Finance 30210
Solutions to Midterm #2 Practice Questions

1) Suppose that we have the following observations of consumer behavior:

   There are two products available to purchase: Cheeseburgers and Milkshakes

   Observation #1: When the price of cheeseburgers was $4 and the price of milkshakes was $2, an individual purchased 5 cheeseburgers and 4 milkshakes.

   Observation #2: When the price of cheeseburgers was $3 and the price of milkshakes was $3, an individual purchased 4 cheeseburgers and 5 milkshakes.

   Are these two observations consistent with a rational choice?

   What we are looking for here is something that indicates the personal preferences of this consumer. Specifically, we know the following:

   If a consumer chooses option A over option B when A is more expensive than B, then the consumer is revealing that he/she strictly prefers option A to option B.

   Take the first observation: With the prices of cheeseburgers and milkshakes given by $4 and $2 respectively, the cost of 5 cheeseburgers and 4 milkshakes is $4(5) + $2(4) = $28. Meanwhile, the cost of the alternative choice (4 cheeseburgers and 5 milkshakes) is $4(4) + $2(5) = $26. Therefore, we can definitely say that the combination (5 CB, 4 MK) is preferred to (4 CB, 5MK).

   Now, take the second observation: With the prices of cheeseburgers and milkshakes given by $3 and $3 respectively, the cost of 5 cheeseburgers and 4 milkshakes is $3(5) + $3(4) = $27 while, the cost of the alternative choice (4 cheeseburgers and 5 milkshakes) is $3(4) + $3(5) = $27. Therefore, we can definitely say that the combination (4 CB, 5 MK) is preferred to (5 CB, 4MK).

   Observation #1: (5 CB, 4 MK) is preferred to (4 CB, 5MK)
   Observation #2: (4 CB, 5 MK) is preferred to (5 CB, 4MK)

   No. This is not consistent with rational behavior!

2) Suppose that the price of good X is $6 and the price of good Y is $2. You have $140 to spend and your preferences over X and Y are defined as

$$U(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}}$$

a) Calculate the marginal utility of X (remember, this is the change in utility resulting from a sight increase in consumption of X)
b) Calculate the marginal utility of Y.
c) Calculate the marginal rate of substitution. What does the marginal rate of substitution measure?
d) Suppose that you chose to consume 10 units of X, 40 units of Y. Is this an optimal choice at the current prices? Explain.
e) If the answer to (d) is no, calculate your optimal choice of X and Y.

The marginal utility of X is the derivative with respect to X

$$U_x(x, y) = \frac{3}{4} x^{-\frac{1}{4}} y^{\frac{1}{4}}$$

The marginal utility of Y is the derivative with respect to Y

$$U_y(x, y) = \frac{1}{4} x^{\frac{3}{4}} y^{-\frac{3}{4}}$$

The MRS is the ratio of the above two expressions

$$MRS = \frac{U_x(x, y)}{U_y(x, y)} = \frac{\frac{3}{4} x^{-\frac{1}{4}} y^{\frac{1}{4}}}{\frac{1}{4} x^{\frac{3}{4}} y^{-\frac{3}{4}}} = 3\left(\frac{y}{x}\right)$$

The marginal rate of substitution measures the rate at witch the consumer is willingh to trade Y for X.

The rule for maximizing utility is

$$MRS = \frac{U_x(x, y)}{U_y(x, y)} = \frac{P_x}{P_y}$$

Here, the ratio of the price of X to the price of Y is 3, but when X = 10, Y = 40

$$MRS = 3\left(\frac{y}{x}\right) = 3\left(\frac{40}{10}\right) = 12 \neq 3$$

This choice can't be optimal!

I am going to answer this the long way….feel free to use shortcuts!

We have the utility function

$$U(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}}$$

Recall the general method for maximization problems:

$$\max_{x,y} \; f(x, y)$$
$$\textit{subject to} \quad g(\mathrm{x}, \mathrm{y}) \geq 0$$

Where $f(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}}$ and $g(x, y) = 140 - 6x - 2y$. Therefore, we can set up the lagrangian as

$$\ell(x, y) = x^{\frac{3}{4}} y^{\frac{1}{4}} + \lambda(140 - 6x - 2y)$$

Taking the first order conditions

$$\ell_x(x, y) = \frac{3}{4} x^{-\frac{1}{4}} y4^{\frac{1}{4}} - 6\lambda = 0$$
$$\ell_y(x, y) = \frac{1}{4} x^{\frac{3}{4}} y^{-\frac{3}{4}} - 2\lambda = 0$$

Solving for lambda…

$$\lambda = \frac{3x^{-\frac{1}{4}} y^{\frac{1}{4}}}{24} = \frac{x^{-\frac{1}{4}} y^{\frac{1}{4}}}{8}$$
$$\lambda = \frac{x^{\frac{3}{4}} y^{-\frac{3}{4}}}{8}$$

Setting the above two equal to each other

$$\frac{x^{-\frac{1}{4}}y^{\frac{1}{4}}}{8} = \frac{x^{\frac{3}{4}}y^{-\frac{3}{4}}}{8}$$

If we simplify this down a bit:

$$\frac{x^{-\frac{1}{4}}y^{\frac{1}{4}}}{8} = \frac{x^{\frac{3}{4}}y^{-\frac{3}{4}}}{8} \quad \text{(Multiply both sides by 8)}$$

$$x^{-\frac{1}{4}}y^{\frac{1}{4}} = x^{\frac{3}{4}}y^{-\frac{3}{4}} \qquad \text{(Divide both sides by } x^{-\frac{1}{4}}\text{)}$$

$$y^{\frac{1}{4}} = xy^{-\frac{3}{4}} \qquad \text{(Divide both sides by } y^{-\frac{3}{4}}\text{)}$$

$$y = x$$

Now use the constraint to solve the rest of the problem.

$$\left.\begin{array}{r} y = x \\ \\ 6x + 2y = 140 \end{array}\right\} \quad 6(x) + 2x = 140 \Rightarrow x = y = 17.5$$

Y = 17.5 , X = 17.5

Note that all income is spent. ($6)(17. 5) + ($2)(17.5) = $140. Further, note that you are spending (3/4) of your income on X, (1/4) of your income on Y.

3) Microeconomics focuses on two primary players: consumers and firms. Both of these players are solving optimization problems.

   a) Briefly, explain the problem that each economic players faces.

      Consumers are endowed with a set of preferences. They have a limited amount of income to spend on various goods and services. They face a set of market prices and must select a choice of consumption to maximize their "well being". The result of this process is a demand curve.

      Firms actually have a couple optimization problems: First, they face a production decision. They are endowed with a technology to convert inputs into output. The face a set of input prices (these are determined in factor markets). They must choose a production process that will generate enough output to match their expected sales and minimize the costs of production.

The second stage is deciding the price to charge. Taking costs as given, the firm must select a price to maximize profits given the properties of the demand they face.

b) Briefly describe the logic behind the solution process.

Both the consumer problem and the producer are solved in virtually identical ways.

Step #1: Given a set of prices, select the proper mix of goods:

$$MRS = \frac{U_x(x,y)}{U_y(x,y)} = \frac{P_x}{P_y} \quad \text{(consumer)}$$

$$TRS = \frac{F_k(k,l)}{F_l(k,l)} = \frac{P_k}{P_l} \quad \text{(firm)}$$

Step #2: Choose an appropriate scale to satisfy your constraint:

$$p_x x + p_y y = I \quad \text{(Consumer)}$$

$$y = k^{\frac{1}{2}} l^{\frac{1}{2}} \quad \text{(Firm)}$$

c) In what ways are these problems similar? Are there any important differences?

The only real notable difference is that the firms decisions are restricted in the short run (capital can't be adjusted). Therefore, while the consumer is always acting efficiently, the firm isn't.

4) Suppose that you have estimated the following demand curve:

$$Q = 125 - 4.5P + .01I$$

Where I represents income and P is price.

a) Suppose that average income is equal to $25,000. Calculate the price elasticity of demand at P=$65. If you were a revenue maximizing firm, would it be optimal to charge a price of $65?

At a price of $65, demand is equal to

$$Q = 125 - 4.5(65) + .01(25{,}000) = 82.5$$

$$\varepsilon = \frac{dQ}{dP}\frac{P}{Q} = 4.5\frac{65}{82.5} = 3.54$$

Demand is very elastic at this price (specifically, bigger than 1). Therefore, this firm should consider lowering its price to attract more sales.

b) Suppose that this market is supplied by perfectly competitive firms with a constant marginal cost of $30 and no fixed costs. Calculate total market sales. Calculate consumer surplus. What would firm profits be?

A perfectly competitive market has no market power and, hence, does not charge a markup above cost (P = MC). At a market price of $30, we have total sales equal to

$$Q = 125 - 4.5(30) + .01(25{,}000) = 240$$

CS = ($83-$30)(1/2)(240) = $6,360

Note that total sales revenue equal $30(240) = $7200, but all of this goes to cover the firms' costs – profits are zero.

c) Now, suppose that, instead, this market was serviced by a monopolist with constant marginal costs equal to 30 and no fixed costs. Repeat part (b).

To solve this, first calculate inverse demand (price as a function of quantity)

$$Q = 375 - 4.5P \Rightarrow P = 83.3 - .22Q$$

Total Revenues for the firm are

$$TR = PQ = (83.3 - .22Q)Q = 83.3Q - .22Q^2$$

Marginal revenues are the derivative of total revenues:

$$MR = 83.3 - .44Q$$

Set Marginal revenue equal to marginal cost and solve for Q

$$83.3 - .44Q = 30 \Rightarrow Q = 120$$

Plug quantity back into the (inverse) demand curve to get price:

$$P = 83.3 - .22(120) = 56.9$$

The firm sets a price of $56.9 and sells 120.

Notice: if we calculate the elasticity of demand at P = $56.9, we get

$$\varepsilon = \frac{dQ}{dP}\frac{P}{Q} = 4.5\frac{56.9}{120} = 2.13$$

Now, plug this into the optimal markup rule:

$$P = \frac{MC}{\left(1+\frac{1}{\varepsilon}\right)} = \frac{\$30}{\left(1-\frac{1}{2.13}\right)} = \$56.9$$

Firm profits are equal to

($56.9 - $30)(120) = $3228

Consumer surplus equals

CS = (83 – 56.9)(1/2)(120) = $1584.

5) Suppose that you operate a water park. You have the following demands for your rides. Rides have a marginal cost of $5.

$$Q = \begin{cases} 50 - P, & \text{(Adults)} \\ 30 - P, & \text{(Children)} \end{cases}$$

a) If you could set different ride prices for adults and children, what would you charge? What would you charge if you were required to charge everybody the same ride price?

First we need to aggregate the two demand curves

$$Q = \begin{cases} 50 - P, & Q \le 20 \\ 80 - 2P, & \text{Q} \ge 20 \end{cases}$$

Now, calculate thee two inverse demand pieces:

$$P = \begin{cases} 50 - Q, & Q \leq 20 \\ 40 - .5Q, & Q \geq 20 \end{cases}$$

Now, calculate TR for each piece

$$TR = PQ = \begin{cases} 50Q - Q^2, & Q \leq 20 \\ 40Q - .5Q^2, & Q \geq 20 \end{cases}$$

Now, calculate marginal revenue for each piece:

$$MR = \begin{cases} 50 - 2Q, & Q \leq 20 \\ 40 - Q, & Q \geq 20 \end{cases}$$

Lastly, set marginal revenue equal to marginal cost. Note that with MC = $5, you will always sell at least 20 tickets, so the only piece of the demand curve that matters is that with Q>20.

$$5 = 40 - Q$$
$$Q = 35$$
$$P = \$22.50$$

Profits = $22.50(35) - $5 (35) = $612.50

However, if we could charge different prices, we can use each demand curve individually.

For Adults:

$$Q = 50 - P$$
$$P = 50 - Q$$
$$TR = 50Q - Q^2$$
$$MR = 50 - 2Q = 5$$
$$Q = 22.5$$
$$P = \$27.50$$

For Kids:

$$Q = 30 - P$$
$$P = 30 - Q$$
$$TR = 30Q - Q^2$$
$$MR = 30 - 2Q = 5$$
$$Q = 12.5$$
$$P = \$17.50$$

Profits = \$27.50 (22.5) + \$17.50(12.5) - \$5 (35) = \$662.50

b) Suppose you could engage in *two part pricing* (i.e a price per ride plus an entry fee. What would you charge for adults and children?

First, recognize that willingness to pay is increasing in the amount that each consumes. Therefore, we want to raise sales as much as possible. This is done by setting a price equal to MC (\$5). AT that price, we have the following:

Adults:

Q = 45

CS = (1/2)(45)(45) = \$1012.50

Children:

Q = 25

CS = (1/2)(25)(25) = \$312.50

Therefore, we could charge children \$437.50 for 25 rides (\$312.50 for admission plus \$125 for the rides) and adults \$1237.50 (\$1012.50 for admission plus \$225 for the rides)

c) Now, suppose that you set *menu prices* (that is, you sell books of tickets – 1 ticket per ride). What ticket packages would you sell?

We only need to worry about adults buying the children's package of 25 rides. If an adult buys 25 rides, he gets

\$312.50 + \$625 = \$937.50 = Total willingness to pay.

Subtracting the \$437.50 package price gives us \$500 of surplus. Therefore, the 45 ride package should cost \$1237.50 - \$500 = \$737.50.

6) What is bundling? Give an example, of how bundling can increase a firms profits. What characteristics of market demand make bundling desirable?

Bundling allows you to take advantage of markets where there might be a large variance in consumer demand across products, but little variance over bundles of the products:

See the lecture slides for a specific example.

7) Explain the concept of *spatial competition*. How can this concept be generalized to talk about product variety choices?

We can think of spatial competition is two ways:

Physical Location: Each consumer pays (in addition to your listed price) a cost of traveling to the store. By locating closer to your customer, his travel costs are lower and so you can charge a higher price.

Product Variety: Now, think of "location" as how close is your product to the consumer's "ideal" product. All the logic from above still applies.

8) Consider the following two industries:

Industry A		**Industry B**	
Firm	Market Share	Firm	Market Share
1	60	1	25
2	10	2	25
3	5	3	25
4	5	4	25

Note: Industry A has an additional 4 firms with a market share of 5% each.

a) Calculate the CR(3) for each industry (the concentration ratio of the top 3 firms)

A = 60+10+5 = 75
B = 25+25+25 = 75

b) Calculate the HHI index of the two industries.

Industry A		**Industry B**	
Market Share	MS Squared	Market Share	MS Squared
60	3600	25	625
10	100	25	625
5	25	25	625
5	25	25	625

Note: the remaining industries in A have a combined squared MS of 100

HHI (A) = 3850
HHI(B) = 2500

c) Why are the two measures different?

Industry A has more total firms, but actually has a more concentrated market that industry B (i.e firm 1 is by far the dominant firm in industry A)

d) In which industry would you expect a higher markup over cost?

Its hard to make a generalization here, but its most likely that Industry A would have the higher markeups.

9) Suppose that you have the following technology for producing output.

$$y = k^{\frac{1}{2}} l^{\frac{1}{2}}$$

The price of labor is \$7 per hour and capital costs \$150 per unit. You are currently using 64 units of capital. You need to produce 100 units of output.

a) Assuming that you cannot adjust your capital stock, how much labor will you need to match your output goal?
b) Calculate your total cost of production.
c) Calculate your average (unit) cost.
d) Calculate your expenditure shares for capital and labor (i.e. what percentage of your costs are labor costs, what percent are capital costs?)
e) Now, suppose that you are allowed to adjust your capital stock as well as labor to meet your output target. Would you choose to increase your capital stock or lower it? Explain.

We are faced with the constraint that

$$k^{\frac{1}{2}} l^{\frac{1}{2}} = 100$$

Given our current capital stock, we can figure out the amount of labor needed to meet our goal.

$$(64)^{\frac{1}{2}} l^{\frac{1}{2}} = 100 \Rightarrow l = 156.25$$

Our total costs are

$$TC = \$7L + \$150K = \$7(156) + \$150(64) = \$10{,}692$$

Average costs are total costs divided by output

$$AC = \frac{\$10{,}692}{100} = \$106.92$$

Note that capital makes up 90% of your budget while labor is only 10%. Given the Cobb Douglas form of production, we know that in the long run, expenditure shares should be 50%, 50% (this is due to the exponents being .5, .5). Therefore, we know that this firm will need to shift to a much less capital intensive technique in the long run.

To do this the long way…..

In the long run, you are free to choose both capital and labor. Therefore, we need to set up the maximization problem

$$\min_{k,l}\left\{\$7L + \$150K\right\}$$

Subject to the constraint that output is at least 100 units. ($k^{\frac{1}{2}}l^{\frac{1}{2}} \geq 1{,}00$)

Setting up the problem:

$$\ell(k,l) = \$7L + \$150K - \lambda\left(k^{\frac{1}{2}}l^{\frac{1}{2}} - 100\right)$$

Take the derivatives with respect to K and L and set them equal to zero

$$\ell_L(k,l) = \$7 - \lambda\left(\frac{1}{2}\right)k^{\frac{1}{2}}l^{-\frac{1}{2}} = 0$$

$$\ell_K(k,l) = \$150 - \lambda\left(\frac{1}{2}\right)k^{-\frac{1}{2}}l^{\frac{1}{2}} = 0$$

Now, solve each for lambda.

$$\frac{\$7}{\left(\frac{1}{2}\right)k^{\frac{1}{2}}l^{-\frac{1}{2}}} = \lambda = \frac{\$150}{\left(\frac{1}{2}\right)k^{-\frac{1}{2}}l^{\frac{1}{2}}}$$

Rearranging, we get

$$\frac{\left(\frac{1}{2}\right)k^{-\frac{1}{2}}l^{\frac{1}{2}}}{\left(\frac{1}{2}\right)k^{\frac{1}{2}}l^{-\frac{1}{2}}} = \lambda = \frac{\$150}{\$7}$$

This simplifies to

$$L = 21K$$

The cost minimizing combination of capital and labor is 25 hours of labor for every unit of capital. Now, use the output constraint

$$100 = k^{\frac{1}{2}}l^{\frac{1}{2}} = k^{\frac{1}{2}}(21k)^{\frac{1}{2}}$$

Solve for k (and l)

$$k = \frac{100}{\sqrt{21}} = 22 \qquad l = 21k = 462$$

IN the long run, your optimal scale is 22 units of capital and 462 hours of labor. Now, if we recalculate your costs

$$TC = \$7L + \$150K = \$7(462) + \$150(22) = \$6{,}534 \qquad AC = \frac{\$6{,}534}{100} = \$65.34$$