

# Consumer Utility Maximization

Recall that the consumer problem can be written in the following form:

$$\begin{array}{l} \underset{x \geq 0, y \geq 0}{\text{Max}} \quad U(x, y) \\ \text{subject to} \\ p_x x + p_y y \leq I \end{array}$$

The diagram illustrates the consumer problem. The utility function  $U(x, y)$  is circled and labeled "Utility Function". The income constraint  $p_x x + p_y y \leq I$  is also circled and labeled "Income Constraint".

That is, the consumer takes prices, income and preferences and maximizes utility through the choice of the two goods (x and y). The resulting choices can be written as demand curves

$$\begin{array}{l} x = x(p_x, p_y, I) \\ y = y(p_x, p_y, I) \end{array}$$

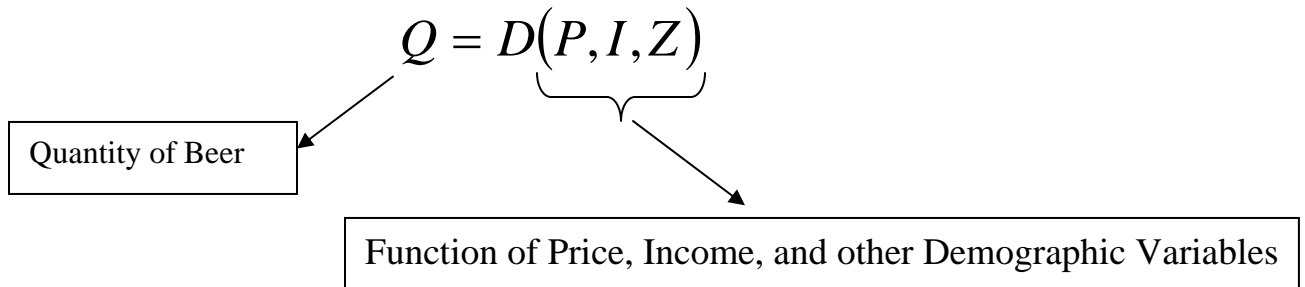
That is, demand for X (and Y) is a function of prices and income.

**PROBLEM:** The above demand curves are based on the assumption that x and y are chosen continuously (i.e. the consumer can select ANY value that satisfies the income constraint)

When the product in question is only available in discrete quantities, the demand curve is not particularly useful....we need to look at the underlying preferences directly!!

# Application: Miller Beer

Miller beer is interested in setting the price for their products to maximize revenues. For simplicity, let's assume that they only have three products: six packs, twelve packs, and cases. At first pass, we could say that all Miller needs to do is estimate a demand curve: That is,



## **Problem: What are your units???**

Suppose that we treat six packs, twelve packs and cases as three different products. We could estimate three demand curves:

$$Q_6 = D(P_6, I, Z)$$
$$Q_{12} = D(P_{12}, I, Z)$$
$$Q_{24} = D(P_{24}, I, Z)$$

That is, the demand for each good as a function of each good's price, income and demographics.

## **Problem: Shouldn't the price of a six pack affect demand for a twelve pack???**

We would need to add all the *cross-prices* (prices of other goods) to the estimated demand curves.

$$Q_6 = D(P_6, P_{12}, P_{24}, I, Z)$$

$$Q_{12} = D(P_6, P_{12}, P_{24}, I, Z)$$

$$Q_{24} = D(P_6, P_{12}, P_{24}, I, Z)$$

**Problem: In any estimation, the independent variables need to be uncorrelated with each other. The positive correlation between the various beer prices will skew the results!!**

We can avoid this problem by estimating demand in ounces as a function of price per ounce

$$Q = D(P, I, Z)$$

However, because we don't sell beer by the ounce, the demand curve itself isn't much use. However, if we could "back out" the underlying preferences, they would be extremely useful. Consider the following example (the prices are, admittedly, not realistic, but are used to keep the math simple).

Suppose that preferences have been identified to be (what a coincidence!). Think of  $x$  as ounces of beer and  $y$  as some other good.

$$U(x, y) = x^{\frac{1}{4}} y^{\frac{3}{4}}$$

We already know what the associated demand curves are

$$x = \frac{I}{4P_x}$$

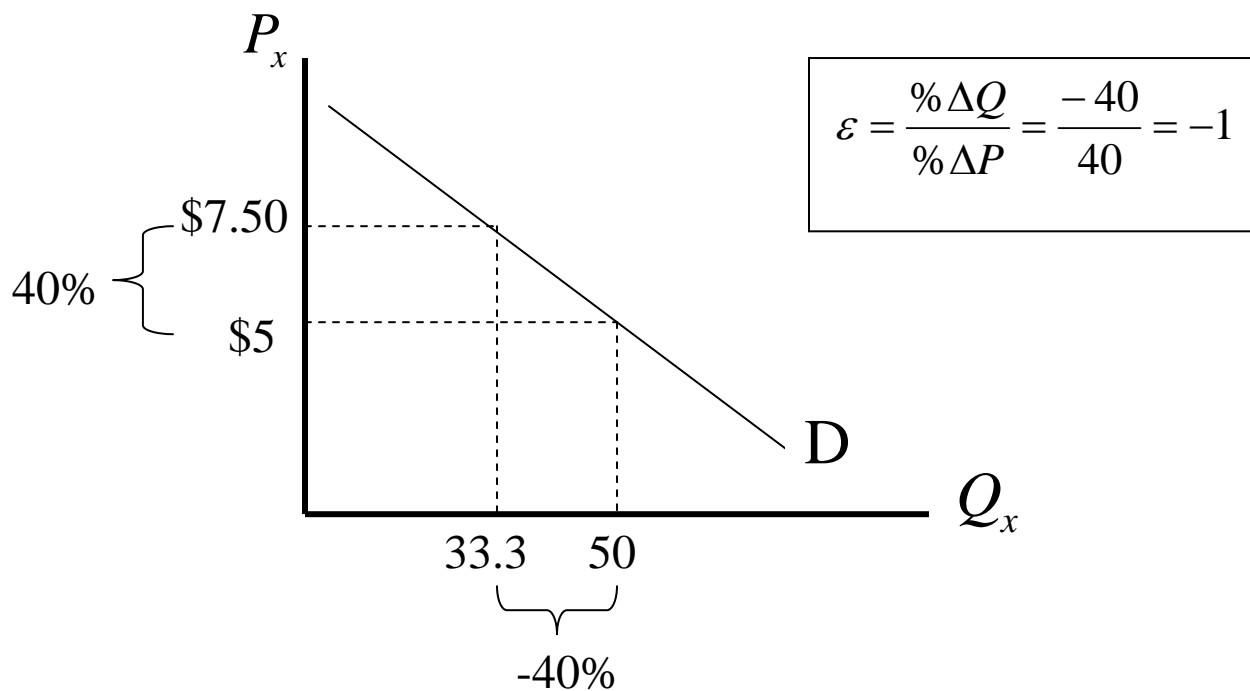
$$y = \frac{3I}{4P_y}$$

So, if income was equal to \$1,000, the price of x was \$5 and the price of y was \$10, this consumer would choose

$$x = \frac{1,000}{4(5)} = 50$$

$$y = \frac{3(1,000)}{4(10)} = 75$$

That is, we have one point on the demand curve for x.



Note that if we changed the price from \$5 to \$7.50, sales drop from 50 to 33.3 and revenues (price times quantity) don't change. This is a consequence of unit elastic demands. However, we are not pricing beer by the ounce, we are pricing particular packages of beer. Consider the following price structure:

- X = 25 , P = \$150 (\$6 per unit of X)
- X = 50, P = \$250 (\$5 per unit of X)
- X = 100, P = \$400 (\$4 per unit of X)

Which package will the consumer choose?

We are still solving a constrained optimization. However, now there are only three available choices ( $X = 25$ ,  $X = 50$ , or  $X = 100$ ). To get the income constraint, we need to find the affordable choice for  $Y$  given the purchases of  $X$ .

$$\text{Max}_{x=25,50,100} x^{\frac{1}{4}} y^{\frac{3}{4}}$$

*subject to*

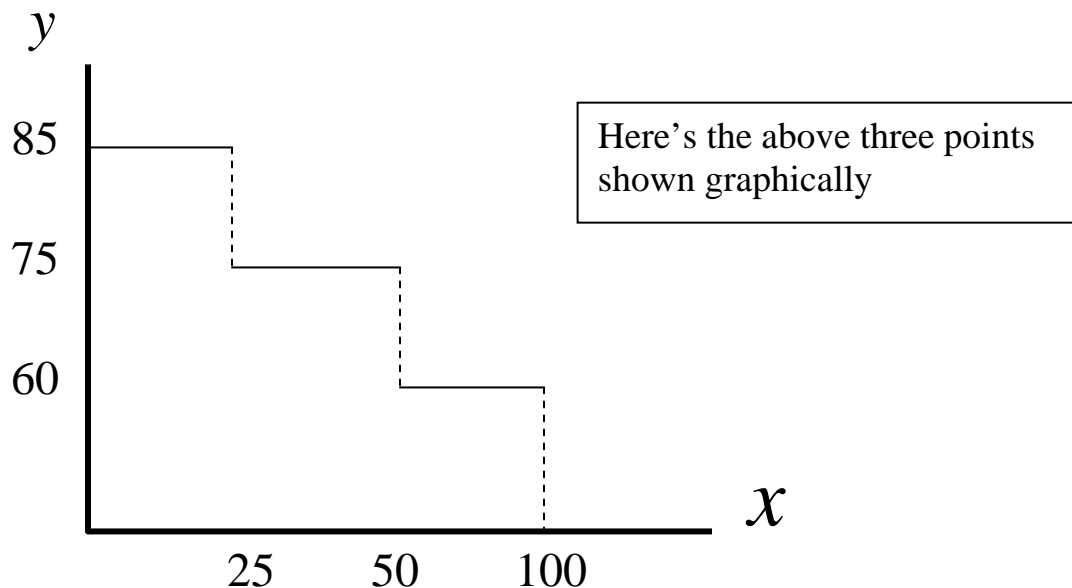
$$y = \frac{I - p_x x}{p_y}$$

Using  $I = \$1,000$  and the price of  $Y$  is  $\$10$

$$x = 25 \rightarrow p_x x = 150 \rightarrow 1000 - p_x x = 850 \rightarrow y = 85$$

$$x = 50 \rightarrow p_x x = 250 \rightarrow 1000 - p_x x = 750 \rightarrow y = 75$$

$$x = 100 \rightarrow p_x x = 400 \rightarrow 1000 - p_x x = 600 \rightarrow y = 60$$



We now only need to calculate the utility associated with the three points above to find the chosen bundle:

$$x = 25, y = 85 \Rightarrow (25)^{\frac{1}{4}} (85)^{\frac{3}{4}} = 62$$

$$x = 50, y = 75 \Rightarrow (50)^{\frac{1}{4}} (75)^{\frac{3}{4}} = 67$$

$$x = 100, y = 60 \Rightarrow (100)^{\frac{1}{4}} (60)^{\frac{3}{4}} = 68$$

At the current set of prices, this consumer will choose  $X = 100$  ( $Y = 60$ ) because it is associated with the highest value of utility! Further, note that with this packaging, beer revenues increase from \$250 (per ounce pricing) to \$400 (package pricing)