Finance 30210  
Problem Set #5

1) For each of the following demand curves, calculate the price elasticity of demand and the income elasticity of demand.
   a) \( Q = 800 - 4P + 2I \)
   b) \( \ln Q = 2.5 - 0.65 \ln P + 0.95I \)
   c) \( Q = 3.6 - 2.1P^3 + 6.7 \ln I \)
   d) \( Q = 4P^{-1.6} I^{45} \)
   e) \( Q = 6e^{-0.75P+0.05I} \)

2) Consider a consumer choosing between three goods.

   \( P = (P1, P2, P3), \ X = (X1, X2, X3) \) (i.e. three prices, three products)

   Each of the following groups represents choices of \( X1, X2, \) and \( X3 \) for various prices of \( X1, X2, \) and \( X3 \). Determine which group is inconsistent with rational choice.

   For example, in Group #1, when the prices of the three goods were $1, $2, and $3 respectively, this consumer chose 3 units of the first good, 2 units of the second good and 1 unit of the third good.

   \[ \begin{align*}
   \text{Group #1:} & & \text{P} & \text{X} \\
   & (1, 2, 3) & (3, 2, 1) \\
   & (2, 1, 2) & (2, 2, 1) \\
   & (3, 5, 1) & (1, 2, 1) \\
   \end{align*} \]

   \[ \begin{align*}
   \text{Group #2:} & & \text{P} & \text{X} \\
   & (3, 4, 1) & (5, 1, 3) \\
   & (2, 3, 2) & (3, 3, 3) \\
   & (5, 3, 1) & (4, 2, 2) \\
   \end{align*} \]

   \[ \begin{align*}
   \text{Group #3:} & & \text{P} & \text{X} \\
   & (4, 3, 2) & (2, 2, 2) \\
   & (5, 3, 3) & (1, 3, 3) \\
   & (5, 2, 3) & (1, 3, 2) \\
   \end{align*} \]

3) Suppose you know that you face the following demand curve:

   \[ Q = 150 - 3P \]

   Calculate the price that maximizes revenues.
4) Suppose that you are currently charging a price of $40. You know that at your current price, income elasticity is equal to 1.5 and price elasticity equals -2.5. If you see a 20% increase in income, calculate the price change required to maintain your current sales level.

5) Consider the following utility functions:

\[
U(x, y) = ax + by \quad \text{(Linear)}
\]

\[
U(x, y) = x^\alpha y^\beta \quad \text{(Cobb-Douglas)}
\]

\[
U(x, y) = (\alpha x^\rho + (1 - \alpha) y^\rho)^{\frac{1}{\rho}} \quad \rho \leq 1 \quad \text{(Constant Elasticity of Substitution)}
\]

For each of the utility functions:

a) Calculate the marginal rate of substitution
b) Calculate the elasticity of substitution

6) Suppose that the price of good X is $4 and the price of good Y is $6. You have $100 to spend and your preferences over X and Y are defined as

\[
U(x, y) = x^\frac{2}{3} y^\frac{1}{3}
\]

Solve for your optimal choice of X and Y.