Finance 30210
Solutions to Midterm #3 Practice Questions

1) Consider the following version of the prisoners dilemma game (Player one's payoffs are in bold):

		Player Two	
		Cooperate	Cheat
Player One	Cooperate	**$15** $15	**$0** $50
	Cheat	**$50** $0	**$10** $10

a) What is each player's dominant strategy? Explain the Nash equilibrium of the game.

Start with player one:

a. If player two chooses cooperate, player one should choose cheat ($15 versus $50)
b. If player two chooses cheat, player one should also cheat ($0 versus $10).

Therefore, the optimal strategy is to always cheat (for both players) this means that (cheat, cheat) is the only nash equilibrium.

b) Why is an infinite horizon required for cooperation to occur? Explain.

If this game is played a multiple but finite number of times, then we start at the end to solve it. At the last stage, this is like a one shot game (there is no future). Therefore, on the last day, the dominant strategy is for both players to cheat. This cheating strategy at the last day unravels back to the beginning.

c) Now, suppose that this game was played an infinite number of times. For what values of the interest rate is the present value of cooperating higher than the value of cheating (so that a cooperative equilibrium could occur)

Once the possibility of an "endgame scenario" disappears, there is an incentive to cooperate.

Suppose that player 2 follows the following strategy:

"I will cooperate today. If player one cooperates, I will trust him and cooperate forever. However, if player 1 cheats, then I will never trust him again."

Now consider player 1's response:

- If I cooperate, I get $15 a year forever.
- If I cheat, I get $50 today and then $10 a year forever.

Player one will cheat if the present value of cheating is greater than the present value of cooperating.

$$\$50 + \frac{\$10}{i} > \$15 + \frac{\$15}{i}$$

Player one will cheat if the interest rate is bigger than 14%.

2) Consider the following pricing game between Dell and Gateway. There are two types of demanders in the market, High and Low.

High demanders value a computer at $4000. There are 100 of these people in the market.

Low demanders value a computer at $1000. There are 200 of these people in the market.

If Dell and Gateway set the same price, they split the market. If they set different prices, the lower price takes the entire market. Assume that the marginal cost of a computer is $500.

First, we need to calculate the profits under each scenario;

Dell Charges $4,000 / Gateway charges $4000

Dell and gateway split the market (50 customers each)

Profit = ($4000 - $500)(50) = $175K

Dell Charges $1,000 / Gateway charges $1000

Dell and gateway split the market (150 customers each)

Profit = ($1000 - $500)(150) = $75K

Dell Charges $1,000 / Gateway charges $4000 (or visa versa)

One firm gets all the sales (300 customers)

Profit = ($1000 - $500)(300) = $150K

(Gateway's payoffs are in bold):

		Dell	
		P = $4,000	P = $1,000
Gateway	P = $4,000	**$175K** $175K	**$0** $150K
	P = $1,000	**$150K** $0	**$75K** $75K

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Note that neither Dell nor Gateway has a dominant strategy. Suppose Dell charges $4,000 Gateway's best response is to swerve charge $4000 ($175 vs. $150). However, if Dell charges $1000, then Gateway should charge $1000 ($75 vs. $0).

Suppose that Dell has adopted the strategy of charging $4000 2/3 of the time and charging $1000 1/3 of the time.

We need to show that if Dell follows the strategy (2/3 ,1/3) then Gateway is indifferent between P = $4000 and P = $1000. If we calculate the expected reward to Gateway from

E(P = $4000) = (2/3)(175) + (1/3)(0) = $117

E(P = $1000) = (2/3)(150) + (1/3)(75) = $125

This can't be an equilibrium because Gateway would have a strict preference for the low price. To find the correct probabilities, we need to set them equal.

. E(P = $4000) = (Ph)(175) + (Pl)(0)
E(P = $1000) = (Ph)(150) + (Pl)(75)

(Ph)(175) = (Ph)(150) + (Pl)(75)

Remember the probabilities need to sum to one

Ph + Pl = 1

Solving for Ph and Pl, we get Ph = ¾ , Pl = 1/4 .

Both players are charging low 25% of the time.. Therefore, the probability that they both charge low is (1/4)(1/4) = 1/16 = 6.25%.

3) Suppose that the elasticity of demand for tennis shoes is 4 (and is constant). Calculate the markup that would be charged is a monopoly controlled the market. How would your answer change if the market was oligopolistic with an HHI index of 5,000.

$$LI = \frac{P - MC}{P} = \frac{\left(\frac{HHI}{10{,}000}\right)}{\varepsilon} = \frac{.5}{4} = .125$$
$$\frac{P - MC}{P} = .125$$
$$P = 1.14MC$$

You should charge a 14% markup.

On the other hand, for a monopolist

$$LI = \frac{P - MC}{P} = \frac{1}{\varepsilon} = \frac{1}{4} = .25$$
$$\frac{P - MC}{P} = .25$$
$$P = 1.33MC$$

A monopolist should charge a 33% markup.

4) Suppose that the (inverse) demand curve for bananas is given by

$$P = 400 - 5Q$$

Where Q is total industry output. The market is occupied by two firms, each with constant marginal costs equal to $5.

a) Calculate the equilibrium price and quantity assuming the two firms compete in quantities. Calculate the elasticity of demand facing each firm. How does this differ from industry elasticity?

First, rewrite the aggregate production as the sum of each firm's output.

$$P = 400 - 5(q_1 + q_2)$$

Now, lets look at the demand facing firm 1 (remember, firm one treats firm two's output as a constant

$$P = (400 - 5q_2) - 5q_1$$

Total revenues equal price times quantity.

$$Pq_1 = (400 - 5q_2)q_1 - 5q_1^2$$

Marginal revenue is the derivative with respect to quantity.

$$MR = (400 - 5q_2) - 10q_1$$

Set marginal revenue equal to marginal cost and solve for quantity. To get market price, remember there are two firms.

$$(400 - 5q_2) - 10q_1 = 5$$
$$q_1 = \frac{395 - 5q_2}{10} = 39.5 - .5q_2$$

This is firm one's best response function. Note that firm two is perfectly symmetric to firm one.

$$q_1 = 39.5 - .5q_2$$
$$q_2 = 39.5 - .5q_1$$

Substitute one into the other to get the equilibrium. Then, substitute into the demand curve to get price (remember, there are two firms)

$$q_1 = q_2 = 26.3$$
$$P = 400 - 5(52.6) = \$137$$
$$\pi_1 = \pi_2 = \$137(26.3) - \$5(26.3) = \$3471.60$$

b) Repeat parts (a) assuming the competition is in prices rather than quantities.

In Bertrand competition, identical products results in marginal cost pricing.

Therefore, price equals \$5, quantity = 79 and profits are zero.

c) Suppose that each firm was capacity constrained. That is, each firm can only produce 100 units. How does this change your answers to (b)?

(*oops…there was a typo in this question….I meant to set the capacity of each firm equal to 10, not 100!!)*

If only 20 units can be produced (10 by each firm), then

$$P = 400 - 5(20) = \$300$$

Each firm produces 10 units and sells them for $300.

5) Explain the similarities/difference between Cournot competition and Bertrand competition. What are the key assumptions/results of each?

Cournot: Simultaneous move game. Each firm has a cost function to determine marginal costs (in the baseline example, marginal costs are constant and equal across firms, but this need not be the case). The firms face a common aggregate demand curve. Each firm chooses production levels conditional on what they expect their rival's production levels to be. The nash equilibrium is the result of all firms playing their best responses.

Bertrand: Simultaneous move game. Each firm has a cost function to determine marginal costs (in the baseline example, marginal costs are constant and equal across firms, but this need not be the case). The firms face a common aggregate demand curve. Each firm chooses *price* conditional on what they expect their rival's *price* to be. The nash equilibrium is the result of all firms playing their best responses.

In the specific case of identical products you could say that Bertrand competition is the "fiercest". Two firms undercut each other until price falls to marginal cost and profits disappear. However, in the general case, cournot competition is the most aggressive. The firm with the cost advantage raises its market share while the weaker firm shrinks. In the Bertrand case, if a firm's costs increase, rivals respond by raising price and maintaining market share rather than stealing from their weaker rival.

6) Explain the following statement: "If firms are competing in quantities, then it pays to be the first to the market. However, if firms are competing in price, its worthwhile to wait for your opponent to make his move"

In cournot competition, the player who selects first has an advantage. The first mover's market share is bigger as are profits. However, in the Bertrand game, the second mover has the advantage. By seeing his opponents price, I firm can come in second and undercut it.

7) What is the chain store paradox? What is the major lesson we get from this game?

The chain store paradox deals with firms that face the treat of potential entry. On the surface, it would seem that establishing a reputation as being a tough competitor (i.e driving your opponents out of business by heavy advertising, low prices, etc) would be worthwhile if it eliminates the threat of entry. However, if there is an endgame, it is never optimal to fight entry in the "last" period. This is similar to the repeated prisoners dilemma game. The threat of fighting entry is never credible so entry always occurs. See the lecture slides for further details.

8) Suppose that the probability of getting in an accident is 3%. The average cost of an accident is $100,000. Suppose that the average car driver has preferences given by

$$U(I) = \sqrt{I}$$

a) Assuming that this individual earns $100,000 per year in income, calculate his expected utility if he buys no insurance.

$$U(\$100{,}000) = \sqrt{100{,}000} = 316$$
$$U(\$0) = 0$$
$$E(utility) = .97(316) + .03(0) = 306$$

$$\sqrt{I} = 306 \rightarrow 306^2 = I = \$94{,}000$$

He would pay up to $6,000.

b) Calculate the cost of this policy to the insurance company.

$$E(\cos t) = .97(0) + .03(100{,}000) = \$3{,}000 .$$

c) We need to worry about the cost to the insurance rising above what the safe driver is willing to pay.

$$E(\cos t) = .50(3{,}000) + .50(X) \geq \$6{,}000$$

Where X is the cost to the insurer of the unsafe driver. Solving for X, we get

X = $9,000.

That is, if the expected cost the insurer of the unsafe driver is greater than $9,000, the market breaks down.

$$E(\cos t) = (1 - p)(0) + p(100{,}000) \geq \$9{,}000$$
$$p = 9\%$$

d) Explain how moral hazard and adverse selection are dealt with in the insurance industry.

Adverse selection takes place before a contract is signed when information is unobservable by one of the parties involved. Moral hazard involves one party changing their behavior after the deal has been made.

The insurance industry looks for identifiable characteristics such as age, marital status, gender to identify unsafe drivers (signaling). The insurance industry avoids moral hazard by charging a deductible.

9) Suppose that the market demand is described by

$$P = 120 - (Q + q)$$

Where Q is the output of the incumbent firm, q is the output of the potential entrant and P is the market price. The incumbent's cost function is given by

$$TC(Q) = 60Q$$

While the cost function of the entrant is given by

$TC(Q) = 60q + 80$ (80 is a sunk cost paid upon entering the market)

a) If the entrant observes the incumbent producing $\overline{Q}$ units of output and expects this level to be maintained, what is the equation for the entrant's residual demand curve?

$$P = (120 - \overline{Q}) - q$$

b) If the entrant maximizes profits using the residual demand in (a), what output will the entrant produce?

Total revenues equals price times quantity

$$TR = Pq$$
$$Pq = (120 - \overline{Q})q - q^2$$

Marginal revenue is the derivative with respect to q.

$$MR = (120 - \overline{Q}) - 2q$$

Set marginal revenue equal to marginal cost (= 60) and solve for q

$$\left(120-\overline{Q}\right)-2q=60$$

$$q=\frac{60-\overline{Q}}{2}=30-\frac{\overline{Q}}{2}$$

c) How much would the incumbent have to produce to keep the entrant out of the market? At what price will the incumbent sell this output?

Profits can be written as follows:

$$\pi=(P-c)q-80=0$$
$$\pi=\left(120-\overline{Q}-q-60\right)q-80=0$$
$$\pi=\left(30-\frac{\overline{Q}}{2}\right)q-80=0$$
$$\pi=\left(30-\frac{\overline{Q}}{2}\right)^2-80=0$$

Solving for Q we get Q = 42, q = 9, P = 69.