1) The two most common paradigms for strategic interaction between firms are Cournot competition and Bertrand competition. Briefly describe the assumptions underlying the two models. What industries would you classify as Bertrand? What industries would you classify as Cournot? In which of the two models is competition the “fiercest”??

Cournot: Simultaneous move game. Each firm has a cost function to determine marginal costs (in the baseline example, marginal costs are constant and equal across firms, but this need not be the case). The firms face a common aggregate demand curve. Each firm chooses production levels conditional on what they expect their rival’s production levels to be. The Nash equilibrium is the result of all firms playing their best responses. Examples of Cournot competition would be petroleum, most of the other commodities, electricity generation, chemicals, and cement.

Bertrand: Simultaneous move game. Each firm has a cost function to determine marginal costs (in the baseline example, marginal costs are constant and equal across firms, but this need not be the case). The firms face a common aggregate demand curve. Each firm chooses price conditional on what they expect their rival’s price to be. The Nash equilibrium is the result of all firms playing their best responses. Examples of Bertrand competition would be the airlines, cellphone service, most of the service industry, and insurance.

In the specific case of identical products you could say that Bertrand competition is the “fiercest”. Two firms undercut each other until price falls to marginal cost and profits disappear. However, in the general case, Cournot competition is the most aggressive. The firm with the cost advantage raises its market share while the weaker firm shrinks. In the Bertrand case, if a firm’s costs increase, rivals respond by raising price and maintaining market share rather than stealing from their weaker rival.

2) Suppose that the (inverse) market demand for fax paper is given by

\[ P = 400 - 2Q \]

Where Q is total industry output. There are two firms that produce fax paper. Each firm has a constant marginal cost of production equal to $40 and they are competing in quantities. That is, they each choose production levels simultaneously.
a) Calculate the best response function for each firm (i.e. each firm’s profit maximizing choice of quantity given the other firm’s production levels)

First, rewrite the aggregate production as the sum of each firm’s output.

\[ P = 400 - 2(q_1 + q_2) \]

Now, let's look at the demand facing firm 1 (remember, firm one treats firm two’s output as a constant)

\[ P = (400 - 2q_2) - 2q_1 \]

Total revenues equal price times quantity.

\[ Pq_1 = (400 - 2q_2)q_1 - 2q_1^2 \]

Marginal revenue is the derivative with respect to quantity.

\[ MR = (400 - 2q_2) - 4q_1 \]

Set marginal revenue equal to marginal cost and solve for quantity. To get market price, remember there are two firms.

\[ (400 - 2q_2) - 4q_1 = 40 \]
\[ q_1 = \frac{360 - 2q_2}{4} = 90 - .5q_2 \]

This is firm one’s best response function. Note that firm two is perfectly symmetric to firm one.

\[ q_1 = 90 - .5q_2 \]
\[ q_2 = 90 - .5q_1 \]

Substitute one into the other to get the equilibrium. Then, substitute into the demand curve to get price (remember, there are two firms)

\[ q_1 = q_2 = 60 \]
\[ P = 400 = 2(120) = 160 \]
\[ \text{profit}_1 = \text{profit}_2 = $160(60) - $40(60) = $7200 \]

b) Calculate the profit maximizing price/quantity for a monopolist facing the same demand curve (and with the same production costs). How does your answer compare to (b)?
For the monopolist, it faces the aggregate demand curve.

\[ P = 400 - 2Q \]

Total revenues equal price times quantity.

\[ PQ = 400Q - 2Q^2 \]

Marginal revenue is the derivative with respect to quantity.

\[ MR = 400 - 4Q \]

Set marginal revenue equal to marginal cost

\[ 400 - 4Q = 40 \]
\[ Q = 90 \]
\[ P = $220 \]

\[ \text{profit} = 220(90) - 40(90) = $16,200 \]

3) Continuing with the above example, you have the demand curve for fax paper.

\[ P = 400 - 2Q \]

a) Suppose that the two firms decide to form a cartel. Calculate the cartel output for each firm.

If the two firms form a cartel, they will collectively produce the monopoly level of output. So, given the answer above,

\[ Q = 90 \]
\[ q_1 = q_2 = 45 \]
\[ P = $220 \]

\[ \text{profit}_1 = \text{profit}_2 = 220(45) - 40(45) = $8,100 \]

b) Calculate the optimal response (i.e. the cartel cheating response).

If firm 1 assumes that firm 2 will maintain the cartel level of production equal to 45, he faces the following residual demand curve.

\[ P = 400 - 2(q_1 + 45) = 310 - 2q_1 \]
If firm 1 profit maximizes, we have

\[ P = 310 - 2q_1 \]
\[ TR = 310q_1 - 2q_1^2 \]
\[ MR = 310 - 4q_1 = 40 \]
\[ q_1 = 67.5 \]
\[ q_1 = 45 \]
\[ P = 400 - 2(67.5 + 45) = $175 \]
\[ \text{profit}_1 = ($175 - 40)67.5 = $9,112.50 \]
\[ \text{profit}_2 = ($175 - 40)45 = $6,075 \]

c) Set up the cartel formation problem as a prisoner’s dilemma problem.

Note that if both sides cheat, we have

\[ q_1 = 67.5 \]
\[ q_1 = 67.5 \]
\[ P = 400 - 2(67.5 + 67.5) = $130 \]
\[ \text{profit}_1 = ($130 - 40)67.5 = $6,075 \]
\[ \text{profit}_2 = ($130 - 40)67.5 = $6,075 \]

So, we have the following (firm 1’s profits in bold)

<table>
<thead>
<tr>
<th>Firm  One</th>
<th>Firm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cooperate</td>
</tr>
<tr>
<td>Cooperate</td>
<td>$8,100</td>
</tr>
<tr>
<td>Cheat</td>
<td>$9,112</td>
</tr>
</tbody>
</table>

Both firms have a (weakly) dominant strategy to cheat, so the Nash equilibrium is (cheat, cheat).

4) Suppose that the (inverse) demand curve for Viagra is given by

\[ P = 200 - 2Q \]

Where \( Q \) is total industry output. The market is occupied by two firms, each with constant marginal costs equal to $8.
a) Calculate the equilibrium price and quantity assuming the two firms compete in quantities.

Repeating the process from (1), we get the following:

\[ q_1 = q_2 = 32 \]
\[ P = 200 - 2(64) = $72 \]
\[ \pi_1 = \pi_2 = 72(32) - 8(32) = $2048 \]

b) How would your answer to (a) change if one of the firm’s costs rose to $10?

Here, we need to actually go through the steps: Assume that firm one’s costs increase to $10.

\[ P = (200 - 2q_2) - 2q_1 \]

Total revenues equal price times quantity.

\[ Pq_1 = (200 - 2q_2)q_1 - 2q_1^2 \]

Marginal revenue is the derivative with respect to quantity.

\[ MR = (200 - 2q_2) - 4q_1 \]

Set marginal revenue equal to marginal cost and solve for quantity. To get market price, remember there are two firms.

\[ (200 - 2q_2) - 4q_1 = 10 \]
\[ q_1 = \frac{190 - 2q_2}{4} = 47.5 - .5q_2 \]

This is firm one’s best response function. Note that firm two has the same problem to solve, but with MC =$8

\[ q_1 = 47.5 - .5q_2 \]
\[ q_2 = 48 - .5q_1 \]

Solving for quantities:
\[ q_1 = 31.3 \]
\[ q_2 = 32.3 \]
\[ P = 200 - 2(63.6) = 78.8 \]
\[ \pi_1 = (78.8 - 10)(31.3) = 2153.44 \]
\[ \pi_2 = (78.8 - 8)(32.3) = 2286.84 \]

Note that firm 2 exploits firm one’s cost increase by grabbing market share.

c) Repeat parts (a) and (b) assuming the competition is in prices rather than quantities.

This is a quick one. With identical products, both firms charge a price equal to marginal cost (in this case, $8) and profits are zero. If firm one’s costs rise to $10, firm 2 charges a price equal to $9.99, takes the entire market and earns profits equal to $(9.99 - 8)(95) = 189.

5) With identical products and no capacity constraints, both firms charge a price equal to marginal cost and earn zero profits. To avoid this, firms have two strategies:

- By restricting capacity, they create excess demand for their products. This will drive the price up

6) Suppose that the market demand is described by

\[ P = 100 - (Q + q) \]

Where \( Q \) is the output of the incumbent firm, \( q \) is the output of the potential entrant and \( P \) is the market price. The incumbent’s cost function is given by

\[ TC(Q) = 40Q \]

While the cost function of the entrant is given by

\[ TC(Q) = 40q + 100 \] (100 is a sunk cost paid upon entering the market)

a) If the entrant observes the incumbent producing \( Q \) units of output and expects this level to be maintained, what is the equation for the entrant’s residual demand curve?
\[ P = (100 - Q) - q \]

b) If the entrant maximizes profits using the residual demand in (a), what output will the entrant produce?

Total revenues equals price times quantity

\[ TR = Pq \]
\[ Pq = (100 - Q)q - q^2 \]

Marginal revenue is the derivative with respect to q.

\[ MR = (100 - Q) - 2q \]

Set marginal revenue equal to marginal cost \((= 40)\) and solve for \(q\)

\[ (100 - Q) - 2q = 40 \]

\[ q = \frac{60 - Q}{2} = 30 - \frac{Q}{2} \]

c) How much would the incumbent have to produce to keep the entrant out of the market? At what price will the incumbent sell this output?

Profits can be written as follows:

\[ \pi = (P - c)q - 100 = 0 \]
\[ \pi = (100 - Q - q - 40)q - 100 = 0 \]
\[ \pi = \left( 30 - \frac{Q}{2} \right)q - 100 = 0 \]
\[ \pi = \left( 30 - \frac{Q}{2} \right)^2 - 100 = 0 \]

Solving for \(Q\) we get \(Q = 40, q = 10, P = 50\)