr>
<td>10</td>
<td>56</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Calculate Marginal product, Average Product, Elasticity of Production.
b) Within what ranges do we see increasing returns, decreasing returns and negative returns?

2) Consider the following short run production function:

\[ Q = 6L^2 - 0.4L^3 \]

a) Find the value of L that maximizes output.
b) Find the value of L that maximizes marginal product.
c) Find the value of L that maximizes average product.

3) Consider the following production function for bus transportation in a particular city:

\[ Q = \alpha L^{\beta_1} F^{\beta_2} K^{\beta_3} \]

Where
- \( L \) = Fuel input in gallons
- \( K \) = Capital input in number of busses
- \( L \) = Labor input in worker hours
- \( Q \) = Output in millions of bus miles

We estimate the various parameters as follows using historical data:

\[ \alpha = .0012 \quad \beta_1 = .45 \quad \beta_2 = .20 \quad \beta_3 = .30 \]

a) Determine output elasticities for Labor, Fuel and Capital.
b) Suppose that labor hours increase by 10%. By what percentage will output increase?
c) Suppose that every year, 3% of the busses are taken out of service? What effect will this have on output?
d) Suppose that we increase all inputs by 10%, what will happen to output?

4) Suppose that you have the following production function:

\[ y = k^{\frac{2}{3}} l^{\frac{1}{3}} \]

Where \( k \) represents the units of capital employed at your production facility, \( l \) is the number of labor hours employed and \( y \) is your total production. You face wage rate equal to $15 per hour and a cost of capital equal to $1,920 per unit. You have 1 unit of capital and that can’t be changed.

a) You are currently employing 8 hours of labor. Calculate your Total costs, Average cost and marginal cost.
b) Calculate your technical rate of substitution. Are you behaving optimally? Explain.
c) Given your fixed capital stock of 1, if you were acting optimally, what should your labor be?
d) Calculate your TC, AC, and Marginal Cost at the optimal choice of labor.

5) Suppose that you are a firm that produces xylophones. You have a production technology to produce xylophones that can be written as:

\[ y = k^{\frac{1}{2}} l^{\frac{1}{2}} \]

Where \( k \) represents the units of capital employed at your production facility, \( l \) is the number of labor hours employed and \( y \) is your total production of xylophones. Assume that labor costs $10 per hour and that capital costs $250 per unit.

a) Suppose that you are currently employing 100 units of capital. If you have expected sales equal to 1,000. Calculate your optimal choice of labor.
b) Given your answer to (a), calculate your marginal and average cost of production.
c) Now, assume that you can adjust your capital as well as labor. Calculate your optimal capital/labor choice.
d) Calculate your long run average cost and marginal cost.
6) Suppose that you have the following production function:

\[ Q = .5LK - .1L^2 - .05K^2 \]

The process per unit of L and K are $20 and $25 respectively. Suppose you are interested in maximizing output given a budget constraint of $500.

a) Write down the lagrangian associated with this optimization.
b) Solve for the optimal values for K and L.
c) Assuming that your budget was increased by 10%, calculate (approximately) the increase in output given the new budget.

7) Suppose that you have estimated the following production function.

\[ \ln Q = 3.2 + .2 \ln k + .7 \ln l \]

Where \( l \) represents unskilled labor (currently being paid minimum wage) and \( k \) represents capital equipment.

a) Will short run marginal costs be increasing or decreasing in Q? Explain.
b) Will long run average costs be increasing or decreasing in Q? Explain.
c) What will this firm’s cost structure look like in the long run (i.e. what percentage of this firms costs will be labor costs? What percentage will be capital costs?)

8) Continuing with the above example, we have the following estimated production function.

\[ \ln Q = 3.2 + .2 \ln k + .7 \ln l \]

Where \( l \) represents unskilled labor (currently being paid minimum wage) and \( k \) represents capital equipment.

a) What will this firm’s elasticity of demand for labor with respect to the wage be in the short term? How about the long term?
b) Suppose that Hillary Clinton is elected President and manages to pass an increase in the minimum wage to $15 per hour. What will happen to employment at this firm in the short run? How about the long run? Explain.
c) How will a $15 per hour wage affect marginal cost in the short run? How about the long run? Explain.
9) Suppose that you have the following production function:

\[ Q = \left( 0.5k^5 + 0.5l^5 \right)^2 \]

a) Calculate the TRS for this production function.
b) If the wage rate is $20 and the cost of capital is $100, calculate the minimum cost of producing 54 units of production.
c) Calculate the elasticity of substitution.
d) Suppose that the price of capital increases to $120. By how much does the ratio of capital to labor utilized change?

10) Suppose that you have the following production function:

\[ Q = 15k + 7l \]

Where \( l \) represents unskilled labor and \( k \) represents capital equipment. You face a wage rate of $8 per hour and a price of capital equal to $20 per unit.

a) Calculate the technical rate of substitution.
b) Calculate the elasticity of substitution.
c) Calculate the optimal choice for capital and labor given a production goal of 140 units of output.
d) What is the elasticity of demand in the short run? How about the long run?
e) Plot out your labor demand curves.