

## Finance 30210 Problem Set #3

- 1) According to a study by Niccie McKay, the average cost per patient day for nursing homes in the US is

$$C = A - .16X + .00137X^2$$

Where  $X$  is the number of patient days per year (in thousands) and  $A$  is a constant that varies by region (but independent of  $X$ ). How big should a nursing home be (in patient days per year) to minimize average costs. Show that your result minimizes rather than maximizes costs.

- 2) Consumer research for Kraft foods has discovered the following relationships between sales of TANG (Yes, the orange drink used by astronauts!) and advertising expenditures for two districts. Sales ( $S$ ) and advertising ( $A$ ) are in millions.

$$S_1 = 10 + 5A_1 - 1.5A_1^2$$

$$S_2 = 12 + 4A_2 - .5A_2^2$$

Solve Kraft's maximization problem; maximize total sales across the two districts subject to a total advertising budget of \$5M. How would a \$1M increase in Kraft's advertising budget influence sales?

- 3) Stafford rug company produces wool rugs and cotton rugs. Total cost (in dollars) is given by

$$C = 7X_1^2 + 9X_2^2 - 1.5X_1X_2$$

Where  $X_1$  represents the number of wool rugs per day and  $X_2$  represents the number of cotton rugs per day. Stafford has commitments to deliver 10 rugs per day, but can deliver any combination of wool or cotton. Calculate the cost minimizing combination of rugs. What would the impact be on Stafford's costs if their orders increased by 50%?

- 4) Consider a consumer choosing between three goods.

$$P = (P_1, P_2, P_3), X = (X_1, X_2, X_3) \text{ (i.e. three prices, three products)}$$

Each of the following groups represents choices of  $X_1$ ,  $X_2$ , and  $X_3$  for various prices of  $X_1$ ,  $X_2$ , and  $X_3$ . Determine which group is inconsistent with rational choice.

For example, in Group #1, when the prices of the three goods were \$1, \$2, and \$3 respectively, this consumer chose 3 units of the first good, 2 units of the second good and 1 unit of the third good.

$$\begin{aligned} \text{Group \#1: } P &= (1, 2, 3), X = (3, 2, 1) \\ P &= (2, 1, 2), X = (2, 2, 1) \\ P &= (3, 5, 1), X = (1, 2, 1) \end{aligned}$$

$$\begin{aligned} \text{Group \#2: } P &= (3, 4, 1), X = (5, 1, 3) \\ P &= (2, 3, 2), X = (3, 3, 3) \\ P &= (5, 3, 1), X = (4, 2, 2) \end{aligned}$$

$$\begin{aligned} \text{Group \#3: } P &= (4, 3, 2), X = (2, 2, 2) \\ P &= (5, 3, 3), X = (1, 3, 3) \\ P &= (5, 2, 3), X = (1, 3, 2) \end{aligned}$$

- 5) Suppose that the price of good X is \$4 and the price of good Y is \$6. You have \$100 to spend and your preferences over X and Y are defined as

$$U(x, y) = x^{\frac{2}{3}} y^{\frac{1}{3}}$$

Solve for your optimal choice of X and Y.